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Local Ring at a Point of an Affine Algebraic Set and Zariski Topology

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Abstract

Subset H of affine $n-$space k^n is called an affine algebraic set if $H = V(J)$ for some ideal J of the polynomial ring $k[T_1,...,T_n]$. If x is an element of an affine algebraic set H, we define a set denoted by θ_x. In this paper we prove that θ_x is a ring, moreover it is a local ring of a point x. Also we define a topology on k^n which is called the Zariski topology on k^n.

Keywords: Affine algebraic set , Irreducible variety, Local ring , Coordinate ring , and Zariski topology.
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1 Introduction

The concept of algebraic sets plays an important role and forms the basic for a vast area of mathematics called Algebraic Geometry. In this section we give some basic definitions.

Definition 1.1 [2]. Let k be a field and let J be an ideal of $k[T_1,...,T_n]$. The set $V(J) = \{m = (x_1,...,x_n) \in k^n : f(x_1,...,x_n) = 0, \forall f \in J\}$ is called an affine algebraic set associated to the ideal J.

Example 1.2. Let $f \in k[T_1,...,T_n]$ and let $J = \langle f \rangle$ (ideal generated by f). Then

$$V(J) = \{m = (x_1,...,x_n) \in k^n : f(m) = 0\}.$$

If f is a polynomial of prime degree , then $V(J)$ is a hyperplane.

Definition 1.3. Let E be a subset of k^n , consider the set $I(E) = \{f \in k[T_1,...,T_n] : f(m) = 0, \forall m \in E\}$. It is easy to show that $I(E)$ is an ideal of $k[T_1,...,T_n]$.

The affine algebraic set $V(I(E))$ which associate to the ideal $I(E)$ is called the affine algebraic set generated by the part E. It is easy to verify that

$$I(V(I(E))) = I(E).$$

Definition 1.4 [2]. An affine algebraic set H is said to be irreducible variety if it cannot be written as union of two proper subsets .one cannot write $H = H_1 \cup H_2$ for some affine algebraic sets H_1, H_2 which are both proper subsets of H .

Counter example 1.5. Take $k = \mathbb{R}$, the set of solutions to $xy = 0$ is reducible variety because it is the union of the solutions to $x = 0$ and the solutions to $y = 0$.

2. Elementary Properties

In this section, we give some basic results which are essential in the sequel.

Proposition 2.1. Let J and J' be two ideals of $k[T_1,...,T_n]$. Then

(a) $J \subseteq I(V(J))$

(b) $J \subseteq J' \Rightarrow V(J') \subseteq V(J) \Rightarrow I(V(J)) \subseteq I(V(J'))$

(c) $V(JJ') = V(J \cap J') = V(J) \cup V(J')$

Proposition 2.2. If $(J_\alpha)_\alpha$ be a family of ideals of $k[T_1,...,T_n]$. Then

$$V(\sum_\alpha J_\alpha) = \bigcap_\alpha V(J_\alpha)$$

Remark 2.5. Every ascending chain $J_1 \subseteq J_2 \subseteq J_3 \subseteq ...$ of ideals of $k[T_1,...,T_n]$ becomes stationary. Also every descending chain $H_1 \subseteq H_2 \subseteq H_3 ...$ of affine algebraic sets of k^n becomes stationary .

Theorem 2.6 [1]. Let H be an algebraic set of k^n, then H is an irreducible variety if and only if $I(H)$ is a prime ideal of $k[T_1,...,T_n]$.

Remark 2.7. After Theorem 2.6 above we saw that , to every irreducible variety H we correspond a prime ideal $I(H)$. is the converse true ? that is ,to every prime ideal J of $k[T_1,...,T_n]$ we correspond an irreducible variety $V(J)$ such that $I(V(J)) = J$. The following counter example tell us that is not true .

Let $k = \mathbb{R}$ the field of real numbers , and let $F = T_1^2 + T_2^2 \in \mathbb{R}[T_1,T_2]$. Take J an ideal generated by F. We have

$$V(J) = \{m = (x_1,x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 = 0\} = \{0\},$$

and $I(V(J)) = I(0) = (T_1,T_2) \neq J$.

Proposition 2.8[5]. Every algebraic set is a uniquely finite union of irreducible varieties .

