Int. J. Open Problems Compt. Math., Vol. 3, No. 5, December 2010
ISSN 1998-6262; Copyright © ICSRS Publication, 2010
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Natural Convection Heat Transfer in a Vertical Conical Annular Porous Medium

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Abstract

In this paper, we study the natural convection heat transfer in a saturated porous medium confined in a vertical annular porous medium. In this study Finite Element Method (FEM) has been used to solve governing partial differential equations. Results are presented interms of average Nusselt number ($\overline{\mathrm{Nu}}$), streamlines and Isothermal lines for various values of Rayleigh number (Ra), Cone angle (C_A) and Radius ratio (R_r).

Keywords: Radius ratio, Cone angle, Streamlines, Nusselt number, Rayleigh number

1 Introduction

Natural convection heat transfer in a saturated porous medium has a number of important and geophysical applications, such as nuclear reactor cooling system and underground energy transport. The problems of free convection about a vertical impermeable flat plate are studied by cheng and Minkowycz [1], Cheng [2], Na and Pop [3] Gorla and Zinalabedini [4]. The vertical cylinder cases are investigated by Minkowycz and Cheng [5], Kumari etal [6], Markin [7] and Basson et.al [8] Cheng et al. [9] use the local non-similarity method to analyze the natural convection of Darcial fluid about a cone. The effect of surface mass flux on a vertical flat plate [10] the similarity solution is possible only when the variations of the wall temperature and the transpiration rate are proportional to power-law of x measured from the leading edge. From practical point of view, however, the uniform mass flux may be easily realized. The effect of uniform surface mass flux on a vertical flat plate with uniform wall temperature is investigated by Merkin [11] and Minkowycz and Cheng [12]. Yücel [13], and Hwang and Chen [14] numerically study the vertical cylinder case.

2 FORMULATION OF THE PROBLEM

The inner surface of the cone is maintained at isothermal temperature T_h and outer surface is at ambient temperature T_∞. It may be noted that, due to axisymmetry, only a section on the annulus is sufficient for analysis purpose.

Continuity equation:

$$\frac{\partial (ru)}{\partial r} + \frac{\partial (rw)}{\partial z} = 0 \tag{1.2.1}$$

The velocity in r and z directions can be described by Darcy law as

Velocity in horizontal direction

$$u = \frac{-K}{\mu}\frac{\partial p}{\partial r} \tag{1.2.2}$$

velocity in vertical direction

$$w = \frac{-K}{\mu}\left(\frac{\partial p}{\partial z} + \rho g\right) \tag{1.2.3}$$

the permeability K of porous medium can be expressed as

$$K = \frac{D_p^2 \phi^3}{180(1-\phi)^2} \tag{1.2.4}$$

The variation of density with respect to temperature can be described by Boussinesq approximation as

$$\rho = \rho_\infty [\, 1 - \beta_T (T - T_\infty) \,] \tag{1.2.5}$$

Momentum Equation:

$$\frac{\partial w}{\partial r}-\frac{\partial u}{\partial z}=\frac{gK\beta}{\nu}\frac{\partial T}{\partial r} \qquad (1.2.6)$$

Energy equation:

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right) \qquad (1.2.7)$$

The continuity equation (1.2.1) can be satisfied by introducing the stream function ψ as

$$u=-\frac{1}{r}\frac{\partial \psi}{\partial z} \qquad (1.2.8)$$

$$w=\frac{1}{r}\frac{\partial \psi}{\partial r} \qquad (1.2.9)$$

The corresponding dimensional boundary conditions are

at $r = r_i$, $T = T_w$, $\psi = 0$ (1.2.10a)

at $r = r_0$, $T = T_\infty$, $\psi = 0$ (1.2.10b)

(except at z = 0)

The new parameters arising due to cylindrical co-ordinates system are

Non-dimensional Radius $\bar{r}=\frac{r}{L}$ (1.2.11a)

Non-dimensional Height $\bar{z}=\frac{z}{L}$ (1.2.11b)

Non-dimensional stream function $\bar{\psi}=\frac{\psi}{\alpha L}$ (1.2.11c)

Non-dimensional Temperature $\bar{T}=\frac{(T-T_\infty)}{(T_w-T_\infty)}$ (1.2.11d)

Rayleigh number $Ra=\frac{g\beta_T \Delta TKL}{\nu\alpha}$ (1.2.11e)

