Int. J. Open Problems Compt. Math., Vol. 3, No. 4, December 2010
ISSN 1998-6262;
www.i-csrs.org

Numerical Solution of Geothermal Heat Exchangers Formulation

MOHAMMED AWWAD ALI AL-DABBAS

Mutah University
Assistant, prof., madabbas@yahoo.com

Abstract

Geothermics has an increasing importance for energy supply worldwide. Hence, there is also an increasing need to model different geothermal scenarios. Depending on the type of problem it may be necessary to take density coupled processes into account. Furthermore, thermal dependence of material properties should be considered.

Special problems occur in cases of fracture flow, which can be of high importance with respect to productivity.

Finally, the simultaneous modeling of heat and mass transport processes may be necessary. Several simulation codes are available which apply different numerical methods. However, the applicability for complex subsurface geometries reduces the number substantially. We will present modeling approaches with ANSAS. These will include applications for deep geothermics (enhanced geothermal systems), the use of mine-water for heating purposes and the numerically efficient modeling of shallow ground heat exchanger arrays.

Key words: borehole**,** Geothermics, ground heat exchanger, finite difference,

1 Introduction

In the past, the main driving force for hydrogeologic studies has been the need to assess the water-supply potential of aquifers. During the past 20 years, however, the emphasis has shifted from water-supply problems to water-quality problems. This has driven a need to predict the movement of contaminants through the subsurface environment. One consequence of the change in emphasis has been a shift in perceived priorities for scientific research and data collection. Formerly, the focus was on developing methods to assess and measure the water-yielding properties of high-permeability aquifers. The focus is now largely on transport

and dispersion processes retardation and degradation of chemical contaminants, the effects of heterogeneity on flow paths and travel times, and the ability of low-permeability materials to contain contaminated groundwater (Konikow, 1996).
The past 20 years or so have also seen some major technological breakthroughs in groundwater hydrology. One technological growth area has been in the development and use of deterministic, distributed-parameter, computer simulation models for analysing flow and solute transport in groundwater systems. These developments have somewhat paralleled the development and widespread availability of faster, larger memory, more capable, yet less expensive computer systems. Another major technological growth area has been in the application of isotopic analyses to groundwater hydrology, wherein isotopic measurements are being used to help interpret and define groundwater flow paths, ages, recharge areas, leakage, and interactions with surface water (Konikow, 1996), (Coplen 1993).
Because isotopes move through groundwater systems under the same driving forces and by the same processes as do dissolved chemicals, it is natural that the groundwater flow and solute-transport models applied to groundwater contamination problems be linked to and integrated with isotopic measurements and interpretations. Many previous applications of isotopic analyses to groundwater systems, however, have assumed overly simplified conceptual models for groundwater flow and transport of dissolved chemicals--either plug flow (with piston-like displacement and no mixing) or a well-mixed reservoir (which unrealistically overestimates the mixing effects of dispersion and diffusion). If the interpretations of isotopic analyses are coupled with more realistic conceptual models of flow and transport, then it is anticipated that the synergistic analysis will lead to a more accurate understanding of the hydrogeologic system being studied. Dinçer and Davis (1984) provide a review of the application of environmental isotope tracers to modelling in hydrology, and Johnson and DePaolo (1994) provide an example of applying such a coupled approach in their analysis of a proposed high-level radioactive waste repository site (Konikow, 1996) (Dinçer and Davis 1984; Johnson and Depaolo 1994).
The purpose of this chapter is to review the state of the art in deterministic modelling of groundwater flow and transport processes for those who might want to merge the interpretation of isotopic analyses with quantitative groundwater model analysis. This chapter is aimed at practitioners and is intended to help define the types of models that are available and how they may be applied to complex field problems. It will discuss the philosophy and theoretical basis of deterministic modelling, the advantages and limitations of models, the use and misuse of models, how to select a model, and how to calibrate a model. However, as this chapter is only a review, it cannot offer comprehensive and in-depth coverage of this very complex topic; but it does guide the reader to references that provide more details. Other recent comprehensive reviews of the theory and practice of deterministic modelling of groundwater processes are provided by

Anderson and Woessner (1992) and Bear and Verruijt (1987), (Konikow, 1996), (Anderson and Woessner 1992; Bear and Verruyt 1987).

2 Problem Formulations

2.1 MODELS

The word model has so many definitions and is so overused that it is sometimes difficult to discern the meaning of the word (Konikow and Bredehoeft 1992). A model is perhaps most simply defined as a representation of a real system or process. A conceptual model is a hypothesis for how a system or process operates. This hypothesis can be expressed quantitatively as a mathematical model. Mathematical models are abstractions that represent processes as equations, physical properties as constants or coefficients in the equations, and measures of state or potential in the system as variables (Konikow, 1996).

Most groundwater models in use today are deterministic mathematical models. Deterministic models are based on conservation of mass, momentum, and energy and describe cause and effect relations. The underlying assumption is that given a high degree of understanding of the processes by which stresses on a system produce subsequent responses in that system, the system's response to any set of stresses can be predetermined, even if the magnitude of the new stresses falls outside of the range of historically observed stresses (Konikow, 1996).

Deterministic groundwater models generally require the solution of partial differential equations. Exact solutions can often be obtained analytically, but analytical models require that the parameters and boundaries be highly idealised. Some deterministic models treat the properties of porous media as lumped parameters (essentially, as a black box), but this precludes the representation of heterogeneous hydraulic properties in the model. Heterogeneity, or variability in aquifer properties, is characteristic of all geologic systems and is now recognised as playing a key role in influencing groundwater flow and solute transport. Thus, it is often preferable to apply distributed-parameter models, which allow the representation of more realistic distributions of system properties. Numerical methods yield approximate solutions to the governing equation (or equations) through the discretisation of space and time. Within the discretised problem domain, the variable internal properties, boundaries, and stresses of the system are approximated. Deterministic, distributed-parameter, numerical models can relax the rigid idealised conditions of analytical models or lumped-parameter models, and they can therefore be more realistic and flexible for simulating field conditions (if applied properly) (Konikow, 1996).

