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Events, Actions and Temporal Logic

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Abstract

In this paper, we have introduced a formalism to represent the temporal relationship between the events and the actions. We have used classes of equivalence to represent the set of actions that happen at the same time or the time of a process. We defined operators who allows us to enumerate all the events that proceed in the future caused by an event e and all the events that proceeded in the past and gave place to an event e. We defined an operator that give us the possibility of representing the continuous evolutions of the universe for varied futures (prediction) or past (diagnostic). It might allow us the representation of the actions and their effects as well as the types of reasoning which are the prediction, the explanation and planning.

Keywords: *Artificial Intelligence, Description Logic, Knowledge Representation, Reasoning on the Actions, Temporal Logic.*

1 Introduction

The changes are caused by events and certain events can be expressed like actions. The concepts of changes and time are closely related. There is a relation between the events, the necessary actions for the realization of these events and the execution time of these actions.
Our objective is to reflect on the actions in order to anticipate, to planify and repair, accordingly, execute actions to prevent some evolutions deemed harmful. Also, to favor some desired evolutions or to remedy a risky situation.
In this paper , we have introduced a formalism to represent the temporal causal

relationship between the events and the actions which are the cause of these events. We proposed a temporal logic to reason on the actions; the events which are the cause of several events as well as the events which are due to several events proceeding in the past. We have used classes of equivalence to represent the set of actions which happen at the same time or the time of a process. We defined operators to enumerate all the events which will proceed in the future caused by an event e and all the events that proceeded in the past and gave place to an event e. Also we have defined an operator to give us the possibility of representing the continuous evolutions of the universe for varied futures (prediction) or past (diagnostic). It might allow the representation of the actions and their effects as well as the types of reasoning as the prediction, the explanation and planning.

2 Language, notation and terminology

Definition 2.1 *Actions* $a_1, a_2, ..., a_m$ *are said to be the cause of an event e if as soon as one of these actions is not carried out, the event is not executed.*

To represent this, we need to introduce the following language which is a first order language with equality :

- Connectors:$\neg, \vee, \wedge, \supset$ *and* $\supset_c$ (causal implication)
- Two signs of quantification noted $\exists$ and$\forall$.
- A symbol of equality, which we will note $\equiv$ to distinguish it from the sign $=$.
- A countable infinite collection of propositional variable.
- A set of operational signs or symbols functional.
- Three unary temporal operators: P_k (past), F_k (future), and P_0 (present).
- The expressions are the symbol strings on this alphabet.
- The set of the formulas noted Φ is by definition the smallest set of expressions which checks the following conditions :
    - Φ contains the propositional variables.
    - A set of elements called symbols of individuals.
    - If A and B are elements of Φ it is the same for $\neg A$ and $A \supset_c B$.
    - If A is an element of Φit is the same for P_k A, $F_k A$ and $P_0 A$.

The language, equally, contains :

- A set of elements called symbols of individuals.
- A set of operative signs or functional symbols.
- A set of relational signs or symbols of predicates.

To introduce causality J. Allen [4] uses the following formula: $Ecause(p_1, i_1, p_2, i_2)$.

It expresses, thus, the fact that p_1 which occurs in i_1 caused the event p_2 which occurs in i_2.

To express that an action a is the cause of an event e, we use the predicate $Ecause(a; e)$ and that if a is an atemporal expression of action type.
An action can be instantaneous as it can be carried out during in a certain interval of time [2], [3]. Consequently, the points and the intervals are necessary to express the execution time of an action.
We call time-element an interval or a point of time. Therefore, an action operates during a time-element [2], [3].
If a is a temporal expression of action type we use the following formulas :

- $t \cdot a$ if a is produced in the past at the element of time t.
- $a \cdot t$ if a it happens in the future at the element of time t.

We will keep the same notations in the case of an event e, ($e \cdot t$ for the future and $t \cdot e$ for the past).

Example 2.2

(a) $Colloquium \cdot May$,means: the colloquium will be held in May.

(b) $May \cdot Colloquium$, means: the colloquium was held in May.

Let T a non empty set of time- elements, A a set of actions, $A \cdot T$ (respectively $T \cdot A$) the set of elements $a \cdot t$ (respectively $t \cdot a$) and Dur; an application from $A \cdot T$ to IR_+ (respectively from $T \cdot A$ to IR_+) defined by [2], [3]:

$$\begin{cases} Dur(a \cdot t) = 0 & \text{if} \quad a \text{ is an instantaneous action, thus, } t \text{ is a point of time .} \\ Dur(a \cdot t) > 0 & \text{if} \quad a \text{ is a durative action, thus, } t \text{ is an interval.} \end{cases}$$

T is, thus, the union of two sets P and I, I is a set whose elements are intervals and P a set whose elements are points of time [5].
If a is an action carried out in t' then the predicate $Ecause(a.t'; e.t)$ expresses the fact that a carried out in t' is the cause of e true in t.

