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Peristaltic Flow of a Dusty Couple Stress Fluid in a Flexible Channel

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Abstract

In this paper, we investigate the peristaltic two-phase fluid flow consisting of couple stress fluid and dusty fluid in a flexible channel. The dusty flow equations are based on Saffmen model. The governing non-linear equations are solved under the usual long wavelength approximation. The fluid and dust velocity components, the flux of the fluid across the channel are calculated for different values of the parameters R, β, α and τ.

Keywords: Couple stress fluids, Peristaltic two-phase fluid flow, Dusty fluid flow, Time average fluxes, Reynolds number, Relaxation time.

1. Introduction

Saffmen [5] has discussed the stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. He made the following assumptions:

i) The dust particles are uniform in size and shape so that their number density and velocity are given as v (x, y, z, t), N(x, y, z, t). Further fro particular situations, the number density can also be assumed to be a constant = N_o.

ii) The volume concentration of the dust is assumed to be so small that the net effect of the dust on the gas is KN (u_p - u) per unit volume.

iii) The Reynold's Number of the relative motion of dust and gas is supposed to be very small compared to unity so that the force between the dust and gas is proportional to the relative velocity.

iv) If the dust particles are assumed to be spheres of radius 'a' the Stoke's drag formula holds good, so that K = 6 Π μ a, μ being the viscosity.

In order to develop a mathematical theory of blood flow in arteries.The studied the oscillatory two-phase flow through a rigid circular pipe and his model differs from Saffaman's model [5] in the introduction of a quantity denoting the volume occupied by the solid particles per unit volume of the mixture. Nag, S.K. [3] studied the two-dimensional flow of unbounded dusty fluid induced by the sinusoidal transverse motion of an infinite wall. He observed that the amplitudes of the oscillatory motion of the fluid and dust particles are large near the wall and damp out far from the wall. Later Nag and Jana [4], observed that both the wave velocity and its damping factor decreased with increase in the mass concentration of the particulate phase. The two-phase flow in a flexible channel on which a traveling sinusoidal wave is imposed on the boundary resulting in a peristaltic flow has been studied by Mallikarja Goud [2]. Dusty Unsteady coquette and poiseuille of couple stress fluid through media has been discussed by Goudru [1]. Exact solutions for the velocities of the fluid and dust particles are obtained by using Laplace transform technique. Dust velocity shear driven rotational waves and associated vertices in a nonuniform dusty plasma has been discussed by P.K.Shukla et al [6].

2. Formulation of the problem

Consider a two-dimensional flow of a dusty viscous incompressible couple stress fluid through a channel bounded by flexible boundaries. The flow is due to the peristaltic action of the boundaries under a constant axial pressure gradient. The governing equations of motion in vector form are

$$\rho\overline{q}_t = -\left[\nabla P\right] + \mu\nabla^2\overline{q} - \eta\nabla^4\overline{q} + KN\left[\overline{q}_t^{\,p} - \overline{q}\right] \tag{2.1}$$

$$\bar{q}_t^p + \left[\bar{q}_t^p.\nabla\right]\bar{q}_t^p = \left[\frac{k\left[\bar{q}-\bar{q}_t^p\right]}{m}\right] \tag{2.2}$$

$$\nabla.\bar{q} = 0 \tag{2.3}$$

$$N_t + \nabla.\left[N\,\bar{q}_t^p\right] = 0 \tag{2.4}$$

Where $\bar{q}$ [u,v], $\bar{q}_t^p$ [u^p ,v^p]denote the velocities of the fluid and dust particles respectively, 'p' is the fluid pressure, 'ρ' is the density of the fluid, 'μ' is the coefficient of viscosity, 'm' is the mass of the dust particle, 'N' is the number density of the particles, 'k' is the stokes resistance coefficient. The particles are assumed to be uniform in size and uniformly distributed in the fluid so that 'N' remains a constant.

Choosing the cartesian coordinate system such that the walls of the channel are

$$y = \pm\, a_0 s\left[x - ct/\lambda\right]$$

Where 'a_0' is the mean depth, 'c' is the wave speed, 'λ' is the wavelength and's' is an arbitrary function twice differentiable varying along channel.