The number and types of equations to be solved are determined by the concepts of the dominant governing processes. The coefficients of the equations are the parameters that are measures of the properties, boundaries, and stresses of the system; the dependent variables of the equations are the measures of the state of the system and are mathematically determined by the solution of the equations. When a numerical algorithm is implemented in a computer code to solve one or more partial differential equations, the resulting computer code can be considered a generic model. When the grid dimensions, boundary conditions, and other parameters (such as hydraulic conductivity and storativity), are specified in an application of a generic model to represent a particular geographical area, the resulting computer program is a site-specific model. The ability of generic models to solve the governing equations accurately is typically demonstrated by example applications to simplified problems. This does not guarantee a similar level of accuracy when the model is applied to a complex field problem (Konikow, 1996).

If the user of a model is unaware of or ignores the details of the numerical method, including the derivative approximations, the scale of discretisation, and the matrix solution techniques, significant errors can be introduced and remain undetected. For example, if the groundwater flow equation is solved iteratively, but the convergence criterion is relatively too coarse, then the numerical solution may converge, but to a poor solution. The inaccuracy of the solution may or may not be reflected in the mass-balance error. Unrecognized errors in numerical groundwater models are becoming more possible as “user-friendly” graphical interfaces make it easier for models to be used (and to be misused). These interfaces effectively place more “distance” between the modeller and the numerical method that lies at the core of the model (Konikow, 1996).

2.2 FLOW AND TRANSPORT PROCESSES

The process of groundwater flow is generally assumed to be governed by the relations expressed in Darcy's law and the conservation of mass. Darcy's law does have limits on its range of applicability, however, and these limits must be evaluated in any application (Konikow, 1996).

The purpose of a model that simulates solute transport in groundwater is to compute the concentration of a dissolved chemical species in an aquifer at any specified time and place. The theoretical basis for the equation describing solute transport has been well documented in the literature (Bear 1997; Domenico and Schwartz 1998). Reilly et al. (1987) provide a conceptual framework for analyzing and modeling physical solute-transport processes in groundwater (Reilly et al. 1987), (Konikow, 1996). Changes in chemical concentration occur within a dynamic groundwater system primarily due to four distinct processes:

1) advective transport, in which dissolved chemicals are moving with the flowing groundwater;

2) Hydrodynamic dispersion, in which molecular and ionic diffusion and small-scale variations in the flow velocity through the porous media cause the paths of dissolved molecules and ions to diverge or spread from the average direction of groundwater flow

3) Fluid sources, where water of one composition is introduced into and mixed with water of a different composition

4) Reactions, in which some amount of a particular dissolved chemical species may be added to or removed from the groundwater as a result of chemical, biological, and physical reactions in the water or between the water and the solid aquifer materials or other separate liquid phases.

The subsurface environment constitutes a complex, three-dimensional, heterogeneous hydrogeologic setting. This variability strongly influences groundwater flow and transport, and such a reality can be described accurately only through careful hydrogeologic practice in the field. Regardless of how much data are collected, however, uncertainty always remains about the properties and boundaries of the groundwater system of interest. Stochastic approaches have resulted in many significant advances in characterising subsurface heterogeneity and dealing with uncertainty (Konikow, 1996) , (Gelhar 1993).

2.3 GOVERNING EQUATIONS

The development of mathematical equations that describe the groundwater flow and transport processes may be developed from the fundamental principle of conservation of mass of fluid or of solute. Given a representative volume of porous medium, a general equation for conservation of mass for the volume may be expressed as:

(rate of mass inflow) – (rate of mass outflow) +

+ (rate of mass production/consumption) = (rate of mass accumulation)

This statement of conservation of mass (or continuity equation) may be combined with a mathematical expression of the relevant process to obtain a differential equation describing flow or transport (Konikow, 1996), (Bear 1997; Domenico and Schwartz 1998; Freeze and Cherry 1979).

2.3.1 GROUNDWATER FLOW EQUATION

The rate of flow of water through a porous media is related to the properties of the water, the properties of the porous media, and the gradient of the hydraulic head, as represented by Darcy's law, which can be written as (Konikow, 1996):

$$q_i = -k_{ij} \frac{\partial h}{\partial x_j}$$

where q_i is the specific discharge, $LT^{-1}; K_{ij}$ is the hydraulic conductivity of the porous medium (a second-order tensor), LT^{-1}; and h is the hydraulic head, L.

A general form of the equation describing the transient flow of a compressible fluid in a non-homogeneous anisotropic aquifer may be derived by combining Darcy's law with the continuity equation. A general groundwater flow equation may be written in Cartesian tensor notation as (Konikow, 1996):

$$\frac{\partial}{\partial x_i}\left(k_{ij} \frac{\partial h}{\partial x_i}\right) = S_s \frac{\partial h}{\partial t} + w^*$$

where S_S is the specific storage, L^{-1}; t is time, T; W* is the volumetric flux per unit volume (positive for outflow and negative for inflow), T^{-1} ; and xi are the Cartesian co-ordinates, L. The summation convention of Cartesian tensor analysis is implied in Eqs.4.2 and 4.3. Eq.4.3 can generally be applied if isothermal conditions prevail, the porous medium only deforms vertically, the volume of individual grains remains constant during deformation, Darcy's law applies (and gradients of hydraulic head are the only driving force), and fluid properties (density and viscosity) are homogeneous and constant. Aquifer properties can vary spatially, and fluid stresses (W*) can vary in space and time (Konikow, 1996).

