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An Ammensal -Enemy Species Pair With Limited and Unlimited Resources Respectively- A Numerical Approach

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Abstract

The Ammensal species (S_1), in spite of its natural resources gets adversely effected due to interaction with the enemy species (S_2). In this paper the mathematical model of Ammensalism between two species is investigated where S_1 with limited resources and S_2 with unlimited resources. This model is characterized by a pair of first order non-linear coupled differential equations. There exist only two equilibrium points and the stability analysis is carried out. The nature of variation of reversal time (t*) of dominance is established by classical Runge-Kutta method of fourth order. Some observations are traced from the relationship between the reversal time of dominance and the carrying capacity of Ammensal species.

Keywords: *Equilibrium point, Equilibrium state, Stability, Carrying capacity, Reversal time of dominance.*

1 Introduction

Ecology is a branch of science which studies the interactions among living beings in the same habitat along with their living styles. Research in theoretical ecology was initiated by Lotka [6] and by Volterra [10].Many mathematicians and ecologists with their zeal and quest followed them contributing their might to the growth of this area of knowledge as reported in the treatises of Meyer [7], Kushing [4], Paul colinvaux

[8], Kapur [3] etc. The ecological interactions can be broadly classified as Prey – predation, competition, Commensalism, Ammensalism, and Neutralism and so on. Lakshmi Narayan and Pattabhi Ramacharyulu [5] studied Prey-predator ecological models with a partial cover for the prey and alternate food for the predator. Some studies on stability analysis of competitive species were carried out by Archana Reddy, Pattabhi Ramacharyulu and Gandhi [2], Acharyulu, K.V.L.N and Pattabhi Ramacharyulu [1] investigated some results on stability of an enemy and Ammensal species pair with limited resources. N.Phani Kumar, N.Seshagiri Rao and N.Ch.Pattabhi Ramacharyulu [9] obtained some results on the stability of a host- a flourishing commensal species pair with limited resources.

The present investigation is related to an analytical study of reversal time (t*) of dominance of Ammensalism between two species. Ammensalism is an ecological relationship between two species where one species (S_1) adversely effects by other species S_2 and S_2 is not effected by S_1: S_1 may be referred as the Ammensal species while S_2 the enemy. The following are Some examples of Ammensalism:

1) penicillum (bread mold), secretes penicillin and kills bacteria. The penicillum does not get any benefit from killing the bacteria.
2) Algal blooms can lead to the death of many species of fish, however the algae do not benefit from the deaths of these individuals.

The Ammensal species (S_1), in spite of its natural resources, declines in its strength from the enemy species (S_2) which is not effected by S_1 .This model is characterized by a coupled pair of first order non-linear differential equations. The two equilibrium points are obtained. The linearised perturbed equations are solved and the trajectories are derived. The variation of reversal time (t*) of dominance is obtained by Runge-Kutta method of fourth order (RK method of fourth order). Some conclusions are identified by the relation between the reversal time of dominance and the carrying capacity of Ammensal species.

Notation adopted:

N_1, N_2 : The populations of the Ammensal (S_1) and enemy (S_2) species respectively at time t.
a_1, a_2 : The natural growth rates of S1 and S_2 .
a_{11} : The self inhibition coefficients of N_1 .
a_{12} : The Ammensal coefficient.
k_{11} (= a_1/ a_{11}); the carrying of capacities of N_1.
t* : The reversal time of dominance of one species over the other
$\bar{N}_i$: The equilibrium values of N_i

U_1 (t), U_2 (t): Small perturbations in N_1 and N_2 over the equilibrilium values
Further both the variables N_1 and N_2 are non-negative and the model parameters a_1, a_2, $a_{11,}$ and a_{12} are assumed to be non-negative constants.

2 The Ammensal Species (S_1) With Limited Resources and The Enemy Species(S_2) With Unlimited Resources

This is the case with $k_{1<\infty}$ and $k_{2\to\infty}$ i.e., when $a_{11} \neq 0$ and $a_{22} = 0$

A) Basic Equation for the growth rate of the Ammensal species (S_1)

$$\frac{dN_1}{dt} = a_1 N_1 - a_{11} N_1^2 - a_{12} N_1 N_2$$

B) Basic Equation for the growth rate of enemy species (S_2) (I)

$$\frac{dN_2}{dt} = a_2 N_2$$

The system has only two equilibrium states defined by $\frac{dN_1}{dt} = 0$, $\frac{dN_2}{dt} = 0$;.

The equilibrium points are obtained as

i) $\bar{N}_1 = 0$; $\bar{N}_2 = 0$ [Fully washed out state] (II)

ii) $\bar{N}_1 = \frac{a_1}{a_{11}}$; $\bar{N}_2 = 0$ [The Ammensal survives and enemy is washed out.] (III)

2.1 The Stability of the equilibrium state 1

To this end, we consider slight deviations U_1 (t) and U1 (t) over the steady state $(\bar{N}_1, \bar{N}_2)$

$$N_1 = \bar{N}_1 + U_1(t), \quad N_2 = \bar{N}_2 + U_2(t),$$

where U_1 (t) and U_2 (t) are so small that the terms other than their first terms can be neglected. Substituting in (I) and neglecting products and higher powers of $U_1$and U_2 we get

$$\frac{dU_1}{dt} = a_1 U_1 \quad ; \quad \frac{dU_2}{dt} = a_2 U_2 \qquad \text{(IV)}$$

The characteristic equation for which is $(\lambda - a_1)(\lambda - a_2) = 0$,

whose roots a_1, a_2 are both positive. Hence the steady state is **unstable** . For solving (IV), we get $U_1 = N_{10}\, e^{a_1 t}$; $U_2 = N_{20}\, e^{a_2 t}$, where N_{10}, N_{20} are initial values of U_1, U_2 respectively.