The equations (2.1)-(2.4) in the component are

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]-\eta\left[\frac{\partial^4 u}{\partial x^4}+\frac{\partial^4 u}{\partial y^4}+2\frac{\partial^4 u}{\partial x^2\partial y^2}\right]+\frac{KN}{\rho}\left[u^p-u\right] \tag{2.5}$$

$$\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{\partial p}{\partial y}+\mu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]-\eta\left[\frac{\partial^4 v}{\partial x^4}+\frac{\partial^4 v}{\partial y^4}+2\frac{\partial^4 v}{\partial x^2\partial y^2}\right]+\frac{KN}{\rho}\left[v^p-v\right] \tag{2.6}$$

$$u_t^p + u^p u_x^p + v^p u_y^p = \frac{k}{m}\left[u-u^p\right] \tag{2.7}$$

$$v_t^p + u^p v_x^p + v^p v_y^p = \frac{k}{m}\left[v-v^p\right] \tag{2.8}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.9}$$

$$N_t + N\left[u_x^p + v_y^p\right] = 0 \tag{2.10}$$

Suffices t, x, y denote differentiation with respect to the respective variable.

The boundary conditions are

$$u = 0 \qquad \text{on } y = \pm\, a_0 s[x] \tag{2.11}$$

$$v = \frac{dy}{dt} \qquad \text{on } y = \pm\, a_0 s[x] \tag{2.12}$$

$$\frac{d^2u}{dy^2} = 0 \quad ; \quad \frac{d^2v}{dy^2} = 0 \qquad \text{on } y = \pm\, a_0 s[x] \tag{2.13}$$

(2.11) corresponds to no slip on the boundary while (2.12) indicate the relative normal velocity is zero on the flexible boundary. (2.13) corresponds to the boundary condition related to couple stress fluids.

We define the non-dimensional variables as

$$x^* = \left[\frac{X - ct}{\lambda}\right] \quad ; \; y^* = \left[\frac{Y}{a_0}\right] \quad ; \; \varepsilon = \left[\frac{a_0}{\lambda}\right]$$

$$u^* = \left[\frac{u - c}{c}\right] \quad ; \; v^* = \left[\frac{v}{c\varepsilon}\right] \quad ; \; p^* = \left[\frac{p}{\rho c^2}\right]$$

$$u^{p^*} = \left[\frac{u^p - c}{c}\right] \quad ; \; v^{p*} = \left[\frac{v^p}{c\varepsilon}\right]$$

Substituting these non-dimensional variables in (2.5)-(2.8), these equations reduces to (after dropping the asterisks)

$$R\varepsilon\left[u\,u_x + \mathrm{v}\,u_y\right] = \left[-Rp + \varepsilon^2 u_{xx} + u_{yy} - RS\,\varepsilon^4 u_{xxxx} - SR\,u_{yyyy}\right] + \left[-2SR\,\varepsilon^2 u_{xxyy}\right] + \left[\frac{\alpha R}{\tau}\right]\left[u^p - u\right] \tag{2.14}$$

$$R\varepsilon^2\left[u\,\mathrm{v}_x + \mathrm{v}\,\mathrm{v}_y\right] = \left[\varepsilon^3 \mathrm{v}_{xx} + \varepsilon \mathrm{v}_{yy} - SR\,\varepsilon^5 \mathrm{v}_{xxxx} - SR\,\varepsilon\,\mathrm{v}_{yyyy}\right] + \left[-2SR\,\varepsilon^3 \mathrm{v}_{xxyy}\right] + \varepsilon\left[\frac{\alpha R}{\tau}\right]\left[\mathrm{v}^p - \mathrm{v}\right] \tag{2.15}$$

$$\varepsilon\left[u^p\,u^p_x + \mathrm{v}^p\,u^p_y\right] = \left[\frac{u - u^p}{\tau}\right] \tag{2.16}$$

$$\varepsilon^2\left[u^p\,\mathrm{v}^p_x + \mathrm{v}^p\,\mathrm{v}^p_y\right] = \left[\frac{\varepsilon\left[\mathrm{v} - \mathrm{v}^p\right]}{\tau}\right] \tag{2.17}$$

