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Semigroup Actions On Intuitionistic Fuzzy 2-Metric Spaces

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Abstract

The purpose of this paper is to investigate the dynamical systems in the context of topological semigroup actions in intuitionistic fuzzy 2-metric spaces.

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1 Introduction

The theory of chaotic dynamical systems is developed in [6,7,11,13]. Attanassov [1,2,3,4,5] introduced the notion of intuitionistic fuzzy sets and developed its theory. Following George and Veeramani [10], Park [15] has defined intuitionistic fuzzy metric space and obtained several classical theorems on it. The concept of intuitionistic fuzzy 2-metric space can be viewed in [14].

The purpose of this paper is to introduce the notion of semigroup actions in intuitionistic fuzzy 2-metric space as a generalization of semigroup action in intuitionistic fuzzy metric space by Yaoyao Lan and Qingguo Li [17]. We also investigate the dynamical systems in the context of semigroup actions in intuitionistic fuzzy 2-metric space and provide results on it.

2 Preliminaries

In this section we recall some useful definitions and results.

Definition 2.1 [14]. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if $*$ satisfies the following conditions:
(i) $*$ is commutative and associative
(ii) $*$ is continuous
(iii) $a * 1 = a$, for all $a \in [0,1]$
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ and $a, b, c, d \in [0,1]$.

Definition 2.2 [14]. A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is continuous t-co-norm if $\diamond$satisfies the following conditions:
(i) $\diamond$ is commutative and associative
(ii) $\diamond$ is continuous
(iii) $a \diamond 0 = a$, for all $a \in [0,1]$
(iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ and $a, b, c, d \in [0,1]$.

Definition 2.3 [8,9]. Let X be a non-empty set. A real valued function d on $X \times X \times X$ is said to be a 2-metric on X if
(i) given distinct elements x, y of X, there exists an element z of X such that $d(x, y, z) \neq 0$
(ii) $d(x, y, z) = 0$ when at least two of x, y, z are equal
(iii) $d(x, y, z) = d(x, z, y) = d(y, z, x)$ for all $x, y, z \in X$
(iv) $d(x, y, z) \leq d(x, y, w) + d(x, w, z) + d(w, y, z)$ for all x, y, z in X.
The pair (X, d) is called a 2-metric space.

Definition 2.4 [14]. An 5-tuple $(X, N, M, *, \diamond)$ is called intuitionistic fuzzy 2-metric space if X is any non-empty set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-co-norm and N, M are fuzzy sets on $X^3 \times (0, \infty)$; N denotes the degree of membership and M denotes the degree of non-membership of $(x, y, z, t) \in X^3 \times (0, \infty)$ satisfying the following conditions:
For all $x, y, z, w \in X$; $s, t, r > 0$,
(1) $N(x, y, z, t) + M(x, y, z, t) \leq 1$
(2) $N(x, y, z, t) > 0$
(3) $N(x, y, z, t) = 1$ if at least two of x, y, z are equal
(4) $N(x, y, z, t) = N(x, z, y, t) = N(y, z, x, t)$
(5) $N(x, y, w, t) * N(x, w, z, s) * N(w, y, z, r) \leq N(x, y, z, t + s + r)$
(6) $N(x, y, z, \cdot) : (0, \infty) \to (0, 1]$ is continuous
(7) $M(x, y, z, t) < 1$
(8) $M(x, y, z, t) = 0$ if at least two of x, y, z are equal
(9) $M(x, y, z, t) = M(x, z, y, t) = M(y, z, x, t)$
(10) $M(x, y, w, t) \diamond M(x, w, z, s) \diamond M(w, y, z, r) \geq M(x, y, z, t + s + r)$
(11) $M(x, y, z, \cdot) : (0, \infty) \to (0, 1]$ is continuous.

Then (N, M) is called intuitionistic fuzzy 2-metric on X denoted by $(N, M)_2$.