Trajectories of Perturbed Species: The trajectories (solution curves of (IV)) in the

$U_1 - U_2$ plane are given by $\left(\frac{U_1}{N_{10}}\right)^{a_2} = \left(\frac{U_2}{N_{20}}\right)^{a_1}$

2.2 Stability of the equilibrium state 2

The corresponding linearised perturbed equations are

$$\frac{dU_1}{dt} = -a_1 U_1 - \frac{a_1 a_{12}}{a_{11}} U_2 \ , \quad \frac{dU_2}{dt} = a_2 U_2 \qquad \text{(V)}$$

The characteristic equation for this system is $(\lambda + a_1)(\lambda - a_2) = 0$, the roots of which are $-a_1$, a_2 and hence the steady state is **unstable .**
For solving of (V), we get

$$U_1 = N_{10}\, e^{-a_1 t} + \frac{N_{20}\, a_1\, a_{12}}{a_{11}\,(a_1 + a_2)} \left[e^{-a_1 t} - e^{a_2 t} \right]; \quad U_2 = N_{20}\, e^{a_2 t}$$

Trajectories of Perturbed Species:
The trajectories (solution curves of (V)) in $U_1 - U_2$ plane are given by

$$x + Py = (1+P)\, y^{-\frac{a_1}{a_2}} \quad \text{where} \quad x = \frac{U_1}{N_{10}}, \quad y = \frac{U_2}{N_{20}} \quad \text{and} \quad P = \frac{a_1\, a_2\, N_{20}}{a_{11}(a_1 + a_2) N_{10}}$$

3 The Solutions Of The Model (I) Obtained By The Classical Runge-kutta Method of Fourth Order

By using single variable genetic algorithm we have tried to find all most all possible solutions in the finite interval with the classical RK method of fourth order. The interval is assumed to range over 0 to 5 for observing the nature of the model. The graphs are presented whenever necessary

CASE(I): The natural growth rate of Ammensal species is less than the natural growth rate of enemy species (i.e. $a_1 < a_2$)
we divide it in to the following three cases

Case(A): when $a_1 < a_2$ and $N_{10} > N_{20}$
Case(B): when $a_1 < a_2$ and $N_{10} = N_{20}$
Case(C): when $a_1 < a_2$ and $N_{10} < N_{20}$

Case(A): when $a_1 < a_2$ and $N_{10} > N_{20}$
i.e the initial strength of the ammensal species is greater than the initial strength of enemy species where the natural growth rate of ammensal species is less than the natural growth rate of enemy species

Table-1

S.No.	a_1	a_{11}	a_{12}	a_2	N_1	N_2	t*
1	**1.803403**	**0.045756**	**3.643864**	**2.859472**	**2.107758**	**1.664007**	**0.03100**
2	0.58562	0.045756	3.643864	1.105563	2.107758	1.664007	0.03400
3	**2.213688**	**0.45081**	**2.450809**	**2.769649**	**1.424785**	**1.074088**	**0.06900**
4	0.052589	1.812028	0.941413	3.293253	3.161425	1.402682	0.08800
5	1.830516	0.409059	2.510159	2.874402	1.424785	0.821149	0.13400
6	**1.862352**	**3.30158**	**1.773829**	**2.933943**	**1.387418**	**0.429724**	**0.2100**
7	1.156148	1.961564	2.911426	3.714524	2.316287	0.331047	0.27300
8	1.706783	4.956027	4.347512	3.970402	2.30452	0.171197	0.29100
9	1.458055	3.716833	2.227686	1.73952	3.652344	0.326983	0.38200
10	**1.460899**	**3.644115**	**2.227686**	**1.73952**	**3.652344**	**0.326983**	**0.38600**
11	1.242465	2.687266	3.017212	2.081598	0.509887	0.159696	0.41400
12	1.242465	0.687956	0.314514	3.085399	0.509887	0.159696	0.48700
13	**1.679163**	**2.276151**	**2.681616**	**1.817272**	**2.030574**	**0.031174**	**1.54600**

Some of the solution curves are illustrated from FIGURE (1) to FIGURE (4) in Table-1 as below and the conclusion is presented in Table-2.