Where

$R = \dfrac{\rho c a_0}{\mu}$; Reynolds number

$S = \dfrac{\eta}{\rho c {a_0}^3}$; Couple stress parameter

$\alpha = \dfrac{Nm}{\rho}$; Dust concentration parameter

$\tau = \dfrac{mc}{k a_0}$; Relaxation time

$p = \varepsilon \dfrac{\partial p_0}{\partial x}$; is the imposed non-dimensional axial pressure gradient

The boundary conditions related to non-dimensional axial velocity relative to the moving frame and transverse velocities are (after dropping the asterisks)

$$u = -1 \quad ; \; v = -s_x \qquad \text{on} \quad y = \pm S[x] \tag{2.18}$$

$$\frac{d^2u}{dy^2} = 0 \quad ; \; \frac{d^2\mathrm{v}}{dy^2} = 0 \qquad \text{on } y = 0 \tag{2.19}$$

3. Method of solution

Under the long wave length approximation (ε<<1), we make use of the regular perturbation scheme to expand u, u^p, v and v^p in powers of ε and consider zeroth order approximations are

Equating the terms independent of ε, the zeroth order equations are

$$u_{0,yy} - RS\,u_{0,yyyy} = Rp \tag{3.1}$$

$$v_{0,yy} - RS\,v_{0,yyyy} = 0 \tag{3.2}$$

$$u_0^p = u_0 \tag{3.3}$$

$$v_0^p = v_0 \tag{3.4}$$

Under the above approximation the velocity of the fluid and the dust particles coincide.

The relative boundary conditions (3.1) and (3.2) are

$$u_0 = -1 \quad ; \quad v_0 = -s_x \quad \text{on} \quad y = \pm s \tag{3.5}$$

$$\frac{d^2u_0}{dy^2} = 0 \quad ; \quad \frac{d^2v_0}{dy^2} = 0 \quad \text{on} \quad y = \pm s \tag{3.6}$$

The equations to the first order in ε are

$$u_{1,yy} - RS\,u_{1,yyyy} + \left[\frac{\alpha R}{\tau}\right]\left[u_1^p - u_1\right] = R\left[u_0 u_{0,x} + v_0 u_{0,y}\right] \tag{3.7}$$

$$v_{1,yy} - RS\,v_{1,yyyy} + \left[\frac{\alpha R}{\tau}\right]\left[v_1^p - v_1\right] = R\left[u_0 v_{0,x} + v_0 v_{0,y}\right] \tag{3.8}$$

$$u_1 - u_1^p = \tau\left[u_0^p\, u_{0,x}^p + v_0^p\, u_{0,y}^p\right] \tag{3.9}$$

$$v_1 - v_1^p = \tau\left[u_0^p\, v_{0,x}^p + v_0^p\, v_{0,y}^p\right] \tag{3.10}$$

The boundary conditions relevant to (3.7) and (3.8) are

$$u_1 = 0 \quad ; v_1 = 0 \quad on \quad y = \pm s \tag{3.11}$$

$$\frac{\partial^2 u_1}{\partial y^2} = 0 \quad ; \frac{\partial^2 v_1}{\partial y^2} = 0 \quad on \quad y = \pm s$$

(3.12)

Solving the above equations subject to the appropriate boundary conditions, we obtain the expressions for zeroth and first order velocity components are

$$u_0 = B_1 + B_2 Ch[ty] + B_3 y^2 + B_4 \tag{3.13}$$

$$v_0 = B_5 \tag{3.14}$$

$$u_1 = E_1 + E_2 y + E_3 Sh[ty] + E_4 Ch[ty] + E_5 \tag{3.15}$$

$$v_1 = E_6 + E_7 y + E_8 Sh[ty] - E_9 Ch[ty] + E_{10} \tag{3.16}$$

The solutions for u_1^p and v_1^p are given by

$$u_1^p = u_1 - m\left[a_{39} + a_{40}\,Ch[ty] + a_{41}\,Ch[ty] + a_{42}\left[Ch[ty]\right]^2 + a_{43}\,y^2 + a_{44}\,y^2Ch[ty] + a_{45}\right]$$

$$+\left[a_{46}\,Ch[ty] + a_{47}\,Sh[ty] + a_{48}y\right] \tag{3.17}$$

$$\text{v}_1^p = \text{v}_1 - m\left[a_{49} + a_{50}\,Ch[ty] + a_{51}\,y^2 + a_{52}\right] \tag{3.18}$$