3 Local Ring at a Point of an Affine Variety and Zariski Topology.

Definition 3.1. Consider the affine space k^n, and let Ω be the set of the algebraic sets of k^n. We have

1. $\bigcap_\alpha V(I_\alpha) = V(\sum_\alpha I_\alpha) \in \Omega$, for each family $(I_\alpha)_\alpha$.
2. $V(I_1) \cup V(I_2) = V(I_1 \cap I_2) = V(I_1.I_2) \in \Omega,$
3. $k^n = V(0) \in \Omega,$
4. $\varnothing = V(\{1\}) \in \Omega.$

So, we define a topology on k^n by taking the affine algebraic sets as the closed sets , this topology is called the Zariski topology on k^n.

The Zariski topology on k^n reflect the algebraic structure of $k[T_1,T_2,...,T_n]$. Then if H is a subset of k^n, we have $H = V(I(H))$ if and only if H is a Zariski- closed set.

Let $H = V(I(H))$ be an affine algebraic set of k^n, we know that

$$I = I(H) = \{f \in k[T_1,...,T_n] : f(H) = 0\},$$

is an ideal of $k[T_1,...,T_n]$. It is obvious that

H is irreducible $\Leftrightarrow I(H)$ is a prime ideal [1] $\Leftrightarrow A = {}^{k[T_1,\ldots,T_n]}\!\!\diagup_{I(H)}$ is an integral domain [3].

The interpretation of the elements of A given by the following explaining: Let $p \in A$, p can be considered as a function $p: H = V(I) \rightarrow k$ defined by $p(x) = P(x_1,\ldots,x_n)$, where P is an arbitrary representative of p .
This function is well defined because if P' is another representative of p , we have $P(x) - P'(x) = (P - P')(x) = 0$ because $P - P' \in I$. Hence $P(x) = P'(x)$.

Definition 3.2 [3]. The elements of the integral domain A above are called the polynomial functions.

For more explaining the interpretation of the elements of A , we know that a function $f : k^n \rightarrow k$ is said to be regular [5], if it can be written as a polynomial, that is, if there is a polynomial p in $k[T_1,T_2,..T_n]$ such that $f(x_1,x_2,..x_n) = p(x_1,x_2,\ldots,x_n)$ for every point $(x_1,x_2,\ldots,x_n)$ of k^n.
Regular functions on affine *n*- space k^n are thus exactly the same as polynomials over k in *n* variables.

Just as continuous functions are the natural maps on Topological spaces , there is a natural class of functions on an algebraic set , called regular functions. A regular function on an algebraic set H contained in k^n is defined to be the restriction of a regular function on k^n .

Just as with the regular functions on affine space , the regular functions on H form a ring , which we denote by $k[H]$. This ring is called the coordinate ring of H . Since regular functions on H come from regular functions on k^n, there should be a relation - ship between their coordinate rings. Specifically , to get a function in $k[H]$ we took a function in $k[T_1,T_2,\ldots,T_n]$, and we said that it was the same as another function if they gave the same values when evaluated on H. This is the same as saying that their difference is zero in H. From this we can see that $k[H]$ is the quotient ${}^{k[T_1,T_2,..T_n]}\!\!\diagup_{I(H)}$ which we denote by A.

Now we discuss the elements of the field of fractions of A , denoted by K ,

$$K = \{ \frac{p}{q} : p \in A, q \in A - \{0\} \}$$

An element φ of K interpreted also as a function $\varphi : H = V(I) \to k$ defined by

$$\varphi(x) = \frac{p(x)}{q(x)}$$

if we can write $\varphi = \frac{p}{q}$, with $q(x) \neq 0$, and it is not defined at a point x if for each writing $\varphi = \frac{p}{q}$ we have $q(x) = 0$.

Let φ be an element of K defined at the point x, this implies that $\varphi = \frac{p}{q}$ such that $q(x) \neq 0$ and that $\varphi(x) = \frac{p(x)}{q(x)}$. The function φ is well defined because if $\varphi = \frac{p'}{q'}$ is an another writing such that $q'(x) \neq 0$. We have $p(x)q'(x) - p'(x)q(x) = (pq')(x) - (p'q)(x) = (pq' - p'q)(x) = 0$ because $pq' - p'q \in I$, hence $\frac{p}{q} = \frac{p'}{q'}$.

Definition 3.3. The elements of the field K above are called algebraic functions .