Example 2.5 [14]. Let (X, d) be a metric space. Denote $a * b = ab$ and $a \diamond b = \min\{1, a + b\}$ for all $a, b \in [0, 1]$ and let N_d and M_d be fuzzy sets on $X^3 \times (0, \infty)$ defined by
$N_d(x, y, z, t) = \frac{ht^n}{ht^n + md(x,y,z)}, M_d(x, y, z, t) = \frac{d(x,y,z)}{kt^n + md(x,y,z)}$
for all $h, k, n, m \in R^+$. Then $(X, N_d, M_d, *, \diamond)$ is an intuitionistic fuzzy 2-metric space.

Remark 2.6. For convenience we denote the intuitionistic fuzzy 2-metric space as X, wherever there is no risk of confusion.

Definition 2.7 [14]. Let $(X, N, M, *, \diamond)$ be an intuitionistic fuzzy 2-metric space and let $r \in (0, 1), t > 0$ and $x \in X$. The set $B(x, r, t) = \{y \in X : N(x, y, z, t) > r$ and $M(x, y, z, t) < 1 - r$ for all $z \in X\}$ is called the open ball with center x and radius r with respect to t.

Definition 2.8 [14]. Let $(X, N, M, *, \diamond)$ be an intuitionistic fuzzy 2-metric space. Then a set $U \subset X$ is said to be an open set if each of its points is the center of some open ball contained in U.

Definition 2.9 [12,16]. A topological semigroup is a semigroup with a Hausdorff topology in which multiplication is continuous in both the variables.

Definition 2.10 [17]. A dynamical system in X is a triple (S, X, π) (denoted by (S, X) in short), where S is a topological semigroup, X is at least Hausdorff and $\pi : S \times X \to X$, $(s, x) \to sx$ is a continuous action on X. Thus
$s_1(s_2x) = (s_1s_2)x$ holds for each (s_1, s_2, x) in $S \times S \times X$.

Definition 2.11 [17]. The orbit of x is the set $Sx = \{sx : s \in S\}$.

Definition 2.12 [17]. If $S = \{f^n : n = 1, 2, ...\}$ and $f : X \to X$ is continuous, then (S, X) is a classical dynamical system in X denoted by (X, f).

Definition 2.13 [17]. For $U \subset X$ and $s \in S$, $s^{-1}U = \{x \in X : sx \in U\}$.

Definition 2.14 [17]. Let S be a topological semigroup. We say that S is a
(1) F-semigroup if for every $s_0 \in S$ the subset $S \backslash Ss_0$ is finite.
(2) C-semigroup if for every $s_0 \in S$ the closure of the subset $S \backslash Ss_0$ is compact in S. (i.e., $S \backslash Ss_0$ is relatively compact).

Example 2.15 [13].
(1) Standard one-parameter semigroup $S = ([0, \infty), +)$ is a C-semigroup.
(2) Every cyclic positive semigroup $P = \{s^n : n \in \mathbb{N}\}$ is a F-semigroup.
(3) Every compact semigroup is a C-semigroup.

Definition 2.16 [17]. The dynamical system (S, X) is called:
(1) topologically transitive (in short: TT) if for every pair of non-empty

subsets U and V in X, there exists $s \in S$ such that $sU \cap V \neq \phi$.
(2) point transitive (PT) if S has a dense orbit, i.e., there is a point $x_0 \in X$ whose orbit is dense in X. Such a point is called transitive point. Notation: $x_0 \in$ Trans (X).
(3) densely point transitive (DPT) if there exists a dense set of transitive points, that is, Trans(X) is dense in X.

Remark 2.17 [17].
(1) Since $s^{-1}(sU \cap V) = U \cap s^{-1}V$, it is equivalent to saying that $U \cap s^{-1}V \neq \phi$.
(2)Always DPT implies PT. In general, TT and PT are independent properties.

Definition 2.18 [17]. X is perfect means that X is a space without isolated points.