FIGURE (1) ; S.N0-3 in TABLE-1

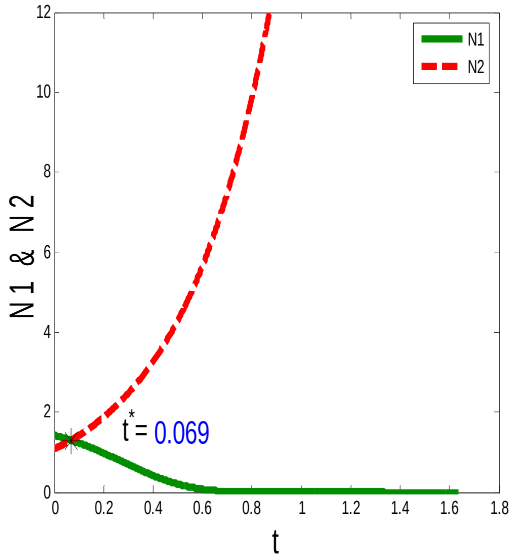


FIGURE (2) ; S.N0-6 in TABLE-1

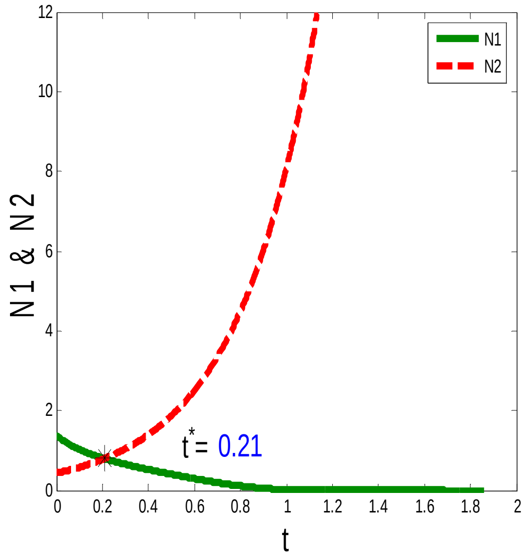


FIGURE (3) ; S.N0-10 in TABLE-1

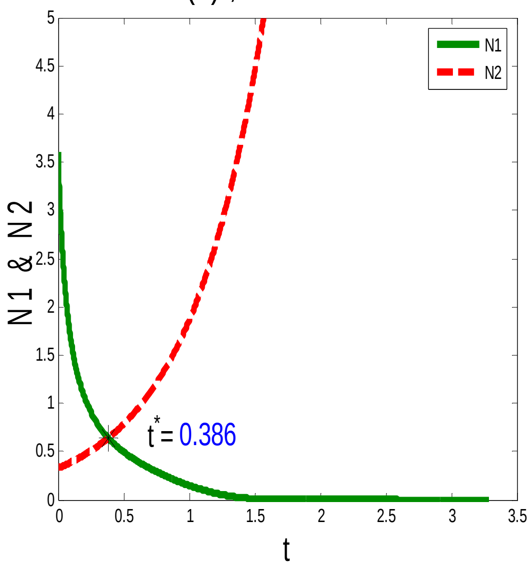
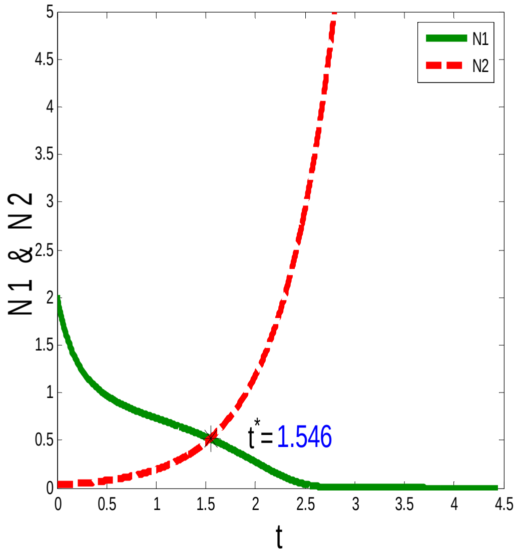


FIGURE (4) ; S.N0-13 in TABLE-1



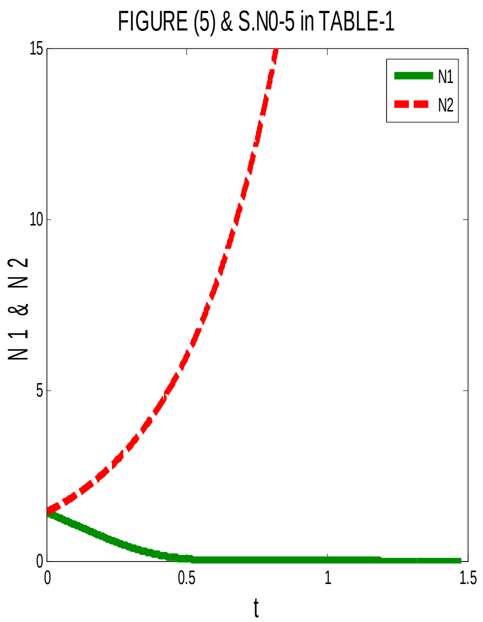
By observing above graphs, we can conclude as below

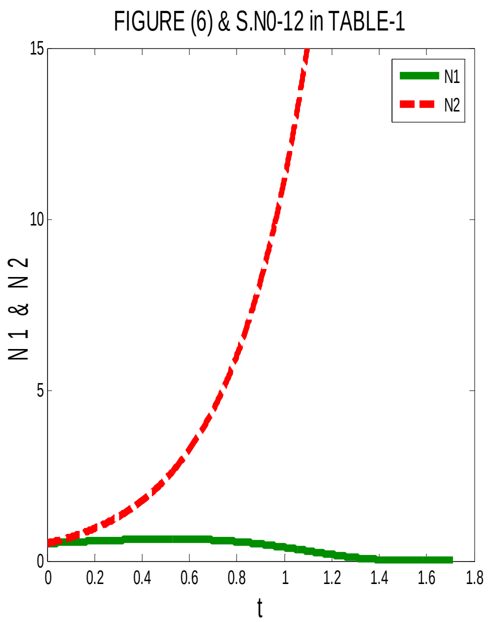
Table- 2

Criterion	Conclusion
$a_1 < a_2$ and $N_{10}>N_{20}$	The enemy(N_2) dominates over the Ammensal (N_1) in natural growth but it's initial strength less than that of Ammensal and Ammensal out numbers the enemy till the reversal time of dominance t* after that enemy out numbers the Ammensal as shown in FIGURE(1) to FIGURE(4)

From the Table-2, it is also observed that the reversal time of dominance has not occurred in cases of Case(B) and Case(C)

CASE(B): when $a_1 < a_2$ and $N_{10} = N_{20}$
For case(B) by considering $N_{10} = N_{20}$ we have obtained the solution curves which do not contain the reversal time of dominance(t*).
for example: consider S.NO- 5 and S.NO- 12 in Table-1 with the corresponding graphs from FIGURE(5) to FIGURE(6) , we can conclude as in the Table-3