Proposition 3.4 : Let $\varphi \in K$, the set

$$H' = \{x \in V(I) : \varphi(x) \text{ is undefined } \}$$

is an algebraic subset include strictly in $V(I) = H$; that is $H' \subset H$.

Proof : Consider the set

$$J = \{q \in A : q\varphi \in A\},$$

we have the following properties :

(i) J is an ideal of A (it is easy to show that).

(ii) $J - \{0\}$ is the set of the possible denominators of the φ because

$$q \in J - \{0\} \Rightarrow q\varphi = p \in A \Rightarrow \varphi = \frac{p}{q} \text{ , also } \varphi = \frac{p}{q} \Rightarrow q \in J - \{0\}.$$

(iii) $J \neq \{0\}$ because if φ was written as $\frac{p}{q}$ we have $q \neq 0$ and $q \in J$.

(iv) $A = \{\varphi : J = A\}$ because for an element of A, we have $J = A$ by the stability of the multiplication in the ring A , hence $A \subset \{\varphi : J = A\}$; also if $\varphi \in \{\varphi : J = A\}$, $1.\varphi = \varphi \in A$.

The sequence

$$\{0\} \to I \to k[T_1,...,T_n] \xrightarrow{s} \overset{\displaystyle J \atop \displaystyle \downarrow}{A} \to \{0\}$$

is exact and $J \neq \{0\} \Rightarrow I \subset s^{-1}(J) \Rightarrow V(s^{-1}(J)) \subset V(I) = H$.

We need ,only , show that $V(s^{-1}(J)) = H'$, for this let $x \in H'$, we have $x \in V(s^{-1}(J)) \Rightarrow \forall Q \in s^{-1}(J), Q(x) = 0 \Rightarrow \forall q \in J, q(x) = 0$, since J is the set of the possible denominators of φ ; then $\varphi(x)$ is undefined and $x \in H'$. Then $V(s^{-1}(J)) \subset H'$.

If $x \in H' \Rightarrow \varphi$ is not defined i.e. for each denominator possible q of φ , $q(x) = 0$, since J is the set of the denominators possible , then for each $q \in J, q(x) = 0,$ i.e. for each representative Q of an element $q \in J, Q(x) = 0$; Since the set of the representatives of the elements of J is precisely $s^{-1}(J)$; hence for all $Q \in s^{-1}(J), Q(x) = 0$, from this we conclude that $x \in V(s^{-1}(J))$. By consequence we have well that $V(s^{-1}(J)) = H' \subset V(I) = H$.

Let $V = V(I)$ be an irreducible variety (I is a prime ideal) , A be the integral domain $k[T_1,...,T_n]/I$ and let K be the set of algebraic functions of A. If x is an element of V we define the set

$$\theta_x = \{\varphi \in K : \varphi(x) \text{ exists }\}$$

Theorem 3. 5. The set θ_x is a local ring .

Proof : It is obvious that θ_x is a sub ring of K. To prove that θ_x is a local ring , define the set

$$\mathfrak{M}(\theta_x) = \{\varphi \in K : \varphi(x) = 0\} = \{\varphi = \frac{p}{q} \in K : p(x) = 0 \wedge q(x) \neq 0\}.$$

In fact $\mathfrak{M}(\theta_x)$ is the unique maximal ideal of the ring θ_x. before to prove that it is a maximal ideal , firstly we prove that the set $\theta_x - \mathfrak{M}(x)$ is the set of invertible elements of θ_x .

Let then $\varphi \in \theta_x - \mathfrak{M}(\theta_x)$; there exist a writing $\varphi = \dfrac{p}{q}$ such that

$\varphi(x)=\dfrac{p(x)}{q(x)}$ with $q(x)\neq 0$. As $\varphi \notin \mathfrak{M}(x)$, we have $p(x)\neq 0$. By consequence there exist a writing $\varphi^{-1}=\dfrac{q}{p}$ such that $p(x)\neq 0, \varphi^{-1}$ exist and φ^{-1} is an element of θ_x which is inverse of φ .

Now if φ is invertible in θ_x , there exist $\varphi' \in \theta_x$ such that $\varphi\ \varphi' = 1$, hence $\varphi\varphi'(x)=\varphi(x)\varphi'(x)=1\neq 0$ and since k is an integral domain $\varphi(x)\neq 0,$ by consequence $\varphi \notin \mathfrak{M}(x)$.