If the aquifer is relatively thin compared to its lateral extent, it may be appropriate to assume that groundwater flow is areally two-dimensional. This allows the three-dimensional flow equation to be reduced to the case of two-dimensional areal flow, for which several additional simplifications are possible. The advantages of reducing the dimensionality of the equation include less stringent data requirements, smaller computer memory requirements, and shorter computer execution times to achieve numerical solutions (Konikow, 1996).

An expression similar to Eq.4.3 may be derived for the two-dimensional a real flow of a homogeneous fluid in a confined aquifer and written as:

$$\frac{\partial}{\partial x_i}\left(T_{ij}\frac{\partial h}{\partial x_i}\right) = S\frac{\partial h}{\partial t} + w$$

where T_{ij} is the transmissivity, L^2T^{-1}; and $T_{ij} \equiv K_{ij}$ b; b is the saturated thickness of the aquifer, L; S is the storage coefficient (dimensionless); and W = W*b is the volume flux per unit area, LT^{-1}.

When Equation is applied to an unconfined (water-table) aquifer system, it must be assumed that flow is horizontal and equipotential lines are vertical, that the horizontal hydraulic gradient equals the slope of the water table, and that the storage coefficient is equal to the specific yield (S_y) (Anderson and Woessner 1992). Note that in an unconfined system, the saturated thickness changes as the water-table elevation (or head) changes. Thus, the transmissivity also can change over space and time (i.e. $T_{ij} \equiv K_{ij}$ b; b(x,y,t) = h _h_b, and h_b is the elevation of the bottom of the aquifer) (Konikow, 1996).

In some field situations, fluid properties such as density and viscosity may vary significantly in space or time. This may occur where water temperature or dissolved-solids concentration changes significantly. When the water properties are heterogeneous and (or) transient, the relations among water levels, hydraulic heads, fluid pressures, and flow velocities are neither simple nor straightforward. In such cases, the flow equation is written and solved in terms of fluid pressures, fluid densities, and the intrinsic permeability of the porous media (Konikow and Grove 1977).

2.3.2 Geothermal energy

Geothermal energy is the heat contained in the solid earth and its internal fluids. This sets it apart from other terrestrial energy sources. It represents a vast supply

which has only started to be tapped by mankind for space heating, process heat and generation of electric power. (Clauser, 2006)
The efficient use of this natural resource can be optimized by applying numerical heat-transport models. In the following we will give an introduction on the modeling of different geothermal utilization scenarios using ANSAS, (Konikow, 1996) (Diersch, 2005; Trefry & Muffels, 2007).

2.3.3 Coupled modeling of flow and heat transport

The computation of heat transport within a porous medium requires the solution of a set of balance equations. For comprehensibility only flow in a saturated medium is discussed. Of course, advanced heat and groundwater modeling codes are also able to compute unsaturated flow, fracture flow and other special processes.
The mass conservation equation of a fluid in a saturated porous medium is given by (Diersch, 2005)

$$S_{\circ} \frac{\partial h}{\partial t} = \nabla \bullet \left(k \left(\nabla h + \chi^{e} \right) \right) + Q$$

where *S*0 is the specific storage due to fluid and medium compressibility (m^{-1}), *h* is the hydraulic head (m), *t* is time (s), χ s the buoyancy coefficient and **e** is the gravitational unit vector . *Q* corresponds to sources and sinks.
K represents the tensor of the hydraulic conductivity (m s^{-1}), defined as where **k** is the permeability tensor $\left(m^{2}\right)$, *f* is the fluid density (kg m^{-3}), *g* is the gravitational force

$$K \equiv \frac{k\rho^{f} g}{\mu^{f}}$$

μ^{f} is the dynamic fluid viscosity .
The Darcy velocity **q** is given by

$$q = -k\left(\nabla h + \chi^{e} \right)$$

The heat transport with conductive and advective parts reads

$$\left(\rho c \right)^{8} \frac{\partial T}{\partial t} = \nabla \bullet \left(\lambda \nabla T - \rho^{f} c^{f} q T \right) + H$$

where *T* is the temperature (K), *c* is the specific heat and λ is the heat conductivity . $(\rho c)^{g}$ is bulk volumetric heat capacity . *H* refers to sources and sinks.

3 Geothermics

Geothermal installations are generally distinguished between shallow (using boreholes with depths up to 400 m) and deep geothermics. The latter is sometimes defined by its direct usability, i.e. that it is not necessary to use heat exchangers (A. Renz., 2009).

3.1 Shallow geothermics

Shallow geothermics is mostly used via borehole heat exchangers, which utilize the temperature difference between the atmosphere and ground. Different technologies exist, e.g., U-shape heat pipe, double U-shape heat pipe, coaxial heat pipes and grounding stakes. Such ground heat exchangers form a vertical borehole system, where a refrigerant circulates in closed pipes exchanging heat with the surrounding aquifer driven alone by thermal conductivity (closed loop system). Unlike those closed systems a combination of extraction and injection wells can be used in an open system, see Figure 2. A procedure of special interest refers to the combined use of solar energy and geothermics, i.e.,. the storage of solar energy in the ground. For instance, in order to give expertise on environmental matters it may be necessary to model the impact of ground heat exchangers on the subsurface temperature. In the following some introductive modeling scenarios are summarized (A. Renz., 2009).