4. Shear stress and flux

The shear stress at the upper wall y =s (x), in the dimensional form is given by

$$T = \frac{\frac{1}{2}\left[\frac{\partial u}{\partial y} + \frac{\partial \text{v}}{\partial x}\right]\left[1 - \left(\frac{ds}{dx}\right)^2\right] + \left[\frac{\partial \text{v}}{\partial y} - \frac{\partial u}{\partial x}\right]\left[\frac{ds}{dx}\right]}{\left[1 + \left(\frac{ds}{dx}\right)^2\right]}$$ and is given by

$$\tau = \frac{\frac{1}{2}[A+B]\left[1-m^2\right] + [C-D]m}{\left[1+m^2\right]}$$

The volume flux of the fluid Q is given by the formula

$$Q = \int_0^S u\,dy$$ and is given by

$$Q = \left[\begin{array}{l} f_{24}S[x] + f_{25}Sh[ts[x]] + f_{26}S[x]^3 + f_{27}S[x] + f_{28}S[x] + \\ \\ f_{29}S[x]^2 + f_{30}Ch[ts[x]] + f_{31}Sh[ts[x]] + f_{32}S[x] \end{array}\right] - [\,f_{30}\,]$$

5. Discussion of the Problem and Numerical results

The axial and the transverse velocity of the fluid u , v as well as the dust particles u^p , v^p p are evaluated analytically to the first order approximation under the long wave length assumption. Their behaviour has been computationally evaluated for variation in the governing parameters R, S, α & τ. In general, U attains maximum on the mid-axis of the channel and gradually reduces to its prescribed value on the flexible boundary.

Fig 1 to 3. We observe that 'u' increases with R for fixed values of S and others parameters α etc and its enhancement in constricted and dilated parts of the channel is rapid in the entire flow field except in the vicinity of the boundary (fig 1). Fig 2 corresponding to the behaviour of u with S, the couple stress parameters for fixed R, α etc .We find that the magnitude of the axial velocity undergoes a slight depreciation with increase in S in the entire flow field both in constricted & dilated parts of the channel. It is interesting to note that in contrast to the earlier cases, the behaviour of u with increase in α, the dust concentration parameter in the constricted part is different from its behaviour in the dilated part, the other parameters R, S etc being fixed. Fig 3 to 4 depicts the transverse velocity for variations in the governing parameters R, S, α & τ. From these profiles we notice that this transverse velocity is upwards in the constricted part pushing the flexible upwards while in the dilated part is downwards with fluid drawn away from the boundary. For higher values of R this change of direction of transverse velocity takes place in the vicinity of the boundary. The magnitude of ‘ v ’ in general, increases with R in both constricted & dilated parts fixing the other parameters (Fig 3). Fig 5 to 6 corresponds the axial dust velocities profiles at the first order approximation. This first order axial dust velocity attains maximum in the mid-axis & gradually reduces to zero on the boundary. It is Interesting to observe that for any fixed S & α. The magnitude of u_1^p enhances rapidly with in an increase in R (Fig 5).For fixed value of R & α, a similar enhancement in $\left|u_1^p\right|$ may be noticed for an increase in S, this enhancement is moderate when R=5 but sufficiently rapid when R takes higher values $R \geq 10$. From fig 7 the magnitude of v_1^p enhances with ‘R’, fixing α &other parameters in both constricted& dilated channel likewise the magnitude of v_1^p with ‘S’ fixing other parameters. The stress on the upper wall enhances with R for fixed S & α. The stress slightly reduces with increase in S while with increase in alpha & slightly enhances for fixed R & S .Like wise the fluid flux enhances with R and slightly reduces with increase in S fixed alpha & R. For a given R & S the fluid flux enhances with α to a slight extent.