The non-dimensional equations for the heat transfer in vertical cone are

Momentum equation:

$$\frac{\partial^2 \overline{\psi}}{\partial \overline{z}^2} + \overline{r}\left(\frac{1}{\overline{r}}\frac{\partial \overline{\psi}}{\partial \overline{r}}\right) = \overline{r} Ra \frac{\partial \overline{T}}{\partial \overline{r}} \tag{1.2.12}$$

Energy equation :

$$\frac{1}{\overline{r}}\left[\frac{\partial \overline{\psi}}{\partial \overline{r}}\frac{\partial \overline{T}}{\partial \overline{z}} - \frac{\partial \overline{\psi}}{\partial \overline{z}}\frac{\partial \overline{T}}{\partial \overline{r}}\right] = \left(\frac{1}{\overline{r}}\frac{\partial}{\partial \overline{r}}\left(\overline{r}\frac{\partial \overline{T}}{\partial \overline{r}}\right) + \frac{\partial^2 \overline{T}}{\partial \overline{z}^2}\right) \tag{1.2.13}$$

The corresponding non-dimensional boundary conditions are

at $\overline{r} = \overline{r}_i$, $\overline{T}=1$, $\overline{\psi} = 0$ (1.2.14)

at $\overline{r} = \overline{r}_0$, $\overline{T}=0$, $\overline{\psi} = 0$ (1.2.14a)

3 SOLUTION OF THE PROBLEM

Let us consider that the variable to be determined in the triangular area as "T". The polynomial function for "T" can be expressed as:

$$T = \alpha_1 + \alpha_2 r + \alpha_3 z \tag{1.2.15}$$

The variable T has the value T_i, T_j & T_k at the nodal position i, j & k of the element. The r and z co-ordinates at these points are r_i, r_j, r_k and z_i, z_j, z_k respectively. Substitution of these nodal values in the equation (1.2.15) helps in determining the constants α_1 , α_2 , α_3 which are:

$$\alpha_i = {}^1/_{2A} \left[(r_j z_k - r_k z_j) T_i + (r_k z_i - r_i z_k) T_j + (r_i z_j - r_j z_i) T_k\right] \tag{1.2.16}$$

$$\alpha_2 = {}^1/_{2A} \left[(z_j - z_k) T_i + (z_k - z_i) T_j + (z_i - z_j) T_k\right] \tag{1.2.17}$$

$$\alpha_3 = {}^1/_{2A} \left[(r_k - r_j) T_i + (r_i - r_k) T_j + (r_j - r_i) T_k\right] \tag{1.2.18}$$

where A is area of the triangle given as

$$2A = \det\begin{vmatrix} 1 & r_i & z_i \\ 1 & r_j & z_j \\ 1 & r_k & z_k \end{vmatrix} \tag{1.2.19}$$

Substitution of α_1, α_2, α_3 in the equation (1.2.15) and mathematical arrangement of the terms results into

$$T = N_i T_i + N_j T_j + N_k T_k \tag{1.2.20}$$

In equation (1.2.20) N_i, N_j, N_k are the shape functions give by

$$N_m = \frac{a_m + b_m r + c_m z}{2A}, \text{m = i, j \& k} \tag{1.2.21}$$

Applying of Galerkin method to momentum equation (1.2.12) yields

$$\{R^e\} = -\int_A N^T \left(\frac{\partial^2 \overline{\psi}}{\partial \overline{z}^2} + \overline{r} \frac{\partial}{\partial \overline{r}} \left(\frac{1}{\overline{r}} \frac{\partial \overline{\psi}}{\partial \overline{r}} \right) + \overline{r} Ra \frac{\partial \overline{T}}{\partial \overline{r}} \right) dv \tag{1.2.22}$$

$$\{R^e\} = -\int_A N^T \left(\frac{\partial^2 \overline{\psi}}{\partial \overline{z}^2} + \overline{r} \frac{\partial}{\partial \overline{r}} \left(\frac{1}{\overline{r}} \frac{\partial \overline{\psi}}{\partial \overline{r}} \right) + \overline{r} Ra \frac{\partial \overline{T}}{\partial \overline{r}} \right) 2\Pi \overline{r} dA \tag{1.2.23}$$

where R^e is the residue. Considering individual terms of equation (1.2.23)