3.2 Open loop

In Figure 3 modeling results for combining an extraction and injection well are shown. At the injection well a special module, which has been programmed for this purpose, adds a variable temperature difference on the temperature of the extracted water. Due to groundwater flow, the temperature at the extraction well increases in time (A. Renz., 2009).

3.3 Geothermal heat pump systems

Geothermal heat pump systems, employing the ground as media of heat exchanger with the surrounding through ground heat exchangers, offer higher energy efficiency and lesser environmental impact than air-cooled systems.

Vertical ground heat exchangers (with U tubes installed inside boreholes) re most common, requiring lesser land field. Fig. 1 shows the general arrangement of a boreholeground heat exchanger and pipe connection in a borefield. However, drilling of deep boreholes involves a high initial cost, which hinders the application of such systems. To reduce cost, precise system design becomes very important (Lee, C.K.., 2008).

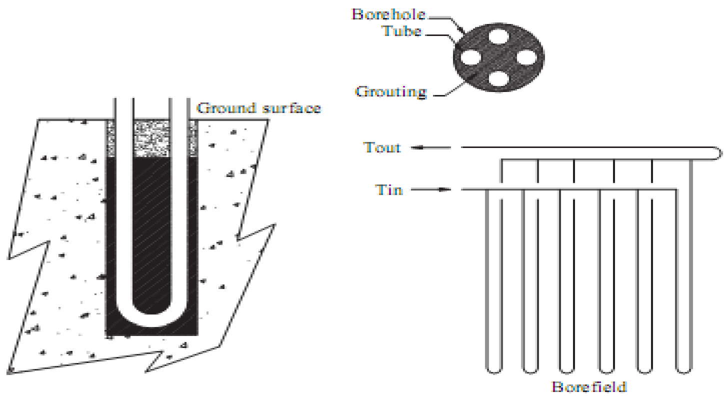

Figure 1 General arrangement of borehole ground heat exchanger

4 Numerical formulation (Lee, C.K.., 2008)

In developing the numerical scheme, the following assumptions were made:

- The ground was homogeneous;
- The thermal properties of all the arterials remain constant within the temperature range investigated;
- The boreholes and the ground had no contact resistance;
- The ground temperature remained unchanged at the to surface and at a distance far below the boreholes and away in the transverse directions;
- Quasi-steady state was maintained inside the boreholes;
- Fluid flow rate in each tube of boreholes was the same;
- Fluid flow rate in each borehole was the same

4.1 Heat transfer around borehole (Lee, C.K.., 2008)

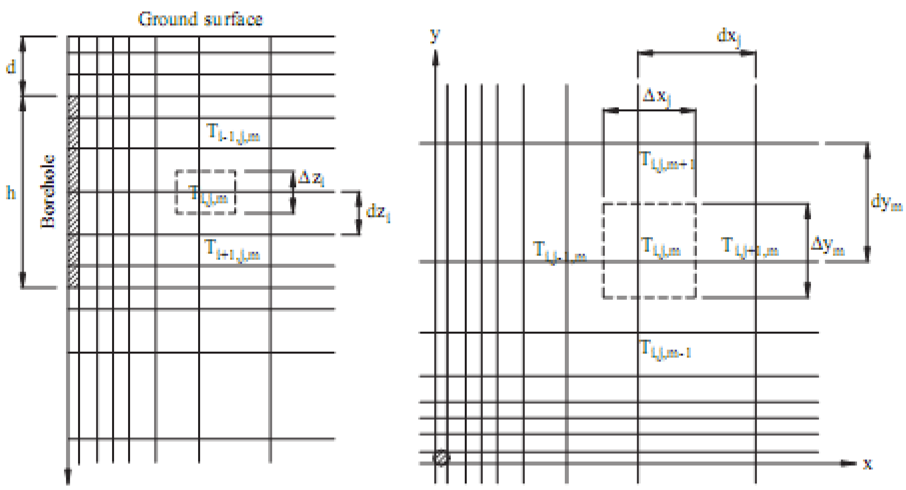

Figure 2: Grid scheme for ground around borehole

Fig. 2 shows the grid scheme used for the ground around borehole. The implicit difference equation with regular grids can be obtained by direct discretization of the governing differential equation for conductive heat transferring the ground as Field ((Lee, C.K.., 2008)

$$\frac{T_{i,j,m}^{n+1}-T_{i,j,m}^{n}}{\Delta t}=a_g\left(\begin{array}{l}\dfrac{T_{i,j+1,m}^{n+1}-2T_{i,j,m}^{n+1}+T_{i,j-1,m}^{n+1}}{(\Delta x)^2}+\dfrac{T_{i,j+1,m}^{n+1}-2T_{i,j,m}^{n+1}+T_{i,j-1,m}^{n+1}}{(\Delta y)^2}\\+\dfrac{T_{i,j+1,m}^{n+1}-2T_{i,j,m}^{n+1}+T_{i,j-1,m}^{n+1}}{(\Delta z)^2}\end{array}\right)$$