Secondly , we prove that $\mathfrak{M}(x)$ is a maximal ideal . Let I be a non trivial ideal of θ_x such that $\mathfrak{M}(x)\subset I$; if this inclusion is strict , then the ideal I contains some invertible elements and we have $I=\theta_x$; this contradicts the hypotheses . Hence $\mathfrak{M}(\theta_x)=I$.

Finally, we prove that $\mathfrak{M}(\theta_x)$ is unique as a maximal ideal of θ_x . Let J be an another maximal ideal of θ_x , we have necessarily that $J\subseteq \mathfrak{M}(x)$,otherwise there exists $\varphi \in J-\mathfrak{M}(\theta_x)\subseteq \theta_x-\mathfrak{M}(\theta_x)$ which will be invertible , then we have $J=\mathfrak{M}(\theta_x)$ which is impossible . As J is a maximal ideal we have $J=\mathfrak{M}(\theta_x)$. By definition, θ_x is a local ring at a point x .

Remark 3.6. For each element x of $V=V(I)$, it is obvious that $\theta_x \supseteq A$. By consequence $A\subseteq \bigcap\limits_{x\in V}\theta_x$. We can asked, is this inclusion strict ? the answer is yes because if we take

$$A=\mathbb{R}[X,Y]\big/(X^2+Y^2-1)=\mathbb{R}[x,y]\ ,$$

where x,y are the classes of X , Y respectively , we have

$$\frac{1}{x-2}\in \bigcap_{x\in V}\theta_x \ \text{ and } \ \frac{1}{x-2}\notin A\ .$$

Proposition 3.7. Let H be an algebraic set , then H is irreducible if and only if every finite intersection of nonempty open sets is a nonempty open set .

Proof : In fact H is irreducible if and only if H is not a finite union of strict closed sets and by taking the complementary , it is equivalent to say that the empty set is not a finite intersection of nonempty open sets , hence every finite intersection of such open sets is a nonempty open .

In return to the local ring $\theta_x = \{\varphi \in K : \varphi(x)$ is defined $\}$ of a point x of irreducible variety V .

Let now U be a nonempty open set of V . We name

$\Gamma(U, O_V)$ the set of defined algebraic functions on U . Then

$$\Gamma(U, O_V) = \{\varphi \in K : \varphi \text{ is defined on } U\}$$

we have immediately the following properties :

(i) $\Gamma(U, O_V) = \bigcap_{x \in U} \theta_x$

(ii) if $U = V$ we have $A \subseteq \Gamma(U, O_V)$

Definition 3.8 [6]. Let $H = V(I)$ and $H' = V(I')$ be two algebraic sets . The application $\varphi : H \to H'$ is said to be a morphism from H to H' if for each $f' \in A' = k[T'_1,\ldots,T'_m]/I'$, $f' o\varphi \in A = k[T_1,\ldots,T_n]/I$.

Remark 3.9. Let $\varphi : H \to H'$ be a morphism from the irreducible variety H to the irreducible variety H' . We can considered the application

$$o\varphi : A' \to A \text{ defined by}$$

$$(o\varphi)(f') = f' o\varphi$$

It is easy to verify that $o\varphi$ is a homomorphism of $k-$algebras , that is

$$(f'_1 + f'_2) o\varphi = f'_1 o\varphi + f'_2 o\varphi \quad \text{and} \quad (f'_1 . f'_2) o\varphi = (f'_1 o\varphi) . (f'_2 o\varphi)$$

So, to a morphism of varieties φ , we have doing a corresponding homomorphism of $k-$algebras $o\varphi$. Is the converse possible ?

Let $\phi : A' \to A$ be a homomorphism of $k-$algebras, we have

$$\phi : \; k[T'_1,\ldots,T'_m]/I' \to k[T_1,\ldots,T_n]/I ,$$

let $t'_1,\ldots,t'_m$ be the classes modulo $T'_1,\ldots,T'_m$ respectively , then we have the functional interpretation

$$t'_j : H' \to k \text{ defined by}$$

$$t'_j(x) = x'_j \;, \text{ where } x = (x'_1,\ldots,x'_m)$$

Suppose that $\phi(t'_j) = \varphi_j$, the φ_j can be considered as functions

$$\varphi_j : H \to k \text{ defined by}$$

$$\varphi_j(x) = \Phi_j$$

where Φ_j is a representative of φ_j in $k[T_1,...,T_n]$.