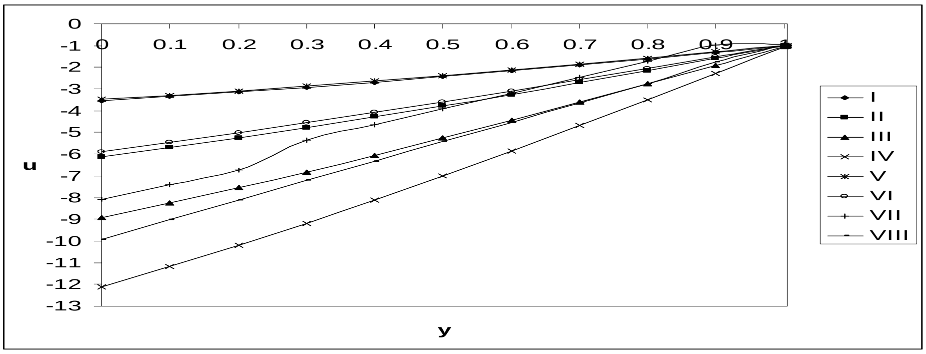

Fig.1.u with R when S=0.2, α =10, Є=0.01, P=1, $x = \dfrac{\pi}{6}$

	I	II	III	IV	V	VI	VII	VIII
R	5	10	15	20	5	10	15	20
β	0.005	0.005	0.005	0.005	-0.005	-0.005	-0.005	-0.00

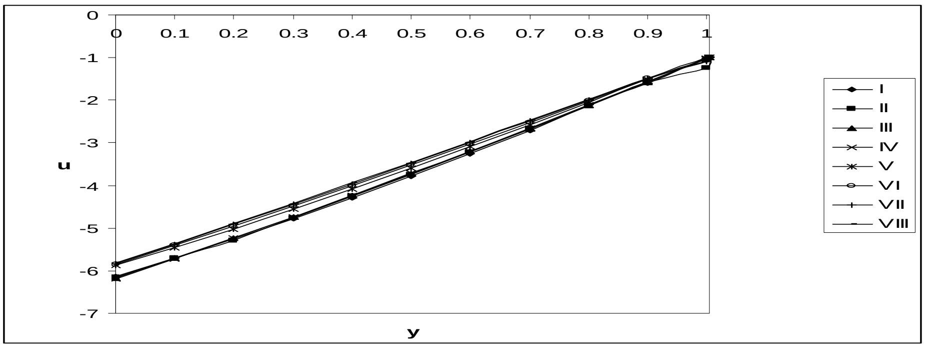

Fig.2.u with S when R =10, α =10, Є=0.01, P=1, x = $\frac{\pi}{6}$

	I	II	III	IV	V	VI	VII	VIII
S	0.2	0.3	0.4	0.5	0.2	0.3	0.4	0.5
β	0.005	0.005	0.005	0.005	-0.005	-0.005	-0.005	-0.005

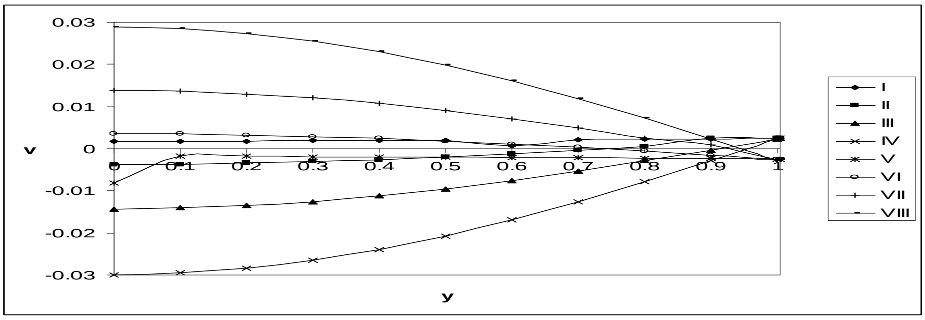

Fig.3.v with R when S=0.2, α =10, Є=0.01, P=1, x = $\frac{\pi}{6}$

	I	II	III	IV	V	VI	VII	VIII
R	5	10	15	20	5	10	15	20
β	0.005	0.005	0.005	0.005	-0.005	-0.005	-0.005	-0.005

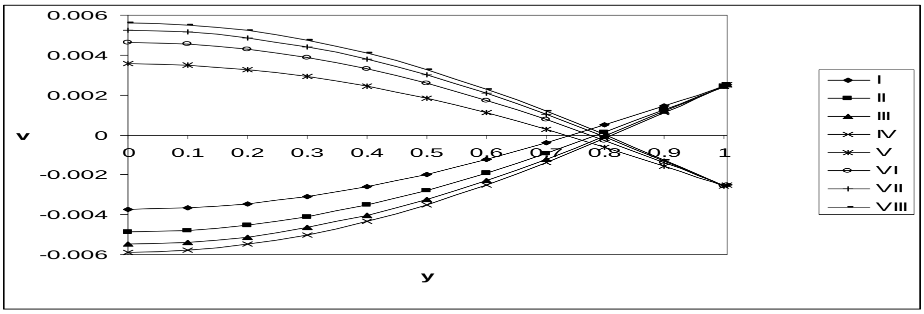

Fig.4.v with R when R =10, α =10, Є=0.01, P =1, x = $\frac{\pi}{6}$

	I	II	III	IV	V	VI	VII	VIII
S	0.2	0.3	0.4	0.5	0.2	0.3	0.4	0.5
β	0.005	0.005	0.005	0.005	-0.005	-0.005	-0.005	-0.005

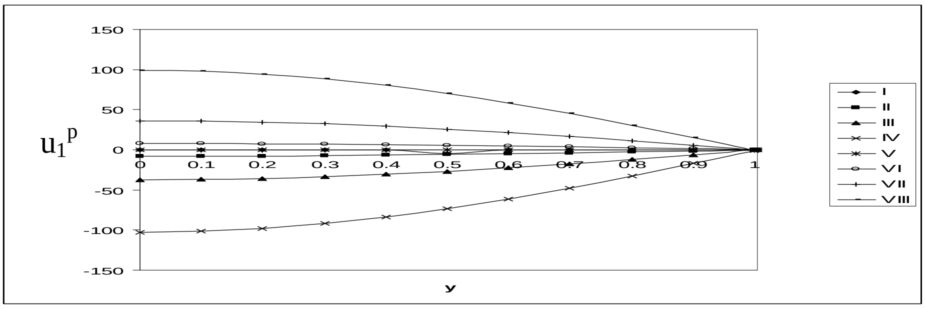