The differentiation of following term results into

$$\frac{\partial}{\partial \overline{r}} \left([N^T] \frac{\partial \overline{\psi}}{\partial \overline{r}} \right) = [N^T] \frac{\partial^2 \overline{\psi}}{\partial \overline{r}^2} + \frac{\partial [N^T]}{\partial \overline{r}} \frac{\partial \overline{\psi}}{\partial \overline{r}} \tag{1.2.24}$$

Thus $$\int_A N^T \frac{\partial^2 \overline{\psi}}{\partial \overline{r}^2} dA = \int_A \frac{\partial}{\partial \overline{r}} \left([N^T] \frac{\partial^2 \overline{\psi}}{\partial \overline{r}^2} \right) 2\Pi \overline{r} dA - \int_A \frac{\partial [N^T]}{\partial \overline{r}} \frac{\partial \overline{\psi}}{\partial \overline{r}} \tag{1.2.25}$$

The first term on right hand side of equation (1.2.24) can be transformed into surface integral by the application of Greens theorem and leads to inter-element requirement at boundaries of an element. The boundary conditions are incorporated in the force vector.

Making use of (1.2.20) produces

$$\int_A N^T \frac{\partial^2 \overline{T}}{\partial \overline{r}^2} 2\Pi \overline{r} dA = -\int_A \frac{\partial N^T}{\partial \overline{r}} \frac{\partial N}{\partial \overline{r}} \begin{Bmatrix} \overline{\psi}_1 \\ \overline{\psi}_2 \\ \overline{\psi}_3 \end{Bmatrix} dA \tag{1.2.26}$$

Substitution of (1.2.21) into (1.2.26) gives

$$= \frac{-1}{(2A)^2} \int_A \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} [b_1 b_2 b_3] \begin{Bmatrix} \overline{\psi}_1 \\ \overline{\psi}_2 \\ \overline{\psi}_3 \end{Bmatrix} 2\Pi \overline{r} dA$$

$$= -\frac{2\Pi \overline{R}}{4A} \begin{bmatrix} b_1^2 & b_1 b_2 & b_1 b_3 \\ b_1 b_2 & b_2^2 & b_2 b_3 \\ b_1 b_3 & b_2 b_3 & b_3^2 \end{bmatrix} \begin{Bmatrix} \overline{\psi}_1 \\ \overline{\psi}_2 \\ \overline{\psi}_3 \end{Bmatrix} \tag{1.2.27}$$

Similarly $$\int_A N^T \frac{\partial^2 \overline{\psi}}{\partial \overline{z}^2} 2\Pi \overline{r} \, dA = -\frac{2\Pi \overline{R}}{4A} \begin{bmatrix} c_1^2 & c_1 c_2 & c_1 c_3 \\ c_1 c_2 & c_2^2 & c_2 c_3 \\ c_1 c_2 & c_2 c_3 & c_3^2 \end{bmatrix} \begin{Bmatrix} \overline{\psi}_1 \\ \overline{\psi}_2 \\ \overline{\psi}_3 \end{Bmatrix} \tag{1.2.28}$$

The third term of equation (1.2.23) is

$$\int_A N^T \bar{r} Ra \frac{\partial \bar{T}}{\partial \bar{r}} 2\Pi \bar{r} dA = Ra \int_A N^T \bar{r} \frac{\partial \bar{T}}{\partial \bar{r}} 2\Pi \bar{r} dA \qquad (1.2.29)$$

In order to get the matrix equation of (1.2.29), the following method can be applied.

Defining the new area ratios as

$$M_k = \frac{area\ \ pij}{area\ \ ijk} \qquad M_i = \frac{area\ \ pjk}{area\ \ ijk} \qquad M_j = \frac{area\ \ pki}{area\ \ ijk}$$

It can be shown that

$$M_i = N_1 \qquad M_j = N_2 \qquad M_k = N_3 \qquad (1.2.30)$$

Replacing shape functions in equation (1.2.29) by (1.1.30) yields

$$\int_A N^T \bar{r} Ra \frac{\partial \bar{T}}{\partial \bar{r}} 2\Pi \bar{r} dA = \bar{r} Ra \int_A \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} \frac{\partial (N)}{\partial \bar{r}} \begin{bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \end{bmatrix} 2\Pi \bar{r} dA \qquad (1.2.31)$$

