# A Two Phase Service $M/G/1$ Vacation Queue With General Retrial Times and Non-persistent Customers

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**Abstract**

*This paper is motivated by an open problem posed by Senthil Kumar and Arumuganathan in their paper published in June 2009 [10]. We study the steady state behavior of an $M/G/1$ retrial queue with non-persistent customers, two phases of heterogeneous service and different vacation policies. We consider the case where the customer after obtaining the first essential service may or may not opt for a second optional service. In [10], the authors have taken the interretrial times to be exponentially distributed. In this paper, we consider a general retrial time distribution with a constant retrial policy. We have obtained the steady state probability generating function of the system size and the orbit size. Also, we obtain expressions for the performance measures of the system. We discuss some particular cases.*

**Keywords:** *Retrial queue, constant retrial policy, non-persistent customers, second phase of optional service, different vacation policies.*

## 1   Introduction

This paper deals with a single server retrial queue in which the retrial time is governed by a non exponential distribution. Retrial queueing systems permit no waiting, in the normal sense. Instead, a customer who finds all servers busy upon arrival, is obliged to leave the service area and to come back to the

system after a random amount of time. Between retrials, a customer is said to be in orbit. These queueing systems are appropriate for communication networks, where, a customer upon getting a busy signal, tries his luck again at a later time. Falin and Templeton [6] proposed the fundamental methods and results of this topic. A comprehensive bibliography is provided by Artalejo [1] and [2]

Fayolle [8] investigated an $M/M/1$ retrial queue with a constant retrial policy, where, the customers in the retrial group form a queue and only the customer at the head of the queue can request for service with a constant rate. Further, he assumed that the retrial times are exponentially distributed random variables. Farahmand [7] generalized this model for an $M/G/1$ queueing system. Artalejo [3] introduced the concept of vacations under a constant retrial policy. Gomez-Corral [9] generalized the results of Farahmand by considering the case where the retrial time is also governed by a generalized distribution.

Choudhury [4] investigated an $M/G/1$ retrial queue with an additional phase of second service and general retrial times with a constant retrial policy. The model with generalized retrial times arises naturally in problems where the server is required to search for customers, that is, this policy is related to many service systems where, after each service completion, the processor will spend a random amount of time in order to find the next item to be processed. The motivation for such types of models comes from Computer and Communication networks, where massages are processed in two stages by a single server. Doshi [5] recognized its applications in a distributed system, where control of two phase execution is required by a central server.

Senthilkumar and Arumuganthan [10] considered an $M/G/1$ retrial queue with two essential phases of service, exponentially distributed interretrial times, non-persistent customers and different vacation policies. They have mentioned in their paper [10] that it would be an interesting problem to generalize their results to the case where the interretrial times are non-exponential.

Motivated by their paper[10], we have considered an $M/G/1$ retrial queue with non-persistent customers, two phases of heterogeneous service (the first one being essential and the second one being optional), with different vacation policies and a generalized retrial time distributions with a constant retrial policy. This paper is therefore a generalization of the paper [4].

The rest of the paper is organized as follows. In section 2, we describe the mathematical model. In section 3, we examine the steady state behaviour and derive the probability generating functions of the system size and orbit size distributions. In section 4, we consider some useful performance measures of the system. In section 5, we discuss some particular cases. In section 6, we present some concluding remarks and in section 7, we propose a few open problems.

# 2   Mathematical Model

We consider a single server queue in which customers arrive according to a Poisson stream of rate $\lambda > 0$. If, upon arrival, the server is free, the service of the arriving customer commences immediately. Otherwise, with a probability $1 - \alpha$, he leaves the system immediately and is lost to the system. With a probability $\alpha$, the customer may join a group of blocked customers called the orbit, from where he repeats his request for service in accordance with an FCFS discipline. We shall assume that only the customer at the head of the orbit is allowed to access the server.

When a service is completed, the access from the orbit to the server is governed by an arbitrary law with a common probability distribution function $R(x)$, Laplace-Stieltjes transform (LST) $R^*(s)$ and the hazard rate function $\eta(x) = \frac{R'(x)}{1-R(x)}$.