Fig.5. u_1^p with R when S=0.2, α =10, Є=0.01, P=1, $x = \dfrac{\pi}{6}$, τ =1

	I	II	III	IV	V	VI	VII	VIII
R	5	10	15	20	5	10	15	20
β	0.005	0.005	0.005	0.005	-0.005	-0.005	-0.005	-0.005

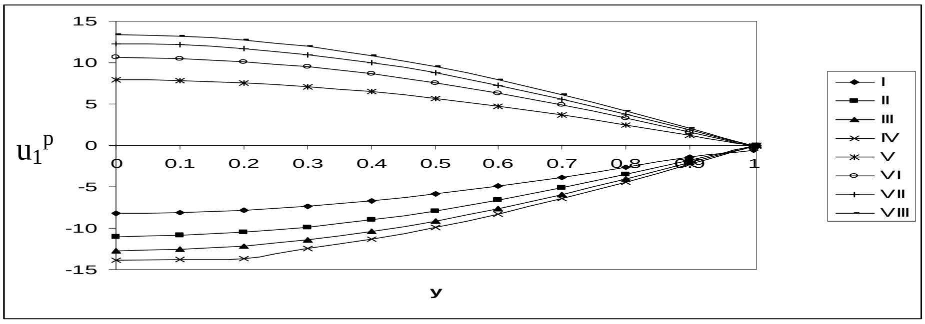

Fig.6. u_1^p with S when R =10, α =10, Є=0.01, P=1, $x = \dfrac{\pi}{6}$, τ =1

	I	II	III	IV	V	VI	VII	VIII
S	0.2	0.3	0.4	0.5	0.2	0.3	0.4	0.5
β	0.005	0.005	0.005	0.005	-0.005	-0.005	-0.005	-0.005

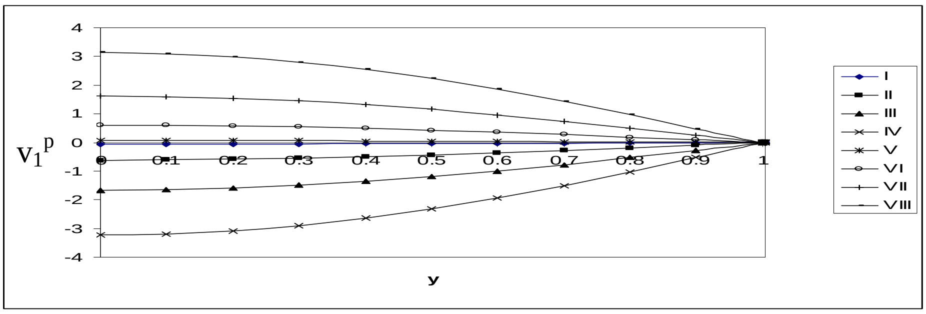

Fig.7. v_1^p with R when S=0.2, α =10, Є=0.01, P=1, $x = \dfrac{\pi}{6}$, τ =1

	I	II	III	IV	V	VI	VII	VIII
R	5	10	15	20	5	10	15	20
β	0.005	0.005	0.005	0.005	-0.005	-0.005	-0.005	-0.005

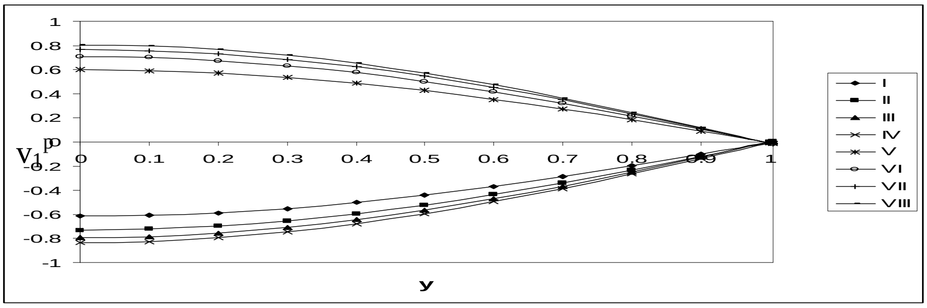

Fig.8. v_1^p with S when R =10, α =10, Є=0.01, P=1, $x = \frac{\pi}{6}$, τ =1

	I	II	III	IV	V	VI	VII	VIII
S	0.2	0.3	0.4	0.5	0.2	0.3	0.4	0.5
β	0.005	0.005	0.005	0.005	-0.005	-0.005	-0.005	-0.005

STRESS AT THE UPPER WALL (τ)

y =1.0043301, t = Π\2, x =2.355, β=0.005