Now we define the application $\varphi : H \to H'$ by

$$\varphi(x) = (\varphi_1(x), \varphi_2(x), ..., \varphi_m(x)) \in k^m .$$

(1) Is $\varphi(x) \in H'$?

Let F' be a polynomial belong to I' , we want to show that $F'(\varphi(x)) = F'((\varphi_1(x),...,\varphi_m(x)) = F'(\Phi_1(x),...,\Phi_m(x)) = 0$, we have

$$F' = \Sigma\, \lambda_{j_1 ... j_m}\ T_1'^{\,j_1} \ldots T_m'^{\,jm}$$

then

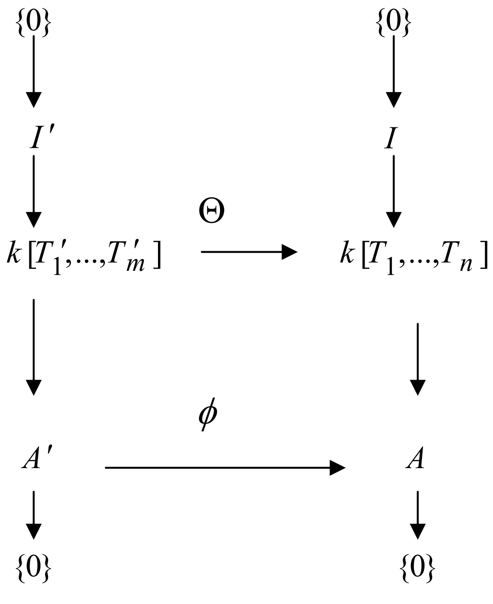
$$F'(\Phi_1(x),...,\Phi_m(x)) = \Sigma\, \lambda_{j_1 \cdots jm}\ \Phi_1^{\,j_1}(x_1,...,x_n) \ldots \Phi_m^{\,jm}(x_1,...,x_n)$$

$$= (\Sigma\, \lambda_{j_1 \cdots jm}\ \Phi_1^{\,j_1} \ldots \Phi_m^{\,jm})(x_1,...,x_n)$$

Consider the application $\Theta : k[T_1',....,T_m'] \to k[T_1,...,T_n]$ defined by

$$\Theta(T_j') = \Phi_j\ ,$$

we have the following commutative diagram



This diagram allowed us to verify that an element F' of I' is such that $s'(F') = 0$, then $\phi o s'(F') = 0 = s\, o\, \Theta(F')$, that is $\Theta(F') \in I$.

So that $F'(\Phi_1(x),...,\Phi_m(x)) = (\Sigma\, \lambda_{j_1 \cdots jm}\ \Phi_1^{\,j1} \ldots \Phi_m^{\,jm})(x_1,...,x_n)$

$$= \Theta(F')(x_1,\ldots,x_n) = 0$$

hence $F'(\varphi(x)) = 0$ and $\varphi(x) \in H'$.

(ii) Is φ a morphism of algebraic sets ?

The answer is yes because each component function is continuous ,since it is a polynomial function .

(iii) Is $o\varphi = \phi$?

Let $f' \in A'$, is $f'o\varphi \in A$? we have the following figure

$$\begin{array}{ccc} H & \xrightarrow{\varphi} & H' \\ & & \downarrow f' \\ & & k \end{array}$$

where φ is given by $\varphi(x_1,\ldots,x_n) = \varphi(x) = (\Phi_1(x),\ldots,\Phi_m(x))$.

As f' is an element of A', we have

$f' = \Sigma \lambda_{j_1 \cdots j_m} t_1'^{j_1} \ldots t_m'^{j_m}$, hence $f'o\varphi(x) = f'(\varphi_1(x),\ldots,\varphi_m(x))$

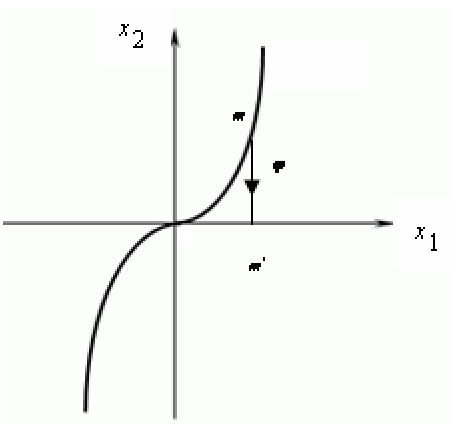
$$f'o\varphi(x) = \Sigma \lambda_{j_1 \cdots j_m} \varphi_1^{j1}(x_1,\ldots,x_n) \ldots \varphi_m^{jm}(x_1,\ldots,x_n)$$

$$= (\Sigma \lambda_{j_1 \cdots j_m} \varphi_1^{jm} \ldots \varphi_m^{jm})(x_1,\ldots,x_n)$$