The actions seem first argument of the Ecause predicate.

The fact that actions $a_1, a_2, ..., a_m$ are the cause of an event e is expressed by the formula: $Ecause(a_1, a_2, ..., a_m; e)$ defined by $Ecause(a_1, a_2, ..., a_m; e) \equiv Ecause(a_1; e) \wedge ... \wedge Ecause(a_m; e)$ where $a_1, a_2, ..., a_m$ are the atemporal expressions of actions type. This formula can be expressed as : $Ecause(a_1, a_2, ..., a_m; e) \equiv ((\exists k)(\neg a_k \supset_c \neg e))$.

Example 2.3 *Ecause(prepare one's paper, traveling, ..., communicate)* $\equiv$ *(¬ traveling)* $\supset_c$ *(¬ communicate)* $\vee$*(¬ no prepare paper)* $\supset_c$ *(¬ communicate).*

If $a_1, a_2, ..., a_m$ are the temporal expressions of actions type carried out respectively in $t_1, t_2, .., t_m$, we use the formula : $Ecause(a_1.t_1, a_2.t_2, ..., a_m.t_m; e.t) \equiv Ecause(a_1.t_1; e.t) \wedge ... \wedge Ecause(a_m.t_m; e.t)$.

Example 2.4 *Ecause(January. prepare one's paper, send paper. April, ..., traveling.15May; Communicate.18 June)* $\equiv$ *Ecause(January. prepare one's paper; communicate.18 June)*$\wedge ... \wedge$ *Ecause (traveling.15 May;communicate.18 June).*

Example 2.5 *The fact of traveling on Monday to communicate on Thursday can be expressed as follows :*

(a) Ecause(traveling .Monday; communicate .Thursday) expresses: the agent will travel on Monday in order to communicate on Thursday.

(b) Ecause(Monday.traveling; communicate.Thursday) expresses: the agent traveled on Monday in order to communicate on Thursday.

(c) Ecause(Monday. traveling; Thursday. communicate) expresses: the agent traveled on Monday and communicated on Thursday.

The actions are the builders of events, so we cannot have:
Ecause(traveling. Monday; Thursday .communicate)

An action a can be primitive as it can be complex. In the case of a complex action to express that the actions $a_{i_1}, ..., a_{i_s}$ carried out in $t_{i_1}, ..., t_{i_s}$ are the cause of a_i realized in t_i and this one will give place to the event e carried out in t we use the following expression

$$\begin{aligned} Ecause(a_i.t_i; t.e). &\equiv Ecause(a_{i_1}.t_{i_1}, a_{i_2}.t_{i_2}, ..., a_{i_s}.t_{i_s}; e.t) \\ &\equiv Ecause(a_{i_1}.t_{i_1}) \wedge Ecause(a_{i_2}.t_{i_2}) \wedge ... \wedge Ecause(a_{i_s}.t_{i_s}; e.t) \\ &\equiv \bigwedge_{j=1}^{j=s} Ecause(a_{ij}.t_{ij}; e.t). \end{aligned}$$

To represent the connection which links a_n to its effect, we define the following application :

$$\begin{array}{rcl} \Psi : A \times A \times ... \times A & \longrightarrow & E \\ (a_1, a_2, \ldots, a_m) & \longmapsto & a_1 \wedge a_1 \wedge \ldots \wedge a_m \equiv e. \end{array}$$

where E is the set of the events, A: the set of the actions and $a_1, a_2, \cdot\cdot\cdot, a_m$ are the actions which are the cause of the achievement of these events.

J. A. Pinto [6] established in his thesis a relation between events, actions and situations but he finds it more convenient to establish a relation between events, actions which occur for the realization of these events and the time when they are carried out. Indeed, there is a relationship between time, the events and the actions which are the cause of their achievement. We can see that in the following diagram:

$$\begin{array}{ccc} A \times A \times ... \times A & \xrightarrow{\Psi} & E \\ \varphi \downarrow & & \uparrow \phi \\ T \times T \times ... \times T & \longrightarrow & T \end{array}$$

T is the set of the time-elements t_i where an action a_i is carried out so that an event e occurs or is true in a time-element t and h is an application defined as follows :

$$\begin{array}{rcl} h : T \times T \times ... \times T & \longrightarrow & T \\ h(t_1, t_2, \cdot, \cdot, \cdot, t_m) = t_1 \oplus t_2 \oplus \cdots \oplus t_m \equiv t. & & \end{array}$$

$\oplus$ is an operator defined on T as follows :

$t_1 \oplus t_2$ is defined if there are two actions a_1 and a_2 taking place in t_1 and t_2 respectively and which are the cause of an event (or fact) e carried out in a point of time t.