Eq. (1) can be re-written in energy balance form with irregular grids as

$$\frac{T_{i,j,m}^{n+1}-T_{i,j,m}^{n}}{\Delta t}=\left(\frac{q_{x+}+q_{x-}+q_{y+}+q_{yx-}+q_{z+}+q_{z-}}{\rho_g c_g \Delta x_j \Delta y_m \Delta z_i}\right)$$

where

$$q_{x+}=k_g\Delta y_m\Delta z_i\frac{T_{i,j+1,m}^{n+1}-T_{i,j,m}^{n+1}}{dx_j}$$

$$q_{x-}=k_g\Delta y_m\Delta z_i\frac{T_{i,j-1,m}^{n+1}-T_{i,j,m}^{n+1}}{dx_{j-1}}$$

$$q_{y+}=k_g\Delta x_j\Delta z_i\frac{T_{i,j,m+1}^{n+1}-T_{i,j,m}^{n+1}}{dy_m}$$

$$q_{y-}=k_g\Delta x_j\Delta z_i\frac{T_{i,j,m-1}^{n+1}-T_{i,j,m}^{n+1}}{dy_{m-1}}$$

$$q_{z+}=k_g\Delta y_m\Delta x_j\frac{T_{i+1,j,m}^{n+1}-T_{i,j,m}^{n+1}}{dz_i}$$

$$q_{z-}=k_g\Delta y_m\Delta x_j\frac{T_{i-1,j,m}^{n+1}-T_{i,j,m}^{n+1}}{dz_{i-1}}$$

The source term q_s equals one-quarter of the borehole load multiplied by a load factor fact (value to be determined in a later section) at each corner point of the borehole and zero elsewhere.

4.2 Heat transfer inside borehole (Lee, C.K.., 2008)

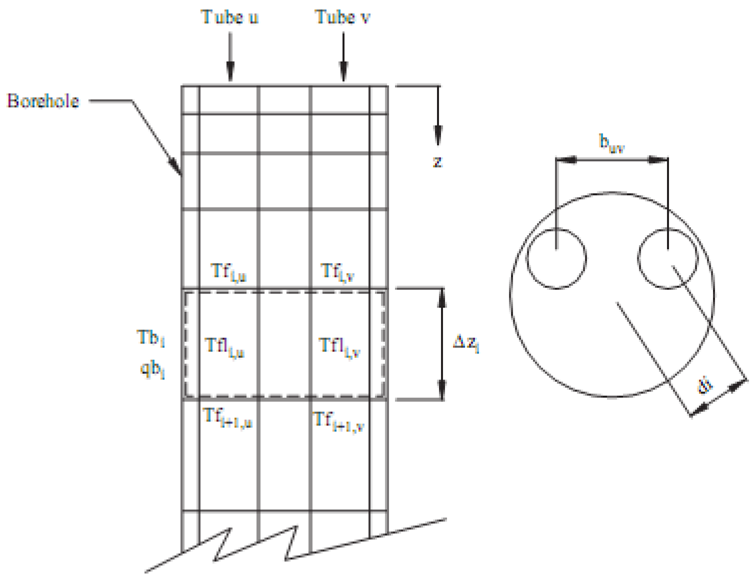

Figure 3: Grid scheme inside borehole

Fig. 3 shows the grid scheme used for the boreholes. With fluid flowing in the tubes, the fluid temperatures along the tubes are strongly coupled. Hence, all grid fluid temperatures have to be solved simultaneously. On the other hand, with no fluid flowing in the tubes, the fluid temperatures along the tubes (not within the same control volume) were weakly coupled (assuming conduction along the tubes to be small compared with that in the transverse direction).
Hence, the numerical formulation is different. The borehole temperature Tbi and loading qbi are defined at mid level of the control volume while ground temperatures are defined at the grid points. Hence, eight round temperatures are required to calculate one borehole temperature, and that the source term qs at particular ground grid point on the borehole surface should be determined from the borehole loading above and below the ground grid point.

4.3 Flow process

For a borehole containing N tubes and assuming steady state conditions,

$$(-1)^{m1} Mc_f \frac{dTf1_u}{dz} = \sum_{v=1}^{u-1} \frac{Tf1_u - Tf1_v}{\overset{\Delta}{R}_{uv}} + \frac{Tf1_u - Tb_v}{\overset{\Delta}{R}_{uv}} + \sum_{v=u+1}^{N} \frac{Tf1_u - Tf1_v}{\overset{\Delta}{R}_{uv}}$$

Where m1 is 1 for downward flowing tubes and 2 for upward flowing tubes By replacing the fluid temperatures with the grid point temperatures, Eq. (3) can be re-written as

$$(-1)^{m1} Mc_f \frac{Tf_{i+1,u} - Tf_{i,u}}{dz_i} = \sum_{v=1}^{u-1} \frac{(Tf_{i+1,u} + Tf_{i,u}) + (Tf_{i+1,u} + Tf_{i,v})}{2\overset{\Delta}{R}_{uv}}$$
$$+ \frac{Tf_{i+1,u} + Tf_{i,u} - 2Tb_v}{\overset{\Delta}{R}_{uv}} + \sum_{vu+11}^{N} \frac{(Tf_{i+1,u} + Tf_{i,u}) + (Tf_{i+1,u} + Tf_{i,v})}{2\overset{\Delta}{R}_{uv}}$$

Eq. (4) represents N_x nzbore coupled equations in N_x nzbore variables T_{fi} ,u by writing Eq. (4) consecutively for $i^{\frac{1}{4}}$ 1 to nzbore and $u^{\frac{1}{4}}$ 1 to N, which are solved iteratively based on prescribed fluid inlet temperature, pipe connection configuration and borehole temperatures.