$= \phi(f')(x)$,because we saw that $\phi(t_j') = \varphi_j$

By consequence $f'o\varphi = \phi(f')$ belong to A and we have proved that $o\varphi = \phi$.

We can conclude that the application $\varphi \mapsto o\varphi$ is a bijection from the set $\mathrm{Mor}(H,H')$ of morphism of the irreducible variety H into the irreducible variety H', onto the set $\mathrm{Hom}(A',A)$ of corresponding $k-$algebras.



Example 3.8 : $\varphi : V(X_2 - X_1^3) \to V(X_2)$

$$o\varphi : {k[X_1, X_2]}/{X_2} \to {k[X_1, X_2]}/{(X_2 - X_1^3)}$$

$$X_1 \mapsto o\varphi(X_1) = X_1$$

Now we discuss the continuity of a morphism of algebraic sets . Let $\varphi : H = V(I) \to H' = V(I')$ be a morphism of algebraic sets . The Zariski topology of k^n induce topological structures on H and H'.We show that φ is a continuous application .

Firstly we remember that , to φ there is a corresponding homomorphism of $k-$algebras

$$\phi : A' = {k[T'_1, ..., T'_m]}/{I'} \to {k[T_1, ..., T_n]}/{I} \text{ defined by}$$

$$f' \mapsto \phi(f') = f' o \varphi$$

and if we suppose that $\phi(t'_j) = t'_j o\varphi = \varphi_j$ we reconstruct an application

$$\Theta : k[T'_1, ..., T'_m] \to k[T_1, ..., T_n] \text{ defined by}$$

$$T'_j \mapsto \phi(T'_j) = \Phi_j \text{ a representative of } \varphi_j .$$

It is evident that $\varphi(x) = (\varphi_1(x), ..., \varphi_m(x))$.

Now let $H'_1 = V'(J')$ be a closed set of H' ; we need to show that $\varphi^{-1}(H'_1)$ is a closed set of H , that is an algebraic set .

We show that $\varphi^{-1}(H'_1) = V((\Theta(J')))$:

Let $x \in \varphi^{-1}(H'_1) = \varphi^{-1}(V'(J'))$, $\forall F \in (\Theta(J'))$, $F = \sum_{i \in I \, finite} F_i \Theta(F'_i)$ with $\forall i, F'_i \in J'$, $F(X) = \sum_i F_i \, \Theta(F'_i(x))$,

$$\Theta(F'_i)(x) = F'_i(\Phi_1(x), ..., \Phi_m(x)) = F'_i(\varphi_1(x), ..., \varphi_m(x)) = F'_i(\varphi(x)),$$

where and as $\varphi(x)$ belong to H'_1 and F'_i belong to J' , we have $F'_i(\varphi(x)) = 0$. By consequence $F(X) = 0$ and $x \in V((\Theta(J')))$. Then $\varphi^{-1}(H'_1) \subseteq V((\varphi(J')))$.

Conversely let $x \in ((\Theta(J')))$. For every F' be an element of J' , $\Theta(F')(x) = 0$ because $\Theta(F') \in \Theta(J')$ where $\Theta(F')(x) = F'(\varphi(x))$; then $\varphi(x) \in H'_1$ that is $x \in \varphi^{-1}(H'_1)$. Consequently, $V((\Theta(J'))) = \varphi^{-1}(H'_1)$ and the application φ is continuous because the inverse image of a closed set of H' is a closed set of H .

4 Conclusion and Open Problem

Having defined the local ring θ_x which will play an important role in Abstract Algebra . This is a suitable point for us to introduce the concept of localization. The local ring θ_x defined above as a subring of the function field K. This paper is about algebraic variety and local ring at a point x of an affine algebraic variety H . The question is , does exist a general notion of an Abstract Algebraic Variety ? If yes , what is its local ring ?

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