Upon completion of the first essential service (FES), a customer may opt for a second optional service (SOS) with probability $r$ or he may decide to leave the system with a probability $1 - r$ $(0 \leq r \leq 1)$. The service time $S_1$ of a customer in the first phase(FES) is assumed to be independent of the service time $S_2$ of the customer in the second phase(SOS). The random variable $S_i$ is assumed to have a distribution function $B_i(x)$ and a LST $B_i^*(s)$, $i = 1, 2$. Let $b_1^i$, $b_2^i$ denote the *ith* moments of $S_1$ and $S_2$ respectively. Then $b_1^i = (-1)^i B_1^{*(i)}(0)$ and $b_2^i = (-1)^i B_2^{*(i)}(0)$.

The total service time $S$ of a customer in the system is given by

$$S = \begin{cases} S_1 + S_2, & \text{with probability r,} \\ S_1, & \text{with probability 1-r} \end{cases}$$

Let $B(x)$, $B^*(s)$, $\nu(x)$ be the distribution function, LST and the hazard rate function respectively of the random variable $S$. $B^*(s) = \{(1 - r) + r B_2^*(s)\} B_1^*(s)$. If $b^i$ denotes the *ith* moment of the random variable S, then

$$
\begin{aligned}
b^1 &= b_1^1 + r b_2^1, \\
b^2 &= b_1^2 + 2r b_1^1 b_2^1 + r b_2^2.
\end{aligned}
$$

Upon completion of a service, the server may remain in the system to serve the next customer (if any), with probability $\beta_0$ or he may proceed on the *kth* vacation scheme with probability $\beta_k$ $(1 \leq k \leq M)$ and $\sum_{k=0}^{M} \beta_k = 1$.

Let $V_k(x)$, $V_k^*(s)$, $\xi_k(x)$ denote the distribution function, LST and the hazard function respectively of the random variable $X_k$, where $X_k$ which denotes the duration of the vacation when the server is on the *kth* vacation scheme. Let $v_k^i$ denote the *ith* moment of the random variable $X_k$. Then

$$v_k^i = (-1)^i V_k^{*(i)}(0), \ k = 1, 2, ..., M.$$

We assume that the retrial time begins at the end of each service completion when the server becomes free or when the server returns from his vacation.

The interarrival times, interretrial times, $S_1, S_2, X_k$ are all assumed to be independent of each other.

Let $C(t)$ denote the state of the server at time t. We define

$$C(t) = \begin{cases} 0, & \text{if the server is idle,} \\ 1, & \text{if the server is busy,} \\ 2, & \text{if the server is on vacation.} \end{cases}$$

Let N(t) be the orbit size at time t. We introduce the supplementary variable

$$Y(t) = \begin{cases} R^0(t), & \text{if } C(t) = 0, \ N(t) \geq 1, \\ S^0(t), & \text{if } C(t) = 1, \\ V_k^0(t), & \text{if } C(t) = 2, \end{cases}$$

where $R^0(t)=$ elapsed retrial time of the customer at the head of the orbit at time t,

$S^0(t) =$ elapsed service time of the customer in service at time t,

$V_k^0(t) =$ elapsed vacation time of the server when he is on the *kth* vacation scheme, k=1,2,...,M.

The process $\{(C(t), N(t), Y(t)), t \geq 0\}$ is a continuous time Markov process. We define the probabilities

$$
\begin{aligned}
P_{0,0}(t) &= Prob\left\{C(t) = 0, \ N(t) = 0\right\}, \\
P_{0,n}(x,t)dx &= Prob\left\{C(t) = 0, \ N(t) = n, \ x \leq R^0(t) < x + dx\right\}, \ x \geq 0, \ n \geq 1, \\
P_{1,n}(x,t)dx &= Prob\left\{C(t) = 1, \ N(t) = n, \ x \leq S^0(t) < x + dx\right\}, \ x \geq 0, \ n \geq 0, \\
P_{2,n}^k(x,t)dx &= Prob\left\{C(t) = 2, \ N(t) = n, \ x \leq V_k^0(t) < x + dx\right\}, \ x \geq 0, \ n \geq 0, \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad k = 1, 2, \cdots, M.
\end{aligned}
$$