Case(C): when $a_1 < a_2$ and $N_{10} < N_{20}$
For case (C) by considering $\mathbf{N_{20}} = N_{10}+1$ in Table-1, we have obtained the solution curves which do not contain the reversal time of dominance(**t***).
For example consider S.NO- 1 and S.NO-13 in Table-1.
The graphs are shown from FIGURE(7) to FIGURE(8) as shown below and the conclusions are stated in Table-3

FIGURE (7) & S.N0-1 in TABLE-1

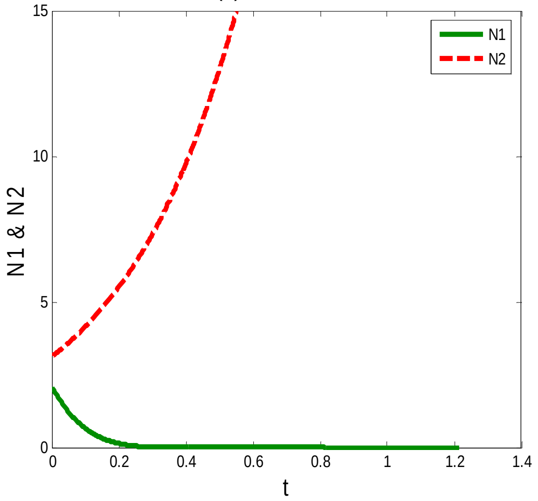
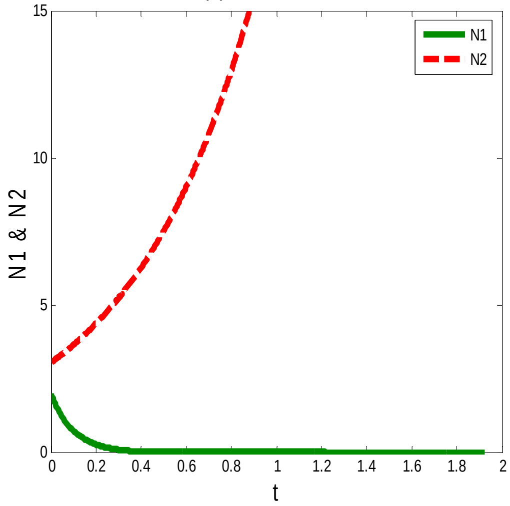


FIGURE (8) & S.N0-13 in TABLE-1



Thus in the cases of B and C, there is no interaction between the two species directly

Table-3

Criterion	Conclusion
$a_1 < a_2$ and $N_{10} < N_{20}$ (or) $N_{10} = N_{20}$	The enemy (N_2) dominates over the Ammensal (N_1) species in natural growth rate and also in its initial population as illustrated in above FIGURES

<u>CASE(II)</u>: The natural growth rate of Ammensal species is greater than the natural growth rate of enemy species (i.e. $a_1 > a_2$)

we divide it in to the following three cases

Case(D): when $a_1 > a_2$ and $N_{10} > N_{20}$
Case(E): when $a_1 > a_2$ and $N_{10} = N_{20}$
Case(F): when $a_1 > a_2$ and $N_{10} < N_{20}$

Case(D): when $a_1 > a_2$ and $N_{10} > N_{20}$
i.e the initial strength of the Ammensal species is greater than the initial strength of enemy species where the natural growth rate of ammensal species is less than the natural growth rate of enemy species

Table-4

S.NO	a_1	a_{11}	a_{12}	a_2	N_{10}	N_{20}	t*
1	4.81661	0.234751	1.483514	0.503332	3.530527	3.358309	0.03200
2	3.971597	1.961564	3.157595	3.714524	2.316287	1.681888	0.03300
3	3.559738	4.156452	4.548582	0.19005	1.894774	1.217921	0.05100
4	**3.607222**	**4.812686**	**4.100668**	**0.19005**	**1.894774**	**1.178971**	**0.05400**
5	4.852398	1.262534	0.262375	0.385846	4.728291	4.13888	0.05800
6	2.327226	0.034941	3.786544	2.161421	0.570694	0.502486	0.06700
7	2.004907	1.961564	0.366506	0.552283	2.316287	1.681888	0.10100
8	1.592739	3.30158	1.773829	1.150753	1.387418	0.429724	0.33500
9	2.940774	0.041736	4.972988	0.143147	2.966858	1.036039	0.40600
10	**1.584124**	**0.409059**	**0.411959**	**1.091212**	**1.424785**	**1.068504**	**0.4100**
11	1.183153	4.24146	3.017212	0.146953	0.314467	0.216725	0.47100
12	2.999389	3.524386	0.24246	2.401495	1.071464	0.27871	0.47200
13	1.390777	1.904476	1.776898	0.75829	1.405592	0.464581	0.50200
14	1.579102	0.485884	1.699869	1.417563	1.384773	0.438546	0.66700
15	1.557829	1.378394	1.773829	1.24957	1.387418	0.302209	0.75100
16	2.198153	0.652309	4.711523	0.19005	1.951794	0.531455	0.88900
17	**4.066173**	**1.262534**	**0.249095**	**0.24901**	**2.328866**	**2.087591**	**1.04300**
18	2.162939	0.042298	4.982126	0.29794	3.256878	0.525939	1.2900
19	1.480018	0.53523	0.371248	1.150753	1.387418	0.247338	1.77700
20	1.592739	3.30158	1.773829	1.150753	1.387418	0.047605	1.82400
21	1.252035	1.725245	2.92523	0.584081	1.675777	0.03608	3.98200
22	**1.232933**	**1.892486**	**3.017212**	**0.677991**	**0.568345**	**0.020049**	**4.20800**
23	1.247447	1.911219	2.92523	0.577784	1.675777	0.021297	4.82100

Some of the solution curves are illustrated from FIGURE(9) to FIGURE(10) as below and the conclusion is presented in Table-5