The borehole loading is then calculated as

$$qb_i = (-1)^{m1} Mc_f \frac{Tf_{i+1,u} - Tf_{i,u}}{dz_i}$$

No flow process with no fluid flowing in the tubes, there is no directional dependence, and the governing differential equation (assuming heat conduction in transverse direction only) becomes

$$-\pi r_{pi}^2 \rho_f c_f \frac{dTfl_{u\,i,u}}{dt} = \sum_{v=1}^{u-1} \frac{Tfl_u - Tfl_v}{\overset{\Delta}{R}n\, f_{uvuv}} + \frac{Tfl_u - Tb_v}{\overset{\Delta}{R}n\, f_{uvuv}}$$
$$+ \sum_{v=u+1}^{N} \frac{Tfl_u - Tb_v}{\overset{\Delta}{R}n\, f_{uvuv}}$$

which can be discredited as

$$-\pi r_{pi}^2 \rho_f c_f \frac{Tfl_{u\,i,u}{}^{n+1} - Tfl_{u\,i,u}{}^{n}}{\Delta t} =$$
$$\sum_{v=1}^{u-1} \frac{Tfl_u{}^{n+1} - Tfl_v{}^{n+1}}{\overset{\Delta}{R}\, n\, f_{uv\,uv}} + \frac{Tfl_u{}^{n+1} - Tb_v{}^{n}}{\overset{\Delta}{R}\, n\, f_{uv\,uv}} + \sum_{v=u+1}^{N} \frac{Tfl_{i,u}{}^{n+1} - Tfl_{i,v}{}^{n+1}}{\overset{\Delta}{R}\, n\, f_{uv\,uv}}$$
$$-Cf\left(Tfl_{i,u}{}^{n+1}\right) - Tfl_{i,v}{}^{n+1} = \sum_{v=1}^{u-1} \frac{Tfl_{i,u}{}^{n+1} - Tfl_{i,v}{}^{n+1}}{\overset{\Delta}{R}\, n\, f_{uv\,uv}} + \frac{Tfl_u{}^{n+1} - Tb_v{}^{n}}{\overset{\Delta}{R}\, n\, f_{uv}} + \sum_{v=u+1}^{N} \frac{Tfl_{i,u}{}^{n+1} - Tfl_{i,v}{}^{n+1}}{\overset{\Delta}{R}\, n\, f_{uv}}$$

To solve $Tfl_{i,u}^{n+1}$, Eq. (7) is re-written as

$$cfTfl_{i,u}^{n} + \frac{Tb_i^n}{\overset{\Delta}{R}\, n\, f_{uv}} = -\sum_{v=1}^{u-1} \frac{Tfl_{i,v}{}^{n+1}}{\overset{\Delta}{R}\, n\, f_{uv\,uv}} \left(\sum_{v=1}^{u-1} \frac{1}{\overset{\Delta}{R}\, n\, f_{uv\,uv}} + Cf\right) \frac{Tfl_u{}^{n+1}{}_v{}^{n}}{\overset{\Delta}{R}\, n\, f_{uv\,uv}} + \sum_{v=u+1}^{N} \frac{Tfl_{i,v}{}^{n+1}}{\overset{\Delta}{R}\, n\, f_{uv\,uv}}$$

Where $Cf = \dfrac{\pi r_{pi}^2 \rho_f C_f}{\Delta t}$

Eq. (8) can be expressed in matrix form as

$$\left[C_f Tfl_{i,u}^{n} + \frac{Tb_i^n}{\overset{\Delta}{Rn}\, f_{uv}}\right] = \left[\overset{*}{Rn}\, f\right]\left[Tfl_{i,v}^{n+1}\right]$$

Where

$$\left[\overset{*}{Rn}\, f\right] = \frac{1}{\left[\overset{\Delta}{Rn}\, f\right]}$$, for $u \neq v$ and

$$\left[\overset{*}{Rn}\, f\right] = \left(\sum_{v=1}^{N} \frac{1}{\left[\overset{\Delta *}{Rn}\, f\right]} + Cf\right)$$

Hence

$$\left[\overset{*}{Rn}\, f\right]^{-1}\left[C_f Tfl_{i,u}^{n} + \frac{Tb_i^n}{\overset{\Delta}{Rn}\, f_{uv}}\right] = \left[Tfl_{i,v}^{n+1}\right]$$

4.4 Borefield performance formulation (Lee, C.K.., 2008)

The inlet fluid temperature can be calculated as

$$T_{in}^{i+} = T_{out}^{n} + \frac{Q}{M_{total} C_f}$$

The load transferred to the borefield is then evaluated as

$$Q_{bc} = M_{total} . C_f \left(T_{in}^{n+} - T_{out}^{i+}\right)$$

Thermal resistance for the entire borefield can be defined as

$$Rb_{bf} = \frac{N_{bore} H \left(T_{in} + T_{out} - 2\overline{Tb}\right)}{2Q_{bf}}$$

5 Results and discussion

5.1 Temperature and loading profile along borehole

It was found that neither the temperature nor the loading was constant along the borehole. The borehole temperature reached a maximum near the top part of the borehole rather than near the middle level of borehole if a finite line source model was used. The bore hole loading decreased with depth up to the bottom end of borehole. This could be explained by considering the fact that the mean fluid temperature inside the borehole decreased with depth (see Fig. 4). Hence, near the top of borehole where the borehole temperature increased with depth, the bore hole loading decreased (due to reduced temperature difference between fluid and borehole). The situation changed only beyond the depth where the borehole temperature decreased with depth at a higher rate than the mean fluid temperature (near the lower part of borehole). Basically, the borehole temperature and loading profiles changed very mildly with time. Indeed, very similar profiles were obtained when using cylindrical coordinate system for a single borehole.