## 3   Steady state analysis

Now, analysis of this queueing model can be performed with the help of the following Kolmogorov forward equations.

$$
\begin{aligned}
\frac{d}{dt}P_{0,0}(t) &= -\lambda P_{0,0}(t) + \beta_0 \int_0^\infty P_{1,0}(x,t)\nu(x)dx \\
&\quad + \sum_{k=1}^M \int_0^\infty P_{2,0}^k(x,t)\xi_k(x)dx.
\end{aligned} \tag{1}
$$

For $x > 0$,

$$
\frac{\partial}{\partial t}P_{0,n}(x,t) + \frac{\partial}{\partial x}P_{0,n}(x,t) = -\left\{\lambda + \eta(x)\right\}P_{0,n}(x,t), \ n \geq 1, \tag{2}
$$

$$
\begin{aligned}
\frac{\partial}{\partial t}P_{1,n}(x,t) + \frac{\partial}{\partial x}P_{1,n}(x,t) &= -\left\{\lambda\alpha + \nu(x)\right\}P_{1,n}(x,t) \\
&\quad + \lambda\alpha(1 - \delta_{n,0})P_{1,n-1}(x,t), \ \ n \geq 0, \tag{3}
\end{aligned}
$$

$$\frac{\partial}{\partial t}P_{2,n}^k(x,t) + \frac{\partial}{\partial x}P_{2,n}^k(x,t) = -\{\lambda\alpha + \xi_k(x)\}P_{2,n}^k(x,t)$$
$$+(1-\delta_{n,0})\lambda\alpha P_{2,n-1}^k(x,t), \quad (4)$$
$$\text{where,} \quad n \geq 0 \text{ and } k = 1, 2, ..., M.$$

The boundary conditions are as follows, for $n \geq 1$,

$$P_{0,n}(0,t) = \beta_0 \int_0^\infty P_{1,n}(x,t)\nu(x)dx$$
$$+ \sum_{k=1}^M \int_0^\infty P_{2,n}^k(x,t)\xi_k(x)dx, \quad (5)$$

$$P_{1,n}(0,t) = \int_0^\infty P_{o,n+1}(x,t)\eta(x)dx + \lambda\int_0^\infty P_{0,n}(x,t)dx, \quad (6)$$

$$P_{2,n}^k(0,t) = \beta_k \int_0^\infty P_{1,n}(x,t)\nu(x)dx, \ n \geq 0, \quad (7)$$

$$P_{1,0}(0,t) = \lambda P_{0,0}(t) + \int_0^\infty P_{0,1}(x,t)\eta(x)dx. \quad (8)$$

Assuming that the system reaches the steady state, the equations (1) to (8) become

$$\lambda P_{0,0} = \beta_0 \int_0^\infty P_{1,0}(x)\nu(x)dx$$
$$+ \sum_{k=1}^M \int_0^\infty P_{2,0}^k(x)\xi_k(x)dx. \quad (9)$$

For $x > 0$,

$$\frac{d}{dx}P_{0,n}(x) = -\{\lambda + \eta(x)\}P_{0,n}(x), \ n \geq 1, \quad (10)$$

$$\frac{d}{dx}P_{1,n}(x) = -\{\lambda\alpha + \nu(x)\}P_{1,n}(x)$$
$$+\lambda\alpha(1-\delta_{n,0})P_{1,n-1}(x), \quad n \geq 0, \quad (11)$$

$$\frac{d}{dx}P_{2,n}^k(x) = -\{\lambda\alpha + \xi_k(x)\}P_{2,n}^k(x)$$
$$+(1-\delta_{n,0})\lambda\alpha P_{2,n-1}^k(x), \quad (12)$$
$$\text{where,} \quad n \geq 0 \text{ and } k = 1, 2, ..., M.$$