Theorem 2.19 [17]. Let (S, X) be a dynamical system.
(1) Let X be perfect and S a F-semigroup. Then PT implies TT. Furthermore, if X is separable and second category, then TT implies DPT and hence also PT.
(2) Every DPT system (S, X) is TT.

Definition 2.20 [17]. A (not necessarily compact) dynamical system (S, X) is called minimal if every point of X is transitive.

3 Semigroup actions on intuitionistic fuzzy 2-metric spaces

Definition 3.1. Let (S, X) be a dynamical system.
(1) A subset A of S acts equicontinuously at $x_0 \in X$ if for every $\epsilon \in (0, 1)$ and $t > 0$, there exists $\delta \in (0, 1)$ such that $N(x_0, x, y, t) > \delta$ and $M(x_0, x, y, t) < 1 - \delta$ imply $N(ax_0, ax, y, t) > \epsilon$ and $M(ax_0, ax, y, t) < 1 - \epsilon$ for every $a \in A$ and $x, y \in X$.
(2) A point $x_0 \in X$ is a point of equicontinuity (notation: $x_0 \in$ Eq(X)) if S acts equicontinuously at x_0. (S, X) is equicontinuous if Eq$(X) = X$.
(3) (S, X) is almost equicontinuous if Eq(X) is dense in X.
Observe that every equicontinuous system is almost equicontinuous.

Theorem 3.2. Let (S, X) be a dynamical system. If A is a relatively compact subset of S, then A acts equicontinuously on X.

Proof. Let $x \in X$ and $s_1 \in S$. Suppose that $s_2 \in S$ is in the neighbourhood of s_1 defined by $N(s_1x, s_2x, z, t) > r$ and $M(s_1x, s_2x, z, t) < 1 - r$ for some $r \in (0, 1)$ and $t > 0$. Now $\pi(s, x) = sx$. By continuity of π given $\epsilon > 0$, for each $s \in S$ there is an open neighbourhood U_s of s and a $\delta_s > 0$ such that if

$s' \in U_s$ and $y \in B(x, \delta_s, t)$, then
$N(s'y, sx, z, t) > \epsilon$ and $M(s'y, sx, z, t) < 1 - \epsilon$
$N(sx, s'x, z, t) > \epsilon$ and $M(sx, s'x, z, t) < 1 - \epsilon$.
Therefore,

$$\begin{aligned} N(s'x, s'y, z, 3t) &\geq N(s'x, sx, z, t) * N(sx, s'y, z, t) * N(s'x, s'y, sx, t) \\ &\geq \epsilon * \epsilon * \epsilon > \epsilon_1 \text{ for some } 0 < \epsilon_1 < 1, \\ M(s'x, s'y, z, 3t) &\leq M(s'x, sx, z, t) \diamond M(sx, s'y, z, t) \diamond M(s'x, s'y, sx, t) \\ &\leq (1-\epsilon) \diamond (1-\epsilon) \diamond (1-\epsilon) < \epsilon_2 \text{ for some } 0 < \epsilon_2 < 1. \end{aligned}$$

Taking $\epsilon' = \max\{\epsilon_1, \epsilon_2\}$ and $t' = 3t$ we have $N(s'x, s'y, z, t') > \epsilon'$ and $M(s'x, s'y, z, t') < 1 - \epsilon'$. From the compactness of $\overline{A}$ it follows that there is a $\delta > 0$ such that if $s' \in S$, $N(x, y, z, t) > \delta$ and $M(x, y, z, t) < 1 - \delta$ then $N(s'x, s'y, z, t') > \epsilon'$ and $M(s'x, s'y, z, t') < 1 - \epsilon'$. Hence $x \in \mathrm{Eq}(X)$. □

Theorem 3.3. If a dynamical system (S, X) is TT, then $\mathrm{Eq}(X) \subset \mathrm{Trans}(X)$.