FIGURE (9) ; S.N0-4 in TABLE-4

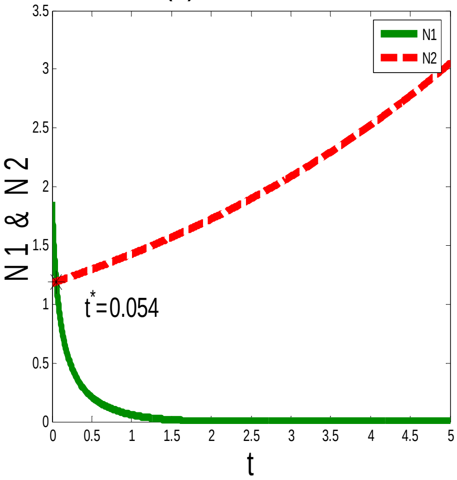


FIGURE (10) ; S.N0-10 in TABLE-4

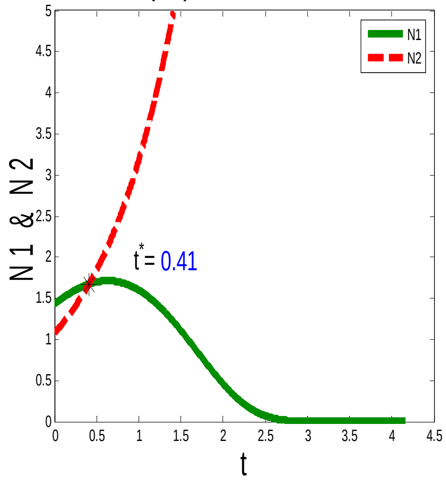


FIGURE (11) ; S.N0-17 in TABLE-4

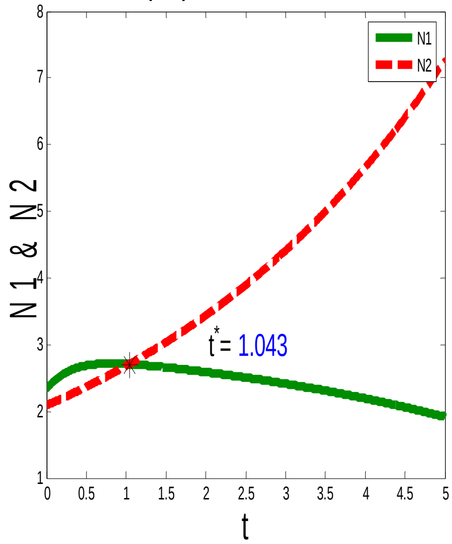


FIGURE (12) ; S.N0-22 in TABLE-4

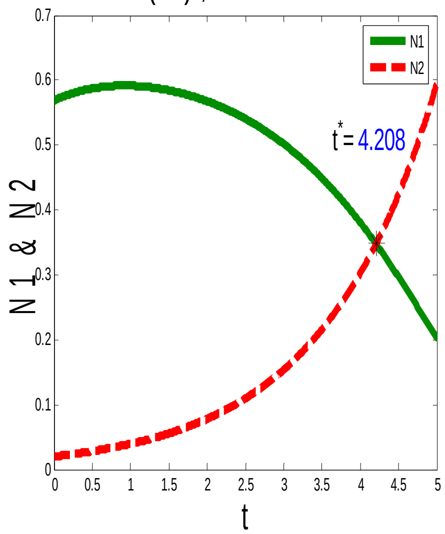


Table-5

Criterion	Conclusion
$a_1 > a_2$ and $N_{10}>N_{20}$	In general by the given condition, the Ammensal (N_1) dominates over the enemy (N_2) in natural growth rate and also it's initial strength is greater than that of an enemy. But from the observations cited in the Table-4,because of unlimited resources of enemy species, the enemy dominates over the Ammensal species .Thus the Ammensal out numbers the enemy till the reversal time of dominance t* after that enemy out numbers the Ammensal as shown from FIGURE(9) to FIGURE(12).

From the Table-2, we have also observed that the reversal time of dominance does not occur in cases of **Case(E):** when $a_1 > a_2$ and $N_{10} = N_{20}$. For case(E) by considering $N_{10} = N_{20}$ in Table 4. We have obtained the solution curves which do not contain the reversal time of dominance (t*). For example: Now consider S.NO-7 and S.NO- 16 in Table-4. The graphs are shown from FIGURE(13) to FIGURE(14) and the conclusions are represented in Table-6.

FIGURE (13) & S.N0-7 in TABLE-4

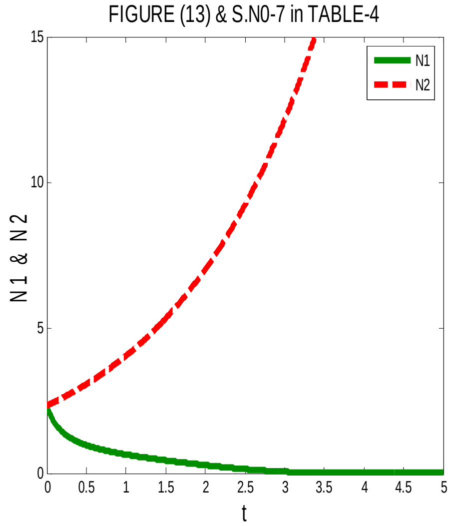
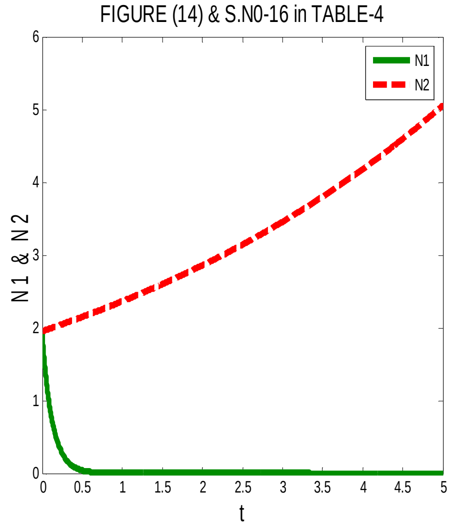


FIGURE (14) & S.N0-16 in TABLE-4



Case(F): when $a_1 > a_2$ and $N_{10} < N_{20}$

For case (F) by considering $N_{20} = N_{10} + 1$ in Table-4, we have obtained the solution curves which do not contain the reversal time of dominance(t*). For example: by considering S.NO-9 and S.NO-18 in Table-4.