The boundary conditions are as follows, for $n \geq 1$,

$$P_{0,n}(0) = \beta_0 \int_0^\infty P_{1,n}(x)\nu(x)dx$$
$$+ \sum_{k=1}^M \int_0^\infty P_{2,n}^k(x)\xi_k(x)dx, \quad (13)$$

$$P_{1,n}(0) = \int_0^\infty P_{o,n+1}(x)\eta(x)dx + \lambda\int_0^\infty P_{0,n}(x)dx, \quad (14)$$

$$P_{2,n}^k(0) \;=\; \beta_k \int_0^\infty P_{1,n}(x)\nu(x)dx, \;\; n \geq 0, \tag{15}$$

$$P_{1,0}(0) \;=\; \lambda P_{0,0} + \int_0^\infty P_{0,1}(x)\eta(x)dx. \tag{16}$$

Define the following partial probability generating functions, for $|z| \leq 1$,

$$P_1(x,z) \;=\; \sum_{n=0}^\infty P_{1,n}(x)z^n, \;\; x > 0, \tag{17}$$

$$P_2^k(x,z) \;=\; \sum_{n=0}^\infty P_{2,n}^k(x)z^n, \;\; x > 0, \tag{18}$$

$$P_1(0,z) \;=\; \sum_{n=0}^\infty P_{1,n}(0)z^n, \tag{19}$$

$$P_2^k(0,z) \;=\; \sum_{n=0}^\infty P_{2,n}^k(0)z^n, \tag{20}$$

$$P_0(x,z) \;=\; \sum_{n=1}^\infty P_{0,n}(x)z^n, \;\; x > 0, \tag{21}$$

$$P_0(0,z) \;=\; \sum_{n=1}^\infty P_{0,n}(0)z^n. \tag{22}$$

From the above equations, (10), (11) and (12),

$$P_0(x,z) \;=\; P_0(0,z)e^{-\lambda x}(1 - R(x)), \tag{23}$$
$$P_1(x,z) \;=\; P_1(0,z)e^{-\lambda\alpha(1-z)x}(1 - B(x)), \tag{24}$$
$$P_2^k(x,z) \;=\; P_2^k(0,z)e^{-\lambda\alpha(1-z)x}(1 - V_k(x). \tag{25}$$

Also from (13), (14), (15) and (16),

$$\begin{aligned}
P_0(0,z) \;=\;& \beta_0 P_1(0,z)B^*(\lambda\alpha(1-z)) \\
&+ \sum_{k=1}^M P_2^k(0,z)V_k^*(\lambda\alpha(1-z)) - \lambda P_{0,0},
\end{aligned} \tag{26}$$

$$P_1(0,z) \;=\; \lambda P_{0,0} + P_0(0,z)\left\{ \frac{z(1 - R^*(\lambda)) + R^*(\lambda)}{z} \right\}, \tag{27}$$

$$P_2^k(0,z) \;=\; \beta_k P_1(0,z)B^*(\lambda\alpha(1-z)). \tag{28}$$

Now,

$$\begin{aligned}
P_0(z) \;=\;& \int_0^\infty P_0(x,z)dx \\
\;=\;& \left\{ \frac{1 - R^*(\lambda)}{\lambda} \right\} P_0(0,z),
\end{aligned} \tag{29}$$

$$P_1(z) = \int_0^\infty P_1(x, z)dx$$

$$= \left\{ \frac{1 - B^*(\lambda\alpha(1 - z))}{\lambda\alpha(1 - z)} \right\} P_1(0, z), \tag{30}$$

$$P_2^k(z) = \int_0^\infty P_2^k(x, z)dx$$

$$= \left\{ \frac{1 - V_k^*(\lambda\alpha(1 - z))}{\lambda\alpha(1 - z)} \right\} P_2^k(0, z). \tag{31}$$

From equation (26),

$$P_0(0, z) = \frac{\lambda R^*(\lambda)(1 - z)}{\gamma(z)[R^*(\lambda) + z(1 - R^*(\lambda))] - z} P_{0,0}, \tag{32}$$

$$\text{where,} \gamma(z) = B^*(\lambda\alpha(1 - z)) \left\{ \beta_0 + \sum_{k=1}^M \beta_k V_k^*(\lambda\alpha(1 - z)) \right\}. \tag{33}$$