The area integration can be evaluate as

$$\int_A M_1^d M_2^e M_3^f = \frac{d!e!f!}{(d+e+f+2)!} 2A \qquad (1.2.32)$$

Application of equation (1.2.32) into (1.2.31) gives rise to:

$$= Ra \frac{A}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \frac{2\Pi \bar{R}^2}{2A} [b_1 + b_2 + b_3] \begin{bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \end{bmatrix} \qquad (1.2.33)$$

$$= \frac{2\Pi \bar{R}^2 Ra}{6} \begin{Bmatrix} b_1\bar{T}_1 + b_2\bar{T}_2 + b_3\bar{T}_3 \\ b_1\bar{T}_1 + b_2\bar{T}_2 + b_3\bar{T}_3 \\ b_1\bar{T}_1 + b_2\bar{T}_2 + b_3\bar{T}_3 \end{Bmatrix} \qquad (1.2.34)$$

Now the momentum equation (1.2.12) can be written in the matrix form as

$$\frac{2\Pi\bar{R}}{4A} \left\{ \begin{bmatrix} b^2 & b_1b_2 & b_1b_3 \\ b_1b_2 & b_2^2 & b_2b_3 \\ b_1b_3 & b_2b_3 & b_3^2 \end{bmatrix} + \begin{bmatrix} c_1^2 & c_1c_2 & c_1c_3 \\ c_1c_2 & c_2^2 & c_2c_3 \\ c_1c_2 & c_2c_3 & c_3^2 \end{bmatrix} \right\} \begin{Bmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \\ \bar{\psi}_3 \end{Bmatrix} + \frac{2\Pi\bar{R}^2 Ra}{6} \begin{Bmatrix} b_1\bar{T}_1 + b_2\bar{T}_2 + b_3\bar{T}_3 \\ b_1\bar{T}_1 + b_2\bar{T}_2 + b_3\bar{T}_3 \\ b_1\bar{T}_1 + b_2\bar{T}_2 + b_3\bar{T}_3 \end{Bmatrix} = 0 \qquad (1.2.35)$$

In simple form the above equation can be represented as:

$$[K_s]\{\bar{\psi}\} = \{f\} \tag{1.2.36}$$

where K_S is stiffness matrix and *f* is the force vector. For equation (1.2.12) they are:

$$[K_s] = \frac{2\Pi \bar{R}}{4A}\left\{\begin{bmatrix} b_1^2 & b_1 b_2 & b_1 b_3 \\ b_1 b_2 & b_2^2 & b_2 b_3 \\ b_1 b_3 & b_2 b_3 & b_3^2 \end{bmatrix} + \begin{bmatrix} c_1^2 & c_1 c_2 & c_1 c_3 \\ c_1 c_2 & c_2^2 & c_2 c_3 \\ c_1 c_3 & c_2 c_3 & c_3^2 \end{bmatrix}\right\} \tag{1.2.37a}$$

$$\{\bar{\psi}\} = \begin{Bmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \\ \bar{\psi}_3 \end{Bmatrix} \tag{1.2.37b}$$

$$\{f\} = \frac{2\Pi \bar{R}^2 Ra}{6}\begin{Bmatrix} b_1 \bar{T}_1 + b_2 \bar{T}_2 + b_3 \bar{T}_3 \\ b_1 \bar{T}_1 + b_2 \bar{T}_2 + b_3 \bar{T}_3 \\ b_1 \bar{T}_1 + b_2 \bar{T}_2 + b_3 \bar{T}_3 \end{Bmatrix} \tag{1.2.37c}$$

The radial distance $\bar{R}$ to the centroid of an element is given by relation

$$\bar{R} = \frac{\bar{r}_1 + \bar{r}_2 + \bar{r}_3}{3}$$

Similarly application of Galerkin method to Energy equation (1.2.13) gives

$$\{R^e\} = -\int_A [N]^T \left[\frac{1}{\bar{r}}\left(\frac{\partial\bar{\psi}}{\partial\bar{r}}\frac{\partial\bar{T}}{\partial\bar{z}} - \frac{\partial\bar{\psi}}{\partial\bar{z}}\frac{\partial\bar{T}}{\partial\bar{r}}\right) - \left(\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left\{\bar{r}\frac{\partial\bar{T}}{\partial\bar{r}}\right\} + \frac{\partial^2\bar{T}}{\partial\bar{z}^2}\right)\right] 2\Pi\bar{r}dA \tag{1.2.38}$$