Proof. Let $x_0 \in \mathrm{Eq}(X)$ and $y \in X$. For the orbit Sx_0 of x_0 and the r-neighbourhood $B(y, r, t) = \{z \in X : N(y, z, sx, t) > r, M(y, z, sx, t) < 1 - r\}$ of y we have to show that $Sx_0 \cap B(y, r, t) \neq \phi$ for $r \in (0, 1)$ and $t > 0$.
Since $x_0 \in \mathrm{Eq}(X)$, there exists a neighbourhood U of x_0 such that $N(sx_0, sx, z, t) > r$ and $M(sx_0, sx, z, t) < 1 - r$ for some $x, z \in U$.
Since X is TT we can choose $s_0 \in S \ni s_0U \cap B(y, r, t) \neq \phi$. This means that $N(s_0x, y, z, t) > r$ and $M(s_0x, y, z, t) < 1 - r$ for some $x, z \in U$. Therefore,

$$\begin{aligned} N(s_0x_0, y, z, 3t) &\geq N(s_0x_0, s_0x, z, t) * N(s_0x, y, z, t) * N(s_0x_0, y, s_0x, t) \\ &\geq r * r * r > r_1 \text{ for some } 0 < r_1 < 1, \\ M(s_0x_0, y, z, 3t) &\leq M(s_0x_0, s_0x, z, t) \diamond M(s_0x, y, z, t) \diamond M(s_0x_0, y, s_0x, t) \\ &\leq (1-r) \diamond (1-r) \diamond (1-r) < r_2 \text{ for some } 0 < r_2 < 1. \end{aligned}$$

Taking $r' = \max\{r_1, r_2\}$ and $t' = 3t$, we then have $N(s_0x_0, y, z, t') > r'$ and $M(s_0x_0, y, z, t') < 1 - r'$. Hence $x_0 \in \mathrm{Trans}(X)$. □

Theorem 3.4. Let S be a C-semigroup. If (S, X) is PT and $\mathrm{Eq}(X) \neq \phi$, then $\mathrm{Trans}(X) \subset \mathrm{Eq}(X)$, that is, every transitive point is an equicontinuous point.

Proof. Let $y \in \mathrm{Trans}(X)$ and $x \in \mathrm{Eq}\ (X)$ be an equicontinuity point. We have to show that $y \in \mathrm{Eq}(X)$. Since $x \in \mathrm{Eq}\ (X)$ for given $\epsilon > 0$ there is a neighbourhood $U(x)$ of x such that $N(sx'', sx', z, t) > \epsilon$ and $M(sx'', sx', z, t) < 1 - \epsilon$ for all $(s, x', x'', z) \in S \times U(x) \times U(x) \times U(x)$. Since $y \in \mathrm{Trans}(X)$ there exists $s_0 \in S$ such that $s_0y \in U(x)$. Thus $U(y) = s_0^{-1}U(x)$ is a neighbourhood of y. Hence for each $(s, y', y'', z) \in S \times U(y) \times U(y) \times U(y)$, $N(ss_0y', ss_0y'', z, t) > \epsilon$ and $M(ss_0y', ss_0y'', z, t) < 1 - \epsilon$.
Since S is a C-semigroup the subset $\mathbb{S} = \overline{S \backslash Ss_0}$ is compact. By Theorem 3.2, $\mathbb{S}$ acts equicontinuously on X. Hence we can choose a neighbourhood $V(y)$ of y such that for all $(\mathtt{s}, y', y'', z) \in \mathbb{S} \times V(y) \times V(y) \times V(y)$, $N(\mathtt{s}y', \mathtt{s}y'', z, t) > \epsilon$ and $M(\mathtt{s}y', \mathtt{s}y'', z, t) < 1 - \epsilon$. Then $W = U(y) \cap V(y)$ is a neighbourhood

of y. Since $S = Ss_0 \cup \mathbb{S}$, we have for each $(s, y', y'', z) \in S \times W \times W \times W$, $N(sy', sy'', z, t) > \epsilon$ and $M(sy', sy'', z, t) < 1 - \epsilon$. This proves that $y \in \mathrm{Eq}(X)$ and hence $\mathrm{Trans}(X) \subset \mathrm{Eq}(X)$. □

Theorem 3.5. Let S be a C-semigroup. If (S, X) is minimal and $\mathrm{Eq}(X) \neq \phi$, then X is equicontinuous.