Hence,

$$P_1(z) = \frac{(1 - B^*(\lambda\alpha(1 - z)))R^*(\lambda)}{\alpha \left\{ \gamma(z)[R^*(\lambda) + z(1 - R^*(\lambda))] - z \right\}} P_{0,0}, \tag{34}$$

$$P_2(z) = \sum_{k=1}^M P_2^k(z)$$

$$= \frac{R^*(\lambda)B^*(\lambda\alpha(1 - z))\sum_{k=1}^M \beta_k(1 - V_k^*(\lambda\alpha(1 - z)))}{\alpha \left\{ \gamma(z)[R^*(\lambda) + z(1 - R^*(\lambda))] - z \right\}} P_{0,0}. \tag{35}$$

The PGF $P(z)$ of the orbit size is given by

$$P(z) = P_{0,0} + P_0(z) + P_1(z) + P_2(z)$$

$$= \frac{\alpha(\gamma(z) - z) + 1 - \gamma(z)}{\alpha \left\{ \gamma(z)[R^*(\lambda) + z(1 - R^*(\lambda))] - z \right\}} R^*(\lambda) P_{0,0}. \tag{36}$$

From the normalizing condition $P(1) = 1$, we have

$$P_{0,0} = \frac{R^*(\lambda) - \rho\alpha}{R^*(\lambda)[1 + \rho(1 - \alpha)]}, \tag{37}$$

$$\text{where,} \ \rho = \lambda \left\{ b^1 + \sum_{k=1}^M \beta_k v_k^1 \right\}. \tag{38}$$

Hence,

$$P(z) = \left\{ \frac{\alpha(\gamma(z) - z) + 1 - \gamma(z)}{\alpha \left\{ \gamma(z)[R^*(\lambda) + z(1 - R^*(\lambda))] - z \right\}} \right\} \frac{R^*(\lambda) - \rho\alpha}{1 + \rho(1 - \alpha)}. \tag{39}$$

Let $K(z)$ be the PGF of the system size .

$$
\begin{aligned}
K(z) &= P_{0,0} + P_0(z) + zP_1(z) + P_2(z) \\
&= \left\{ \frac{(\alpha-1)(\gamma(z)-z) + (1-z)B^*(\lambda\alpha(1-z))}{\alpha\{\gamma(z)[R^*(\lambda)+z(1-R^*(\lambda))]-z\}} \right\} \\
&\quad \times \frac{R^*(\lambda)-\rho\alpha}{1+\rho(1-\alpha)}.
\end{aligned} \tag{40}
$$

Remark: It should be noted that the expressions for the PGFs $P(z)$ and $K(z)$ are given in terms of $P_{0,0}$. Therefore, a necessary condition for the steady state to be attained is given by $P_{0,0} > 0$. Since $R^*(\lambda)(1+\rho(1-\alpha)) > 0$, $R^*(\lambda) - \rho\alpha$ should be greater than zero. Therefore a necessary condition for the steady state to exist is given by $R^*(\lambda) > \lambda\alpha \left\{ b_1^1 + rb_2^1 + \sum_{k=1}^M \beta_k v_k^1 \right\}$.

By an argument similar to that used by Gomez-Corral [9], it should not be difficult to prove that the above condition is also sufficient for the existence of the steady state.

# 4    Performance measures

a) The expected number of customers in the system is given by

$$
\begin{aligned}
E[L] &= \lim_{z\to 1} \frac{d}{dz} K(z) \\
&= \frac{\lambda^2\alpha\gamma_* \{(1-\alpha)R^*(\lambda)+\alpha\}}{2(1+\rho(1-\alpha))(R^*(\lambda)-\rho\alpha)} + \frac{\rho\alpha(1-R^*(\lambda))}{R^*(\lambda)-\rho\alpha} \\
&\quad + \frac{\lambda b^1}{1+\rho(1-\alpha)},
\end{aligned}
$$

$$
\text{where, } \gamma_* = b^2 + 2b^1 \sum_{k=1}^M \beta_k v_k^1 + \sum_{k=1}^M \beta_k v_k^2.
$$

b)

$$
\begin{aligned}
\text{The blocking probability} &= 1 - \{P_{0,0} + P_0(1)\} \\
&= \frac{\rho}{1+\rho(1-\alpha)}.
\end{aligned}
$$