Considering the terms individually of the above equation

$$\int_A [N]^T \frac{\partial\bar{\psi}}{\partial\bar{z}}\frac{\partial\bar{T}}{\partial\bar{r}} 2\Pi\, dA = \int_A \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} \frac{\partial[N]}{\partial\bar{z}}\{\bar{\psi}\}\frac{\partial[N]}{\partial\bar{r}}\{\bar{T}\} 2\Pi\,\bar{r}dA \tag{1.2.39}$$

$$= \frac{2\Pi A}{3} \text{ X } \frac{1}{4A^2}\left[c_1\bar{\psi}_1 + c_2\bar{\psi}_2 + c_3\bar{\psi}_3\right]\ [b_1, b_2, b_3]\begin{bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \end{bmatrix} \tag{1.2.40}$$

$$= \frac{2\Pi}{12A}\begin{Bmatrix} c_1\bar{\psi}_1 + c_2\bar{\psi}_2 + c_3\bar{\psi}_3 \\ c_1\bar{\psi}_1 + c_2\bar{\psi}_2 + c_3\bar{\psi}_3 \\ c_1\bar{\psi}_1 + c_2\bar{\psi}_2 + c_3\bar{\psi}_3 \end{Bmatrix}\ [b_1, b_2, b_3]\begin{bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \end{bmatrix} \tag{1.2.41}$$

Following the same above steps

$$\int_A [N]^T \frac{\partial\bar{\psi}}{\partial\bar{r}}\frac{\partial\bar{T}}{\partial\bar{z}} 2\Pi dA = \int_A \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}\frac{\partial[N]}{\partial\bar{r}}\{\bar{\psi}\}\frac{\partial[N]}{\partial\bar{z}}\{\bar{T}\} 2\Pi dA$$

$$\int_A N^T \frac{\partial \bar{\psi}}{\partial \bar{r}} \frac{\partial \bar{T}}{\partial \bar{z}} 2\Pi \, dA = \frac{2\Pi}{12\,A} \begin{Bmatrix} b_1\bar{\psi}_1 + b_2\bar{\psi}_2 + b_3\bar{\psi}_3 \\ b_1\bar{\psi}_1 + b_2\bar{\psi}_2 + b_3\bar{\psi}_3 \\ b_1\bar{\psi}_1 + b_2\bar{\psi}_2 + b_3\bar{\psi}_3 \end{Bmatrix} \left[c_1, c_2, c_3\right] \begin{bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \end{bmatrix}$$

The remaining two terms of Energy equation can be evaluated in similar fashion of equation (1.2.23)

$$\int_A N^T \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial \bar{T}}{\partial \bar{r}} \right) 2\Pi \, \bar{r} dA = -\frac{2\Pi \, \bar{R}}{4A} \begin{bmatrix} b_1^2 & b_1 b_2 & b_1 b_3 \\ b_1 b_2 & b_2^2 & b_2 b_3 \\ b_1 b_3 & b_2 b_3 & b_3^3 \end{bmatrix} \begin{bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \end{bmatrix}$$

$$\int_A N^T \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} 2\Pi \, \bar{r} dA = -\frac{2\Pi \, \bar{R}}{4A} \begin{bmatrix} c_1^2 & c_1 c_2 & c_1 c_3 \\ c_1 c_2 & c_2^2 & c_2 c_3 \\ c_1 c_3 & c_2 c_3 & c_3^2 \end{bmatrix} \begin{bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \end{bmatrix}$$

Thus the stiffness matrix of Energy equation is given by

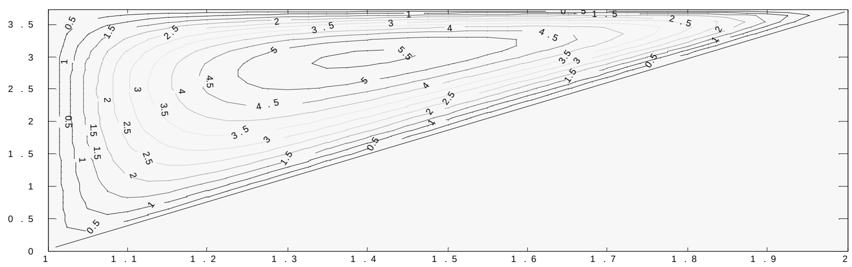
$$\left[\frac{2\Pi}{12A} \begin{Bmatrix} c_1\bar{\psi}_1 + c_2\bar{\psi}_2 + c_3\bar{\psi}_3 \\ c_1\bar{\psi}_1 + c_2\bar{\psi}_2 + c_3\bar{\psi}_3 \\ c_1\bar{\psi}_1 + c_2\bar{\psi}_2 + c_3\bar{\psi}_3 \end{Bmatrix} \left[b_1, b_2, b_3\right] - \frac{2\Pi}{12A} \begin{Bmatrix} b_1\bar{\psi}_1 + b_2\bar{\psi}_2 + b_3\bar{\psi}_3 \\ b_1\bar{\psi}_1 + b_2\bar{\psi}_2 + b_3\bar{\psi}_3 \\ b_1\bar{\psi}_1 + b_2\bar{\psi}_2 + b_3\bar{\psi}_3 \end{Bmatrix} \left[c_1, c_2, c_3\right] \right] \begin{bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \end{bmatrix}$$