The graphs are shown as in FIGURE(15) and FIGURE(16) respectively and the conclusions are given in Table-6

FIGURE (15) & S.N0-9 in TABLE-4

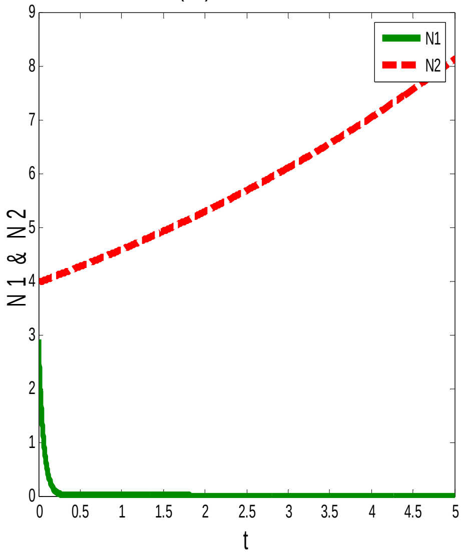


FIGURE (16) & S.N0-18 in TABLE-4

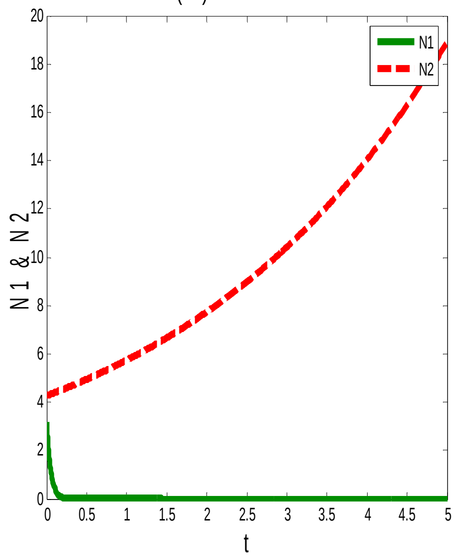


Table-6

Criterion	conclusion
$a_1 > a_2$ and $N_{10} = N_{20}$ (OR $N_{10} < N_{20}$)	Because of the unlimited resources of enemy species, the Ammensal (N_1) could not dominate over the enemy (N_2) even though its natural growth rate is greater than enemy species and its initial strength is less than or equal to that of enemy species as shown in FIGURE (15) and FIGURE (16)'

4 Effect of Reversal Time Of Dominance On The Carrying Capacity Of Ammensal Species

The interaction between these two species is observed by the relation between the carrying capacity of Ammensal species with limited resourses and reversal time of dominance in various cases with respect to the change in growth rate of the ammensal and enemy species.

We have discussed it in the following 8 cases. The graphs are presented from FIGURE (17) to FIGURE(25) and the conclusions are given in Table-15 when the reversal time of dominance increases i.e t* increases.

CASE((i): when the growth rate of Ammensal species is increasing and its natural growth rate is less than the natural growth rate of enemy species.
[i.e **a_1 increases and a_1<a_2**] **(in Table-7 and FIGURE-17)**

CASE(ii): when the growth rate of Ammensal species is decreasing and its natural growth rate is less than the natural growth rate of enemy species
[i.e **a_1 decreases and a_1< a_2**] **(in Table-8 and FIGURE-18)**

CASE(iii): when the growth rate of Ammensal species is increasing and its natural growth rate is greater than the natural growth rate of enemy species
[i.e **a_1 increases and a_1> a_2**] **(in Table-9 and FIGURE-19)**

CASE(iv): when the growth rate of Ammensal species is decreasing and its natural growth rate is greater than the natural growth rate of enemy species
[i.e **a_1 decreases and a_1> a_2**] **(in Table-10 and FIGURE-20**)

CASE(v): when the growth rate of enemy species is increasing and its natural growth rate is greater than the natural growth rate of Ammensal species
[i.e **a_2 increases and a_2> a_1**] **(in Table-11 and FIGURE-21)**

CASE(vi): when the growth rate of enemy species is decreasing and its natural growth rate is greater than the natural growth rate of Ammensal species
[i.e **a_2 decreases and a_2> a_1**] **(in Table-12 and FIGURE-22)**

CASE(vii): when the growth rate of enemy species is increasing and its natural growth rate is less than the natural growth rate of Ammensal species
[i.e **a_2 increases and a_2< a_1**] **(in Table-13and FIGURE-23)**

CASE(viii): when the growth rate of enemy species is decreasing and its natural growth rate is less than the natural growth rate of Ammensal species
[i.e **a_2 decreases and a_2< a_1**] **(in Table-14 and FIGURE-24)**

The solutions of the above cases are mentioned from Table-**7** to Table-14y and drawn the graphs as shown from FIGURE (17) to FIGURE(25) .The conclusions are specified in the Table-15