c) The expected waiting time in the system

$$
E[W] = \frac{E[L]}{\lambda\alpha}.
$$

d) The expected number of customers in the orbit is given by

$$
E[L_q] = \frac{\lambda^2\alpha\gamma_* \{(1-\alpha)R^*(\lambda)+\alpha\}}{2(1+\rho(1-\alpha))(R^*(\lambda)-\rho\alpha)} + \frac{\rho\alpha(1-R^*(\lambda))}{R^*(\lambda)-\rho\alpha}.
$$

e) The expected waiting time in the orbit is

$$E[W_q] = \frac{E[L_q]}{\lambda\alpha}.$$

f) The steady state distribution of the server state is given by

$$Prob\{\text{server is idle}\} = P_{0,0} + P_0(1) = \frac{1 - \rho\alpha}{1 + \rho(1 - \alpha)},$$

$$Prob\{\text{server is busy}\} = P_1(1) = \frac{\lambda b^1}{1 + \rho(1 - \alpha)},$$

$$Prob\{\text{server is on kth vacation}\} = \frac{\lambda\beta_k v_k^1}{1 + \rho(1 - \alpha)},$$

$$Prob\{\text{server is on vacation}\} = \sum_{k=1}^{M} P_2^k(1) = \frac{\rho - \lambda b^1}{1 + \rho(1 - \alpha)}.$$

# 5  Particular cases

**Case i:** If all the customers are persistent (i.e. $\alpha = 1$) and there is no vacation scheme then PGF of the orbit size is given by

$$P(z) = \frac{(R^*(\lambda) - \rho)(1 - z)}{R^*(\lambda)(1 - z)\gamma(z) - z(1 - \gamma(z))},$$

$$\text{where } \gamma(z) = B^*(\lambda(1 - z))$$

$$\text{and } B^*(\lambda(1 - z)) = \{(1 - r) + rB_2^*(\lambda(1 - z))\} B_1^*(\lambda(1 - z)).$$

The above result is the same as that given in Corollary 3.1 of [4]

**Case ii:** If there is no second phase of service (i.e r $= 0$), no vacation schemes and $\alpha = 1$, then the PGF of system size distribution is given by

$$K(z) = \frac{(R^*(\lambda) - \rho)(1 - z)B^*(\lambda - \lambda z)}{R^*(\lambda)(1 - z)B^*(\lambda - \lambda z) - z(1 - B^*(\lambda - \lambda z))}.$$

This is the same as the equation (16) of Gomez- Corral [9]

# 6  Conclusion

In this paper, we have taken the first step in solving the open problem mentioned in [10]. We have extented their results to the case where the interretrial times are non-exponentially distributed. We have obtained expressions for the PGFs of the system size and orbit size distributions in the steady state. We

have derived expressions for the blocking probability, distribution of the sever state, expected number of customer, in the orbit and the mean response time. We have been able to obtain the results of [4] and [9] as special cases of our model.

# 7    Open Problem

In this paper, we have taken the interretrial times to be non-exponentially distributed. However, we have assumed that only the customer at the head of the orbit is allowed to make a retrial. This restrictive assumption was necessary because the inherent difficulty in non-exponential retrial times is caused by the fact that the queueing model must keep track of the elapsed retrial time for each of the customers (possibly, a large number) in orbit. However, it remains an interesting problem to consider extensions of the model to the case of generally distributed retrial times without a constant retrial policy.

Another interesting problem is obtained by including the concept of reneging customers. A customer, who makes a retrial attempt and fails to obtain service may leave the system with some probability $\theta_0$ or he may join the orbit with a probability $1 - \theta_0$. Such a customer is said to renege. It would be an interesting problem to add this feature to the model presented in this paper.

In the area of computer and communication networks, the concept of feedback is becoming increasingly important. A customer after obtaining service may decide to join the orbit for obtaining service again. Such a customer is called a feedback customer. It would be interesting to allow customer feedbacks in the model presented in this paper.

The problem of having more than one optional service at the second phase would also be an interesting problem to solve.

It would be also interesting to determine a control policy which gives the best estimate for the probabilities $\beta_i$ and which minimizes the total cost of the service system.

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