5.2 Ground temperature profiles around borehole

Figs. 5 show the ground temperature profiles at various distances from the borehole after 1, 10 year using ANSAS programm. The temperature profiles at the borehole were different from those away from borehole. At distance far from the borehole, the depth of maximum temperature shifted to mid-level of the borehole. This meant that the thermal interference effect between two boreholes in a bore field would depend both on the borehole spacing and depth. To apply the method of superposition to estimate the performance of a bore field, individual finite difference scheme would be needed for each borehole for precise simulation. A simpler way was to discretize the entire bore field, and perform the simulation for all bore holes simultaneously. This was easily achieved when using rectangular coordinate system by expanding the original grid system in transverse directions to cover the entire bore

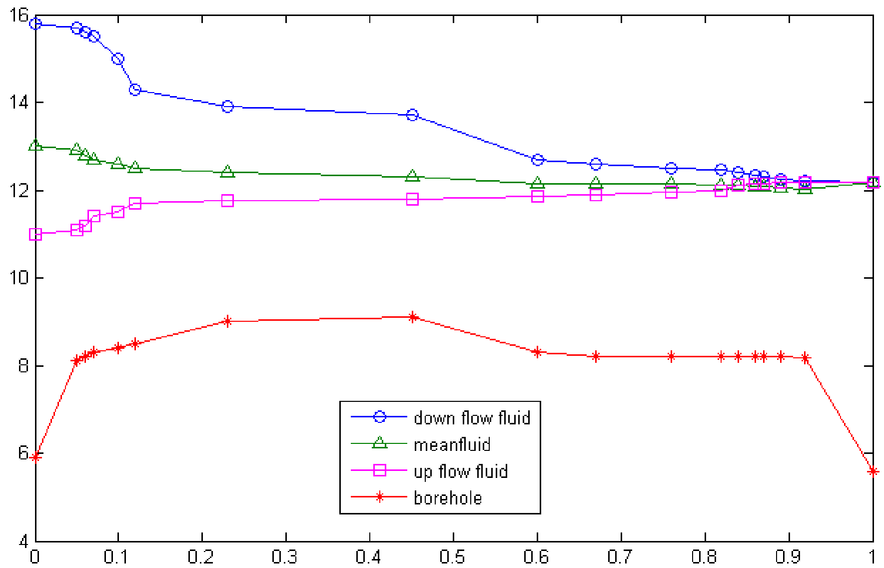

Fig 4. Comparison of fluid temperature rise and borehole temperature rise after 1 year using ANSAS programm.

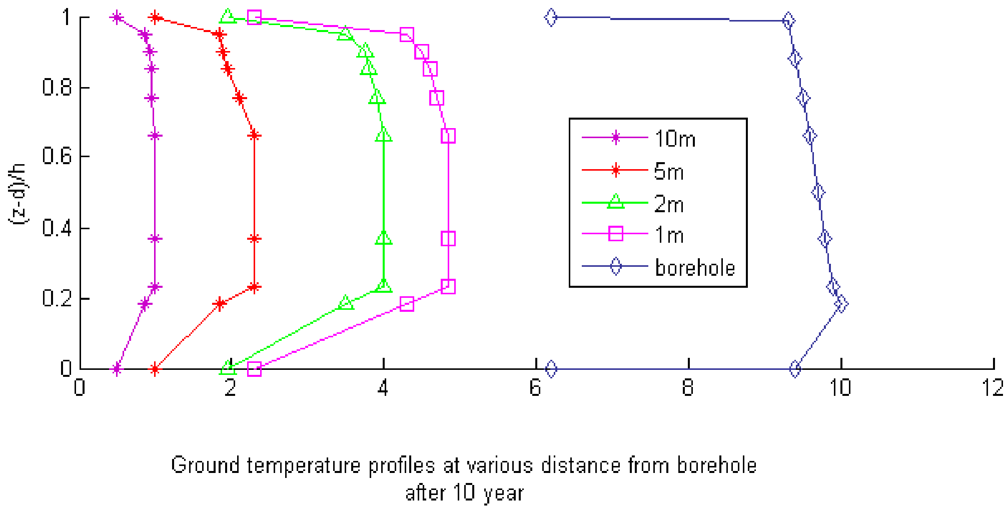

Fig. 5. Ground temperature profiles at various distances from borehole after 10 years using ANSAS programm.

6 Conclusions

A numerical model was developed to predict heat extraction and injection rates of a ground heat exchanger.

_ it is based on simulation code for the analysis of underground heat and water movement, in which circulatory water model in the heat exchanger and the ground surface heat flux model are incorporated.

_ an estimation method for the soil thermal properties based on a ground investigation and theoretical formulas was proposed.

References

1. L.F. KONIKOW, 1996, "Use of Numerical Models to Simulate Groundwater Flow and Transport ", US Geological Survey, Reston, Virginia, USA
2. Lee, C.K.. "Computer simulation of borehole ground heat exchangers for geothermal heat pump systems", Renewable Energy, 200806
3. Renz. "Numerical Modeling of Geothermal Use of Mine Water: Challenges and Examples", Mine Water and the Environment, 03/2009
4. Nam, Y.. "Development of a numerical model to predict heat exchange rates for a ground-source heat pump system", Energy & Buildings, 2008
5. C. Clauser. "8.1 The Earth's thermal regime", Landolt-Brnstein - Group VIII Advanced Materials and Technologies, 2006