Table-7

t*	a_1	a_{11}	a_{12}	a_2	N_1	N_2	k_{11}
0.088	**0.052589**	1.812028	0.941413	3.293253	3.161425	1.402682	**0.029022**
0.273	**1.156148**	1.961564	2.911426	3.714524	2.316287	0.331047	**0.589401**
0.414	**1.242465**	2.687266	3.017212	2.081598	0.509887	0.159696	**0.462353**
0.487	**1.242465**	0.687956	0.314514	3.085399	0.509887	0.159696	**1.806024**
1.546	**1.679163**	2.276151	2.681616	1.817272	2.030574	0.031174	**0.737720**

Table-8

t*	a_1	a_{11}	a_{12}	a_2	N_1	N_2	k_{11}
0.069	**2.213688**	0.45081	2.450809	2.769649	1.424785	1.074088	**4.910468**
0.21	**1.862352**	3.30158	1.773829	2.933943	1.387418	0.429724	**0.564079**
0.291	**1.706783**	4.956027	4.347512	3.970402	2.30452	0.171197	**0.344385**
1.546	**1.679163**	2.276151	2.681616	1.817272	2.030574	0.031174	**0.73772**

Table-9

t*	a_1	a_{11}	a_{12}	a_2	N_1	N_2	k_{11}
0.058	**4.852398**	1.262534	0.262375	0.385846	4.728291	4.13888	**3.84338**
4.208	**1.232933**	1.892486	3.017212	0.677991	0.568345	0.020049	**0.651489**
4.821	**1.247447**	1.911219	2.92523	0.577784	1.675777	0.021297	**0.652697**

Table-10

t*	a_1	a_{11}	a_{12}	a_2	N_1	N_2	k_{11}
0.088	**0.052589**	1.812028	0.941413	3.293253	3.161425	1.402682	**0.029022**
1.043	**4.066173**	1.262534	0.249095	0.24901	2.328866	2.087591	**3.220644**
1.29	**2.162939**	0.042298	4.982126	0.29794	3.256878	0.525939	**51.13573**
1.824	**1.592739**	3.30158	1.773829	1.150753	1.387418	0.047605	**0.482417**
3.982	**1.252035**	1.725245	2.92523	0.584081	1.675777	0.03608	**0.725714**
4.821	**1.247447**	1.911219	2.92523	0.577784	1.675777	0.021297	**0.652697**

Table-11

t*	a_1	a_{11}	a_{12}	a_2	N_1	N_2	k_{11}
0.034	0.58562	0.04576	3.64386	**1.10556**	2.10776	1.66401	**12.79876**
0.382	1.458055	3.716833	2.227686	**1.73952**	3.652344	0.326983	**0.392284**
0.386	1.460899	3.644115	2.227686	**1.73952**	3.652344	0.326983	**0.400893**
1.546	1.679163	2.276151	2.681616	**1.817272**	2.030574	0.031174	**0.73772**

Table-12

t*	a_1	a_{11}	a_{12}	a_2	N_1	N_2	k_{11}
0.291	1.706783	4.956027	4.347512	**3.970402**	2.30452	0.171197	**0.344385**
0.487	1.242465	0.687956	0.314514	**3.085399**	0.509887	0.159696	**1.806024**
1.546	1.679163	2.276151	2.681616	**1.817272**	2.030574	0.031174	**0.73772**

Table-13

t^*	a_1	a_{11}	a_{12}	a_2	N_1	N_2	k_{11}
0.406	2.940774	0.041736	4.972988	**0.143147**	2.966858	1.036039	**70.46133**
0.471	1.183153	4.24146	3.017212	**0.146953**	0.314467	0.216725	**0.278949**
0.889	2.198153	0.652309	4.711523	**0.19005**	1.951794	0.531455	**3.369803**
1.043	4.066173	1.262534	0.249095	**0.24901**	2.328866	2.087591	**3.220644**
1.29	2.162939	0.042298	4.982126	**0.29794**	3.256878	0.525939	**51.13573**
4.821	1.247447	1.911219	2.92523	**0.577784**	1.675777	0.021297	**0.652697**

Table-14

t^*	a_1	a_{11}	a_{12}	a_2	N_1	N_2	k_{11}
0.033	**3.971597**	**1.961564**	**3.157595**	**3.714524**	**2.316287**	**1.681888**	**2.024709**
0.472	**2.999389**	**3.524386**	**0.24246**	**2.401495**	**1.071464**	**0.27871**	**0.851039**
0.667	**1.579102**	**0.485884**	**1.699869**	**1.417563**	**1.384773**	**0.438546**	**3.249957**
0.751	**1.557829**	**1.378394**	**1.773829**	**1.24957**	**1.387418**	**0.302209**	**1.130177**
1.824	**1.592739**	**3.30158**	**1.773829**	**1.150753**	**1.387418**	**0.047605**	**0.482417**
4.208	**1.232933**	**1.892486**	**3.017212**	**0.677991**	**0.568345**	**0.020049**	**0.651489**
4.821	**1.247447**	**1.911219**	**2.92523**	**0.577784**	**1.675777**	**0.021297**	**0.652697**

