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An Enemy- Ammensal Species Pair With Limited Resources –A Numerical Study

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Abstract

This paper deals with a mathematical model of an Ammensal-enemy species pair surviving with limited resources. This model is characterized by a pair of first order non-linear coupled differential equations. The nature of variation of reversal time (t*) of dominance for stability is established by classical Runge-Kutta method of fourth order. Some observations are made from the relationship between the carrying capacities of Ammensal species and enemy species through the reversal time of dominance.

AMS Classification: 92 D 25, 92 D 40.
Keywords: *Equilibrium point, Equilibrium state, stability, carrying capacity, Reversal time of dominance.*

1 Introduction

Mathematics is the science of calculations. Ever since its inception it has been extending its yeoman services for the development of science and technology in general and engineering in particular. It has become the backbone of modern scientific development. It has extended its sphere with manifold dimensions and every branch of mathematics has its own importance. One of such branches is Mathematical modeling. Mathematical modeling is an important interdisciplinary activity which involves the study of some aspects of diverse disciplines. Biology, Epidemiodology, Physiology, Ecology, Immunology, Bio-Economics, Genetics, Pharmacokinetics are some of those disciplines. This mathematical modeling has raised to the zenith in recent years and spread to all branches of life and drew the attention of every one. Research in theoretical ecology was initiated by Lotka [7] and by Volterra [10].Many mathematicians and ecologists with their zeal and

quest followed them contributing their might to the growth of this area of knowledge as reported in the treatises of Meyer [8], Kushing [4], Paul colinvaux [9], Kapur [4] etc. The ecological interactions can be brodly classified as Prey – predation, competition, Commensalism, Ammensalism, and Neutralism and so on. Lakshmi Narayan and Pattabhi Ramacharyulu [6] studied Prey-predator ecological models with a partial cover for the prey and alternate food for the predator. Some studies on stability analysis of competitive species was carried out by Archana Reddy, Pattabhi Ramacharyulu and Gandhi [3]. Acharyulu,K.V.L.N and Pattabhi Ramacharyulu[1] investigated some results on stability of an enemy and Ammensal species pair with various resources. Acharyulu and Pattabhi Ramacharyulu[1,2] investigated some results on stability of an enemy and Ammensal species pair with limited resources with all the four equilibrium points and the criteria for the stability are discussed in all cases

The present investigation is related to an numerical study on reversal time (t*) of dominance with carrying capacities of Ammensalism between two species. Ammensalism is the interaction between two species where one species restricts the success of the other and the other species has no effect on it. In other words it is a type of symbiosis. In this process one organism expels a chemical compound in its normal metabolism and that becomes detrimental to another organism. A common example for this is the bread mould pencillium and another example is Black wall nut tree.

The Ammensal species (S_1), in spite of its natural resources, declines in its strength from the enemy species (S_2) which is not effected by S_1. .This model is characterized by a coupled pair of first order non-linear differential equations. The variation of reversal time (t*) of dominance is obtained by Runge-Kutta method of fourth order (RK method of fourth order). Some conclusions are made on the relation between the carrying capacites of Ammensal species and enemy species with respect to the reversal time of dominance.

Notation adopted:

$N_1(t)$: The population of the Ammensal species.

$N_2(t)$: The population of the enemy species

a_i : The natural growth rate of N_i, i = 1, 2

a_{ii} : The rate of decrease of N_i, due to insufficient resources of N_i, i = 1, 2

$K_{ii} = \frac{a_i}{a_{ii}}$: Carrying capacities of N_i, i = 1, 2

t* : The reversal time (t*) of dominance of one species over the other

where the variables N_1 and N_2 are non-negative and the model parameters a_1, a_2, a_{11}, and a_{12} are assumed to be non-negative constants.

2 The Ammensal species (S_1) and the enemy species(S_2) with limited resources

3 The solutions of the model (I) obtained by the classical Runge-kutta method of fourth order (considered $a_1 > a_2$)

4 Effect of reversal time of dominance on the carrying capacities of Ammensal species and enemy species

5 Figures from (13) to (18)

6 Conclusions

The conclusions of above observations are cited in the following Table-10

Table-10

S.NO.	CRITERION	CONCLUSION
1	**t* increases and k_{11} also increases (no restiction on N_{10} and N_{20}) (Table-4 & Figure-13)**	K_{22} has varied fluctuations over the given interval
2	**t* increases and k_{11} increases (withN_{10}>N_{20}) (Table-5 & Figure-14)**	K_{22} rises and falls sharply initially, after a marginal growth increases considerably and maintains the same value with a slight and steady decrease.
3	**t* increases and k_{11} increases (withN_{10}<N_{20}) (Table-6 & Figure-15)**	K_{22} grows and falls initially and remains the same in the remaining part ot the taken interval .
4	**t* decreases and k_{22} increases (withN_{10}>N_{20}) (Table-7 & Figure-16)**	K_{11} falls down sharply intially and has a very nominal growth in the interval considered .
5	**t* increases and k_{22} increases (withN_{10}<N_{20}) (in Table-8 & Figure-17)**	K_{11} has ups and downs in the given interval
6	**t* decreases and k_{22} increases (withN_{10}<N_{20}) (in Table-9 & Figure-18)**	K_{11} comes down steadily with negligible growth towards the end of the specified interval. .

7 Open Problems

i) The same technique can also be applied in the ecosystem consisting of a prey ,a predator with limited resources and an enemy Ammensal to the prey with various resources

ii) In this paper we have investigated some results on the stability in a Mathematical model of two species by the classical Runge-Kutta method of fourth order. One can apply the same technique of Picard's method for a four species ecosystem.

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