6. Hans-Joachim Baues. "The cohomology of homotopy categories and the general linear group", K-Theory, 1989
7. http://jesr.journal.fatih.edu.tr/constrained-hadi.pdf
8. http://e-research.tnstate.edu/cgi/viewcontent.cgi?article=1003&context=math_wp
9. N. C. Bacalis. "Properties of hydrogen terminated silicon nanocrystals via a transferable tight-binding Hamiltonian, based on ab-initio results", Journal of Mathematical Chemistry, 07/12/2009
10. http://www.rudolfhaken.com/shared/HakenClarinetConcerto/03HakenClarinetCtoOrchScore.pdf
11. Cao, M.. "Sensitivity of fundamental mode shape and static deflection for damage identification in cantilever beams", Mechanical Systems and Signal Processing, 201102
12. Auzinger, W.. "Efficient mesh selection for collocation methods applied to singular BVPs", Journal of Computational and Applied Mathematics, 20050801
13. http://csse.usc.edu/csse/TECHRPTS/PhD_Dissertations/files/LeeMing_Dissertation.pdf
14. http://ayso67.org/03_game_pdfs/GU14%20Schedule.pdf
15. Antoine Saucier. "Sinusoidal oscillations radiating from a cylindrical source in thermal conduction or groundwater flow: Closed-form solutions", International Journal for Numerical and Analytical Methods in Geomechanics, 2010
16. http://mosaic.math.tamu.edu/~maguiar/thesis2.pdf
17. Sedghi, M.M.. "Three-dimensional semi-analytical solution to groundwater flow in confined and unconfined wedge-shaped aquifers", Advances in Water Resources, 200906
18. D. Cabello. "DT-CNN emulator: 3D heat equation solver with applications on the non-destructive soil inspection", 2008 11th International Workshop on Cellular Neural Networks and Their Applications, 07/2008
19. J. K. PERCUS. "Spatial Structure of a Fermion Fluid", Annals of the New York Academy of Sciences, 4/1987
20. M. R. Sheffer. "Three-dimensional modelling of streaming potential", Geophysical Journal International, 6/2007
21. Winfried Auzinger. "Analysis of a New Error Estimate for Collocation Methods Applied to Singular Boundary Value Problems", SIAM Journal on Numerical Analysis, 2005
22. Fernando Pardo. "FPGA computation of the 3D heat equation", Computational Geosciences, 12/24/2009
23. C. Chauviere. "A New Method for Micro-Macro Simulations of Viscoelastic Flows", SIAM Journal on Scientific Computing, 2002
24. Vicente Muňoz. "Geometric structures on loop and path spaces", Proceedings Mathematical Sciences, 09/2010

25. Bayn. "Linear Algebra", Essentials of Mathematical Methods in Science and Engineering, 07/07/2008
26. Zhai, S.. "Pure ice sublimation within vials in a laboratory lyophiliser; comparison of theory with experiment", Chemical Engineering Science, 200502
27. Ashisha Kumar. "Mixed norm estimate for Radon transform on weighted L p spaces", Proceedings Mathematical Sciences, 09/2010
28. Konikow, L.F.. "Groundwater flow and transport modeling", Journal of Hydrology, 19880730

Nomenclature

a_g thermal diffusivity of ground (m2/s)	H length of borehole (m)
B_{uv} centre to centre distance between tube u and v in a borehole (m)	h heat transfer coefficient (W/m2)
$c^{\frac{1}{4}}{}_f$ pr2	k thermal conductivity (W/ m_k)
pir $f_{cf} = D_t$ (W/ m_k)	M : mass flow rate of fluid inside tube of borehole (kg/s)
c_f specific heat capacity of fluid (W/kg K)	M: total mass flow rate of fluid through a borefield (kg/s)
q_i centre to centre distance between tube and borehole (m)	N number of tubes inside each borehole
d_x ground grid spacing in x direction (m)	N_{bore} : number of boreholes in a borefield
d_y ground grid spacing in y direction (m)	nzbore number of ground grid points along borehole depth
d_z ground grid spacing in z direction (m)	p_r Prandtl number
Q applied load to a borefield (W)	q_{z+} heat load into control volume of ground from downstream z direction (W)
Q_{bf} load transferred from borefield to ground (W)	q_{z-} heat load into control volume of ground from upstream z direction (W)
q_b borehole loading per unit length (W/m)	R_{uvo} thermal interference coefficient between tube u and v according to Eq. (A.1) (m_k /W)
Q fact load factor to be multiplied in calculating the source term	R_{uvo-} element of inverse matrix of R_{uvo} between tube u and v (W/ m_k)
q_s source term in the finite difference equation(W)	R_{UVD} thermal interference coefficient between tube u and v according to Eq. (3) (m_k /W)
qt tube loading per unit length inside borehole (W/m)	R_{bbf} thermal resistance of entire borefield (m_k /W)
q_{x+} heat load into control volume of ground	Re Reynolds number

from downstream x direction (W)	
q_{x-} heat load into control volume of ground from upstream x direction (W)	R_{nfuv} _thermal interference matrix element between tube u and v according to Eq. (9) at no fluid
q_{y+} heat load into control volume of ground from downstream y direction (W)	flow (W/ m_k)
q_{y-} heat load into control volume of ground from upstream y direction (W)	R_{nfuvD} thermal interference coefficient between tube u and v according to Eq. (6) at no fluid flow
R_b thermal resistance between fluid and grouting inside borehole (m_k /W)	Tb Borehole temperature (K)
$r_{\max}$ margin from borefield boundary beyond which the ground temperature was assumed unchanged	T_{fi} fluid temperature at centre of control volume inside borehole (K)
(m)	Tin fluid temperature entering a borefield (K)
R_{pi} inner radius of tube (m)	Tout fluid temperature leaving a borefield (K)
R_{po} outer radius of tube (m)	t time (s)
T ground temperature (K)	$t_{\max}$ maximum time encountered in the analysis (s)
Greek symbols	**Subscripts**
Δ x length of control volume of ground in x direction (m)	b borehole
Δ y length of control volume of ground in y direction (m)	f fluid
Δ z length of control volume of ground in z direction (m)	g ground
Δ t discretization time step (s)m dynamic viscosity (kg/ms)	i, j, m ground discretization step designation in z, x and y directions
ρ density (kg/m3)	p tube inside borehole