This operator has the following characteristics:

$\star$ The operator $\oplus$ is internal if $t \in T$ (the agent must act so that the event takes place in time-element t belonging to T).

$\star$ The operator is commutative if the order of the actions does not intervene (the agent is free to start with any action). We denote: $t_1 \oplus t2 \equiv t_2 \oplus t_1$.

The intervening order of the actions in some events plays a significant role; like carrying out an action before another, reproduction of an action (process)or to carry out several actions at the same time. This led us to introduce operators on the actions. These operators define constraints over time.

We define on T a relation of precedence noted R_c as follow : $t_1\ R_c\ t_2$ or rather t_1 precedes t_2 if the action a_1 must occur before the action a_2 (a_1 and a_2 being the actions which are the cause of e). The relation (T, R_c) is a strict order temporal framework. (T, R_c) and has the property of discretion, than (T, R_c) is a discrete temporal framework provided with a strict order.

An event can be the cause of one or more events in the future as it is often due to one or more events which proceeded in the past.

To express that the following operator is defined :

$$\begin{array}{rccl} \otimes : & \mathbb{Z} \times T & \rightarrow & T \\ & (k,t) & \mapsto & \otimes(k,t) \equiv k \otimes t \end{array}$$

- If $k = 0$, then $k \otimes t = {}_0t$ where ${}_0t = t_1 \oplus t_2 \oplus \cdots \oplus t_m$ is time-element where the event e occurs at the present and where m is the number of actions which are the cause of e true in ${}_0t$. We denote $e = P_0 e$.

- If $k > 0$ then $k \otimes t{=}_k t$ where ${}_kt$ is time-element where the event $F_k e$ will occur in the future and which is due to e carried in ${}_0t = t_1 \oplus t_2 \oplus \cdots \oplus t_m$.

- If $k < 0$ then $k \oplus t{=}^k t$ where ${}^kt$ is time-element where the event denoted $P_k e$ which occurred in the past and gave place to e in ${}_0t = t_1 \oplus t_2 \oplus \cdots \oplus t_m$. Here, m is the intervening number of actions so that e is true in ${}_0t$, consequently, $F_k e$ (respectively $P_k e$) is true in ${}_kt$ (respectively in ${}^kt$). $|k|$ is the number of events $F_k e$ (respectively $P_k e$).

The operator F_k will allow us to enumerate all the events that proceed in the future whereby e is the cause and the operator $P_k e$ will allow us to enumerate all the events which proceeded in the past and which gave place to e. The operator $\otimes$ may give us the possibility of representing the continuous evolutions of the universe for varied futures (prediction) or past (diagnostic). It may allow the representation of the actions and their effects as well as the types of reasoning which are the prediction, the explanation and planning.

3 Temporal Logic L_c

We propose a temporal logic to reason on the actions.

3.1 deductive system

The axioms of the temporal logic L_c are:

(i) Axioms of propositional logic [7].

(ii) (a) $F_k(A \supset_c B) = (F_k A) \supset_c (F_k B)$ where $F_k(A \supset_c B)$ is the event which will occur in the future and which will take place only if $A \supset_c B$ takes place ($A \supset_c B$ is due to m actions $a_1, a_2, \cdots, a_m$)

(b) $P_k(A \supset_c B) = (P_kA) \supset_c (PkB)$ where P_kA is an event which occurred in the past and which gave place to $(A \supset_c B)$

(c) $P_0(A \supset_c B) = (P_0A) \supset_c (P_0B)$.

The axioms (ii) : (a), (b) and (c) express the distributivity of the operators F_k, P_k and P_0 with regard to the causal implication.

The rules of deductions are :

(i) The modus ponens [7].

(ii) Temporal generalization: If A is a theorem, F_kA, P_kA and P_0A are equally theorems.

The theorems of L_c are by definition all the formulas deductible from the axioms by using the rules of deductions. In particular all the theorems of propositional calculus are theorems.

3.2 semantic of L_c

In the semantic of propositional calculus, an assignment of values of truth V is an application, that each propositional variable associates a value of truth. An assignment of value of truth describes a state of the world.