Proof. By definition of minimal, we have $\mathrm{Trans}(X) = X$. Using Theorem 3.4, if $\mathrm{Eq}(X) \neq \phi$ then every transitive point is an equicontinuity point. Thus $\mathrm{Eq}(X) = X$. □

Definition 3.6. (Sensitive dependence on initial conditions). A dynamical system (S, X) has sensitive dependence on initial conditions or more briefly, is sensitive, if $\exists\ \epsilon \in (0, 1)$ and $t > 0$ such that for every $x \in X$ and every neighbourhood U of x, $\exists\ (s, y, z) \in S \times U \times U$ with $N(sx, sy, z, t) < \epsilon$ and $M(sx, sy, z, t) > 1 - \epsilon$. When (S, X) is not sensitive, we say that (S, X) is nonsensitive.

Remark 3.7.
(1) The definition of sensitive dependence on initial conditions plays an important role in classical chaotic systems. Note that the above form is just a generalization of existing definition for (X, f) when $S = \{f^n : n = 1, 2, ...\}$.
(2) Spelling out the property of nonsensitive we have: for every $\epsilon \in (0, 1)$ and $t > 0\ \exists\ x \in X$ and a neighbourhood U of x, such that for each $(s, y, z) \in S \times U \times U$, $N(sx, sy, z, t) \geq \epsilon$ and $M(sx, sy, z, t) \leq 1 - \epsilon$. We observe that trivially (S, X) is nonsensitive whenever X has no isolated points.
(3) Without loss of generality, we sometimes use the open ball $B(x, r, t)$ instead of the neighbourhood U of x in Definition 3.6.

Theorem 3.8. For a PT dynamical system (S, X) with no isolated points, being nonsensitive is equivalent to the following property: for every $\epsilon \in (0, 1)$ and $t > 0$, there exists a transitive point $x_0 \in X$ and a neighbourhood U of x_0 such that for every $y \in U$ and every $s \in S$, $N(sx_0, sy, z, t) \geq \epsilon$ and $M(sx_0, sy, z, t) \leq 1 - \epsilon$.

Proof. Let ϵ' be given and let x and U be as in the definition of nonsensitive. Since (S, X) is PT, there is a point $x_0 \in X$ whose orbit is dense, i.e., there exists a $s_0 \in S$ such that $s_0 x_0 \in U$. Denote $x_1 = s_0 x_0$. Then $N(sx, sx_1, z, t) \geq \epsilon'$ and $M(sx, sx_1, z, t) \leq 1 - \epsilon'$.
On the other hand, $\exists\ r \in (0, 1)$ and $t > 0$ such that $B(x, r, t) \subset U$. Let $V = B(x, r, t)$. Then $\forall\ y \in V \subset U$ and $s \in S$, $N(sx, sy, z, t) \geq \epsilon'$ and $M(sx, sy, z, t) \leq 1 - \epsilon'$. Therefore,

$$\begin{aligned} N(sx_1, sy, z, 3t) &\geq N(sx_1, sx, z, t) * N(sx, sy, z, t) * N(sx_1, sy, , sx, t) \\ &\geq \epsilon' * \epsilon' * \epsilon' \geq \epsilon_1 \text{ for some } \epsilon_1 \in (0, 1) \text{ and} \end{aligned}$$

