Int. J. Open Problems Compt. Math., Vol. 2, No. 4, December 2009
ISSN 1998-6262; Copyright ©ICSRS Publication, 2009
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A Note on the Paper "Charactrerizatios on 2-Isometries"

Hossein Abbassi[1] and Kourosh Nourouzi[2]

[1,2] Department of Mathematics, K. N. Toosi University of Technology,
P.O. Box 16315-1618, Tehran, IRAN
e-mail: nourouzi@kntu.ac.ir

Abstract

In the main theorem of the paper "Characterizations on 2-isometries, J. Math. Anal. Appl., Vol. 340, (2008), pp. 621–628", the authors assume an unnecessary property which is shown in this note it is automatically valid in every 2-normed space.

Keywords: *Linear 2-normed space, 2-isometry*

Let X be a real linear space with $\dim X > 1$. A function $\|\cdot,\cdot\| : X \times X \to \mathcal{R}$ is said to be a 2-norm on X if for every $\alpha \in \mathcal{R}$ and $x, y, z \in X$:
(a) $\|x, y\| = 0 \Leftrightarrow x$ and y are linearly dependent,
(b) $\|x, y\| = \|y, x\|$,
(c) $\|\alpha x, y\| = |\alpha| \|x, y\|$,
(d) $\|x, y + z\| \leq \|x, y\| + \|x, z\|$.
Then $(X, \|\cdot,\cdot\|)$ is called a linear 2-normed space [2].

In [1], the authors gave some characterizations on 2-isometries defined between 2-normed spaces. In Theorem 2.7 which is the main theorem of [1], to show that the given function is a 2-isometry, among other hypotheses the authors also assume the following property:

- *for every positive real number α and x, y, z belong to linear 2-normed space X with $y \notin \{x, z\}$, there exists $w \in X$ such that*

$$\| x - w, y - w \| = \alpha = \| y - w, z - w \| .$$

This property is valid in no linear 2-normed space. Since for a given linear 2-normed space X, it suffices to choose $y = \emptyset$, $x \neq y$ and $z = 2x$. Then

$$\| y - w, z - w \| = \| z - y, w - y \| = 2 \| x - w, y - w \|,$$

for all $w \in X$. Nevertheless, the mentioned property is valid in every linear 2-normed space X if $y \notin \{x, z\}$ is replaced by $\| x - z, y - z \| \neq 0$. To see this, let $\alpha \in \mathcal{R}^+$ be given. Putting

$$\alpha_1 := \alpha \| x - z, y - z \|^{-1}, \quad w_1 := z + \alpha_1 (x - z), \quad w_2 := x - \alpha_1 (x - z),$$

we obtain

$$\| y - z, w_1 - z \| = \alpha_1 \| y - z, x - z \| = \alpha, \tag{1}$$

and

$$\| y - x, w_2 - x \| = \alpha_1 \| y - x, z - x \| = \alpha. \tag{2}$$

On the other hand, by ([1]; Lemma 2.2) , the equalities (1) and (2) imply

$$\| y - z, p - z \| = \alpha, \qquad (p \in \{w_1 + \lambda(y - z) : \lambda \in \mathcal{R}\}) \tag{3}$$

and

$$\| y - x, q - x \| = \alpha, \qquad (q \in \{w_2 + \lambda(x - y) : \lambda \in \mathcal{R}\}). \tag{4}$$

If $\alpha' := 2\alpha_1 - 1$, then

$$w := w_1 - \alpha'(y - z) = x - \alpha_1(x - z) + \alpha'(x - y) = w_2 + \alpha'(x - y),$$

and therefore

$$w \in \{w_1 + \lambda(y - z) : \lambda \in \mathcal{R}\} \cap \{w_2 + \lambda(x - y) : \lambda \in \mathcal{R}\}.$$

Now from (3) and (4) it implies that

$$\| y - x, w - x \| = \| y - z, w - z \| = \alpha,$$

or equivalently

$$\| x - w, y - w \| = \| y - w, z - w \| = \alpha.$$

References

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