S	I	II	III	IV	V	VI
0.2	1.488	2.844	4.348	6.132	6.878	7.624
0.3	1.431	2.788	4.324	6.165	7.023	7.881
0.4	1.398	2.759	4.314	6.187	7.107	8.028
0.5	1.376	2.741	4.309	6.202	7.162	8.123

	I	II	III	IV	V	VI
α	10	10	10	10	20	20
R	5	10	15	20	20	20

Table.1

FLUID FLUX (Q)

t = Π\2, x =2.355, β=0.005

S	I	II	III	IV	V	VI
0.2	-2.379	-3.714	-5.175	-6.865	-7.466	-8.066
0.3	-2.353	-3.695	-5.186	-6.923	-7.615	-8.307
0.4	-2.339	-3.686	-5.193	-6.957	-7.700	-8.443
0.5	-2.329	-3.681	-5.199	-6.979	-7.755	-8.531

	I	II	III	IV	V	VI
α	10	10	10	10	20	20
R	5	10	15	20	20	20

Table.2

6. Open Problem

In this paper we have studied the peristaltic flow of a dusty couple stress fluid a flexible channel by perturbation method. Instead of perturbation method, one can adopt some other techniques to investigate the behavior the peristaltic flow of a dusty couple stress fluid a flexible channel.

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