$M(sx_1, sy, z, 3t) \leq M(sx_1, sx, z, t) \diamond M(sx, sy, z, t) \diamond M(sx_1, sy, , sx, t)$
$\leq (1-\epsilon') \diamond (1-\epsilon') \diamond (1-\epsilon') \leq \epsilon_2$ for some $\epsilon_2 \in (0,1)$.
Taking $\epsilon = \max\{\epsilon_1, \epsilon_2\}$ and $t' = 3t$, $N(sx_1, sy, z, t') \geq \epsilon$ and $M(sx_1, sy, z, t') \leq 1-\epsilon$. Since X has no isolated points, the point x_1 is also transitive and the proof is complete. □

Theorem 3.9. Let S be a C-semigroup. Assume that a TT dynamical system (S, X) is separable and second category. Then (S, X) is almost equicontinuous if and only if it is nonsensitive.

Proof. Clearly an almost equicontinuous system is always nonsensitive. Conversely suppose that (S, X) is nonsensitive, for any $\epsilon \in (0,1)$ $\exists$ $x \in X$ and a neighbourhood U of x such that for all $(s, y, z) \in S \times U \times U$ and $t > 0$ $N(sx, sy, z, t) \geq \epsilon$ and $M(sx, sy, z, t) \leq 1-\epsilon$.
Suppose that $\{U_n\}_{n=1}^{\infty}$ is a countable base, $\exists$ $n_0 \in \mathbb{N}$ such that $y \in U_{n_0} \subset U$. Without loss of generality, we can assume that $sU_n \subset B(sx, 1/n, t)$, for all $(s, n) \in S \times \mathbb{N}$. Let $V_n = S^{-1}U_n$ and $\mathbb{V} = \bigcap\limits_{n\in\mathbb{N}} V_n$.
Clearly every V_n is open and meets every open subset of X since (S, X) is TT. This means that each V_n is dense in X. Since X is second category, by Baire Theorem [14], $\mathbb{V} = \bigcap\limits_{n\in\mathbb{N}} V_n$ is also dense. Now it remains to show that $\mathbb{V} \subset \text{Eq}\ (X)$. Given $x \in \mathbb{V}$ $\exists$ $s_0 \in S$ such that $s_0x \in U_n$. Let $V = s_0^{-1}U_n$. Hence for $y \in V$ and each $s = s's_0 \in Ss_0$ we have
$N(sx, sy, z, t) = N(s's_0x, s's_0y, z, t) \geq \frac{1}{n} \geq r_1$ for some $r_1 \in (0,1)$ and
$M(sx, sy, z, t) = M(s's_0x, s's_0y, z, t) \leq 1 - \frac{1}{n} \leq r_2$ for some $r_2 \in (0,1)$.
Taking $r = \max\{r_1, r_2\}$, $N(sx, sy, z, t) \geq r$ and $M(sx, sy, z, t) \leq 1-r$. But $\overline{S \setminus Ss_0}$ is compact because S is a C-semigroup. Then by Theorem 3.2 the set $\overline{S \setminus Ss_0}$ acts equicontinuously on X. This means that if W is an open neighbourhood of x, for all $y \in W$ and for each $s \in \overline{S \setminus Ss_0}$, $N(sx, sy, z, t) \geq r$ and $M(sx, sy, z, t) \leq 1-r$ holds. Let $O = W \cap V$ be an open neighbourhood of x. Then $N(sx, sy, z, t) \geq r$ and $M(sx, sy, z, t) \leq 1-r$ for all $s \in S$ and all $y \in O$. Hence $x \in \text{Eq}\ (X)$. This completes the proof. □

4 Open Problems

1. Efforts can be made to apply this theory to fuzzy n-normed linear space in particular to fuzzy 2-normed linear space.
2. We have discussed some types of transitivity in a dynamical system for intuitionistic fuzzy 2-metric space. One can work in this direction by generalizing the concept of transitivity to include more cases and the same can be done for F-Semigroup and C-Semigroup.

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