In the case of L_c, we choose as variable propositional the actions which are the cause so that an event e is true in a time-element t.

Let V_c the valuation defined on the framework temporal (T, R_c) :

$$\begin{array}{rccl} V_c : & A & \to & P(T) \\ & ai & \mapsto & V_c(ai) = T^i = \{t_i / a_i true in t_i\} \end{array}$$

t_i is the time-element when the action a occurs so that the event e is true in ${}_0t = t_1 \oplus t_2 \oplus \cdots \oplus t_m$.

The action a_i thus, occurs only once in T then $T^i = t_i$.

The case of an action which reproduces in T will be studied later on.

If T^i is empty then, a_i is not true in t_i, consequently e has will not be carried out in ${}^0t$.

Definition 3.1

1. $V_cP_0e = V_c(e) = V_c(a_1 \wedge \cdots \wedge a_m) =_{def} V_c(a_1) \oplus \cdots \oplus V_c(a_m) \equiv \{t_1\} \oplus \{t_2\} \oplus \cdots \oplus \{t_m\} \equiv \{{}_0t\}$

2. $V_c\{\neg a_i\} = T - V_c\{a_i\} = T - T_i$

3. *As e is due to the actions* $a_1, a_2, \cdots, a_m$*, thus, if there is k such as an action* a_k *does not take place, this would inevitably involve non-achievement of e (or that e will not be true in* $\{_0t\}$ *accordingly :*

   $V_c\{e\} = V_c\{a_1 \wedge \cdots \wedge \neg a_k \wedge \cdots \wedge a_m\}= V_c\{a_1\} \oplus \cdots \oplus V_c\{\neg a_k\} \oplus \cdots \oplus V_c\{a_m\}= T_1 \oplus \cdots \oplus \{T_k\} \oplus \cdots \oplus T_m \equiv T - V_c(e)$.

4. *The event e can give place to several events in the future noted* $F_k e$, $k \geq 1$*, and each event will occur in a time-element* $_kt$ *with the following condition:*

   $t_i\ R_c\ {_0t}\ R_c\ {_kt}$ *and* $_0t\ = t_1 \oplus t_2 \oplus \cdots \oplus t_m$ *then* $V_c(F_k e) = \{_kt\ /t_i\ R_c\ {_0t}\ R_c\ {_kt},\ {_0t}\ = t_1 \oplus t_2 \oplus \cdots \oplus t_m\}$.

5. *the event e can be due to several events* $P_k e$ *which occurred in the past and each event* $P_k e$ *occurred in a time-element* $_kt$ *with the following condition :*

   $t_i\ R_c\ {_0t}\ R_c\ {_kt}\ {_0t}\ = t_1 \oplus t_2 \oplus \cdots \oplus t_m$ *and therefore :* $V_c(P_k e) = \{^kt\ /t_i\ R_c\ {_0t}\ R_c\ {^kt},\ {_0t}\ = t_1 \oplus t_2 \oplus \cdots \oplus t_m\}$.

6. $Vc(A \supset_c B) = \{t/t_A\ R_c\ t_B\ R_c\ t,\ {_0t} = t_1 \oplus t_2 \oplus \cdots \oplus t_m\}$,

   indeed $(A \supset_c B)$ *is true in a certain time-element t pertaining to* T *only if* A *is true in one time-element* t_A *of* T*; but* A *true in* t_A *is the cause of* B *true in* t_B*, thus, to have* B *in* t_B *it is enough to have* A *in* t_A *and this will give* $A \supset_c B$ *true in* t.

7. $V_c\{A \cap B\} = V_c(A) \wedge V_c(B)$.

8. $V_c(A \sqcup B) = V_c(A) \vee V_c(B)$.

We also define the valuation in the following cases :

- Case of the events that require the realization of several actions at the same time. For that we define on A a relation defined as follows :

$$a_1 R_c a_2 \Leftrightarrow V_c(a_1) = V_c(a_2) \Leftrightarrow t_1 = t_2.$$

It will ,thus, be said that a_1 and a_2 are in relation if they occur in even time. R_c is a relation of equivalence.

We have the following diagram [8]:

$$\begin{array}{ccc} A & \xrightarrow{V_c} & P(T) \\ s\downarrow & & \uparrow i \\ A/R_c & \xrightarrow{\overline{V_c}} & ImV_c \end{array}$$

$\overline{V}_c(\overline{a}) = V_c(a)$, $i(t_1) = \{t_1\}$ and $s(a) = \overline{a} = \{a' \in A/a'R_ca\}$ is the class of equivalence of a, it contains all the actions which occur at the same time as a, $ImV_c = \{s(a), a \in A\}$is a subset of $P(T)$ and A/R_c is the set of the classes of equivalence of the elements of A, it, thus ,contains the sets of the actions which occur in even-time.