5 Figures

FIGURE-17; FROM THE DATA OF TABLE-7

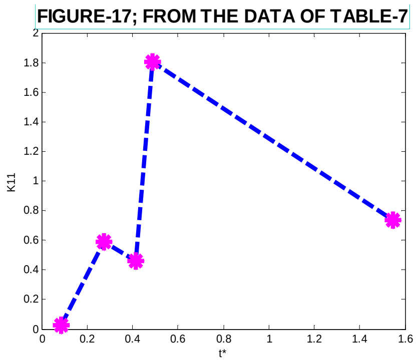


FIGURE-18; FROM THE DATA OF TABLE-8

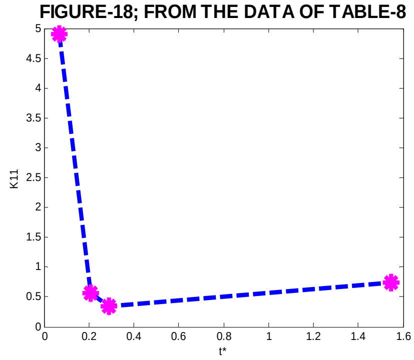


FIGURE-19 FROM THE DATA OF TABLE-9

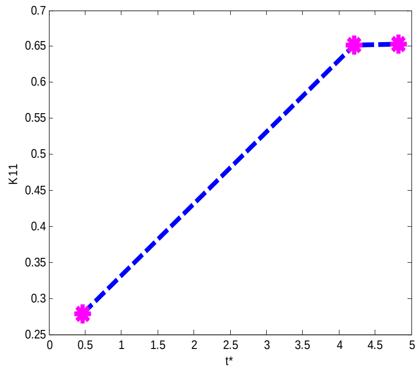
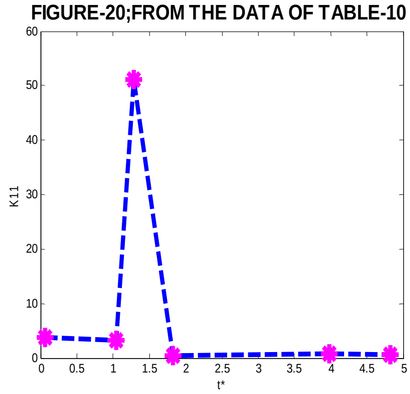
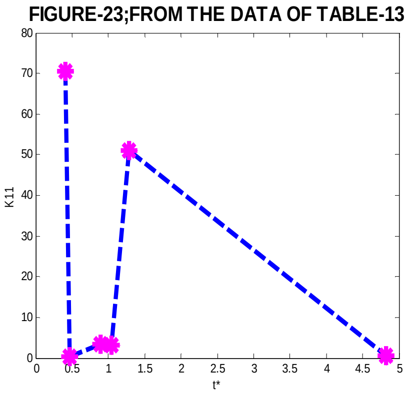
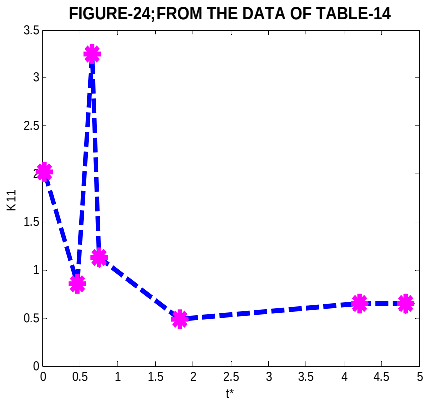
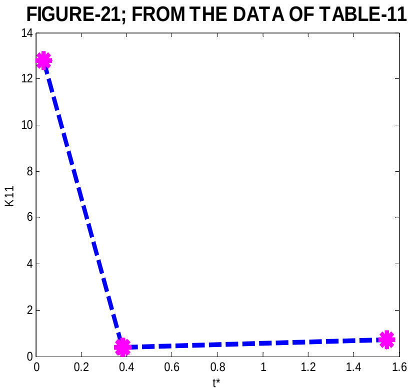


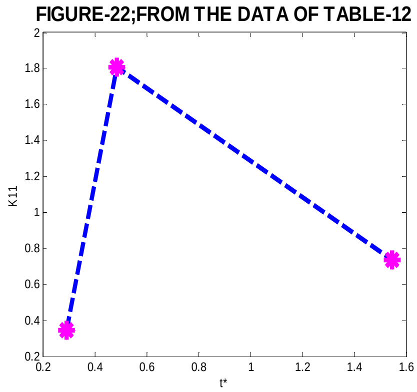
FIGURE-20;FROM THE DATA OF TABLE-10











Thus the study reveals that the carrying capacity of Ammensal species under goes several variations with the increase of t* as described in the above figures. It is only due to the interaction with the unlimited resourses of enemy species.

The above observations are summarized briefly in the following Table-15.

6 Conclusions of Above Mentioned Cases:

Table-15

S.NO.	CRITERION (when t* increases)	CONCLUSION
1	**a_1 increases and $a_1 < a_2$** (Table-7 and FIGURE-17)	k_{11} increases and decreases alternately in the specified interval .
2	**a_1 decreases and $a_1 < a_2$** (Table-8 and FIGURE-18)	k_{11} falls down steeply and then recovers slightly in the considered interval.
3	**a_1 increases and $a_1 > a_2$** (Table-9 and FIGURE-19)	k_{11} flourishes at steady pace and it has a very nominal flourishment towards the end of taken interval.
4	**a_1 decreases and $a_1 > a_2$** (Table-10and FIGURE-20)	k_{11} rises sharply and falls back for a small period of t* and it has negligible increase in the rest of the interval.
5	**a_2 increases and $a_2 > a_1$** (Table-11and FIGURE-21**)**	k_{11} declines at the beginning and then increases very marginally in the prescribed interval.
6	**a_2 decreases and $a_2 > a_1$** (Table-12 and FIGURE-22)	k_{11} soars high and then declines steadily in the observed interval
7	**a_2 increases and $a_2 < a_1$** (Table-13and FIGURE-23)	k_{11} reaches the nadir from its zenith initially. it scores high and drops down at regular pace after a small breath.
8	**a_2 decreases and $a_2 < a_1$** (Table-14 and FIGURE-24)	k_{11} oscillates at the beginning and looses strength for a while. It recovers very slightly from the middle of the interval.

7 Open Problems

i) In this paper we have investigated some results on the Reversal time of dominance in a mathematical model of two species by the classical Runge-Kutta method of fourth order. One can apply the same technique for a three species ecosystem.

ii) Instead of Runge- Kutta method of Fourth order, one can adopt picard's method to investigate the stability of the two species ecosystem with various resources.

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