We can, thus, represent the set of the actions occurred at the same time by the class of equivalence of an action that is the representative of the class.

- Case of an action which is repeated in different time-element (process). Let

$$f: \begin{array}{ccc} T & \rightarrow & A \\ t_i & \mapsto & a_i \end{array}$$

We define on T a relation :

$$t_1 R_c t_2 \Leftrightarrow a_1 = a_2$$

it will ,thus, be said that t_1 and t_2 are in relation if the same action a occurs in t_1 and t_2. R_c is a relation of equivalence. We have the following diagram [8] :

$$\begin{array}{ccc} T & \xrightarrow{f} & A \\ s\downarrow & & \uparrow i \\ T/R_c & \xrightarrow{\overline{f}} & Imf \end{array}$$

$T/R_c = \{\overline{t}/t\ inT\}$, is the set of the classes of equivalence, Imf is the set of images of the elements of T, $\overline{t} = \{t_i\ inT/tR_ct_i\}$ is the class of equivalence of t, it contains all the time-elements t_i where an action a produced in t and is reproduced in other time-element t_i (process).

Therefore, we represent the set of the time-elements when an action is repeated by the class of equivalence of a time-element that is the representative of the class. For this case one defines a valuation

$$V_c: \begin{array}{ccc} A & \rightarrow & P(T) \\ a & \mapsto & V_c(a) = \{t_i \text{ a true in } t_i\} \end{array}$$

- Case of the concurrent actions. Let a and a' two actions concurrent for the realization of an event e. We have two possibilities for the choice of the actions.

  (i) The agent is interested by the first action carried out (temporal choice)

  (ii) The agent is interested by the simplest action to carry out.

For that we define on A the following relation : $a\ R_c\ a' \Leftrightarrow a'$ is negligible in front of a for the realization of e. In the first case a' negligible in front of a , it expresses the fact that action a is the first carried out. So, it is the action chosen by the agent. On the other hand, the agent is interested by the simplest action to carry out, a' negligible in front of a will express the fact that a is simpler action to realize than a'.

We define a valuation:

$$\begin{array}{rcll} V_c: & A & \rightarrow & P(T) \\ & a & \mapsto & V_c(a) = \{t_a / atrueint_a\} \end{array}$$

$V_c(a) = \{ta\}$ if a' is negligible in front of a if not $V_c(a) = \emptyset$.

We can generalize this with several actions a_1,a_2, $\cdots$, a_m.
$V_c(a_i) = \{t_{a_i}\}$ if a_j is negligible in front of a_i for any $j \neq i$ if not $V_c(a_i) = \emptyset$.

4 Conclusion and Open Problem

In this paper, we have introduced a formalism to represent the temporal causal relationship between the events and the actions which are the cause of these events. We have used classes of equivalence to represent the set of actions which happen at the same time or also the time of a process. We defined operators that allow enumerate all the events that proceed in the future and whose event e is the cause and all the events that proceeded in the past and which gave place to an event e. Also we have defined an operator which could give us the possibility of representing the continuous evolutions of the universe for varied futures (prediction) or past (diagnostic). It might allow us the representation of the actions and their effects as well as the types of reasoning which are the prediction, the explanation and planning.

F.Baader and al propose a formalism of action based on description logics [9] They make a first proposal for an action formalism in which the states of the world, the pre and post-conditions can be described by using Dl-concepts. The idea to investigate action formalisms based on description logics was inspired by the expressivity space between existing action formalisms. To represent the temporal dimension, classical Description Logics are extended with temporal constructors; thus a uniform representation for states, actions and plans is provided. H.Strass and M.Thielscher study the integration of two prominent fields of logic- based AI: action formalisms and non-monotonic reasoning. The resulting framework allows an agent employing an action theory as internal world model to make useful default assumptions. They show that the mechanism behaves properly in the sense that all intuitively possible conclusions can be drawn and no implausible inferences arise. In particular, it suffices to make default assumptions only once (in the initial state) to solve projection problems [10].

H. Liu, have investigated updates of ABoxes in DLs and analyzed their computational behavior. The main motivation for this endeavor is to establish the theoretical foundations of progression in action theory based on DLs and to provide support for reasoning about action in DLs [11].

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