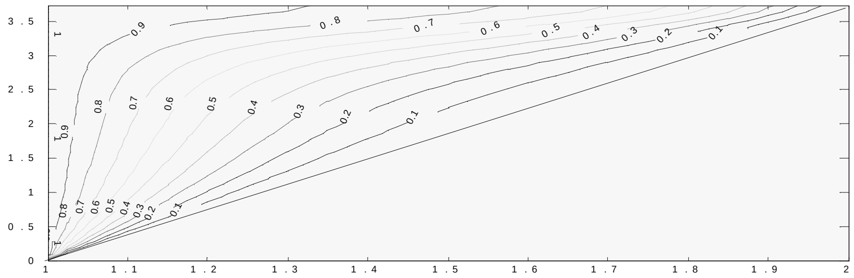
$$+ \frac{2\Pi \, \bar{R}}{4\,A} \left\{ \begin{bmatrix} b_1^2 & b_1 b_2 & b_1 b_2 \\ b_1 b_2 & b_2^2 & b_2 b_3 \\ b_1 b_3 & b_2 b_3 & b_3^3 \end{bmatrix} \begin{bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \end{bmatrix} + \begin{bmatrix} c_1^2 & c_1 c_2 & c_1 c_3 \\ c_1 c_2 & c_2^2 & c_2 c_3 \\ c_1 c_3 & c_2 c_3 & c_3^2 \end{bmatrix} \begin{bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \end{bmatrix} \right\} = 0 \qquad (1.2.42)$$

4 DISCUSSION OF THE PROBLEM

Results are obtained in terms of Nusselt number (Nu) at hot wall for various parameters such as Cone angle (C_A), Radius ratio (R_r) and Rayleigh number (Ra), when heat is supplied to vertical conical annular.

a)





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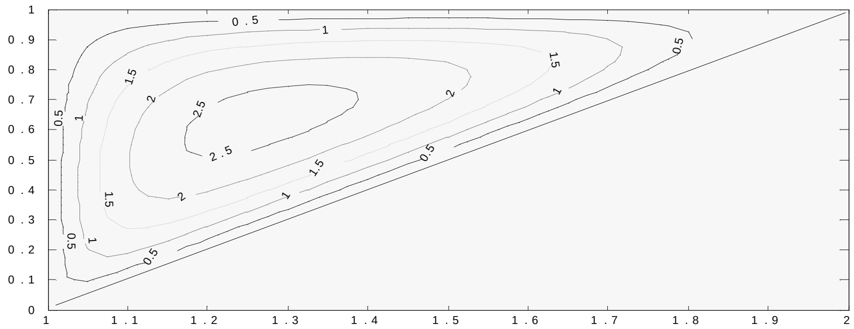


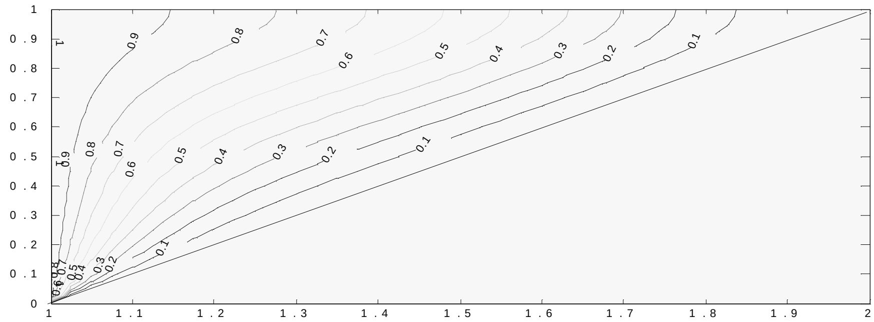
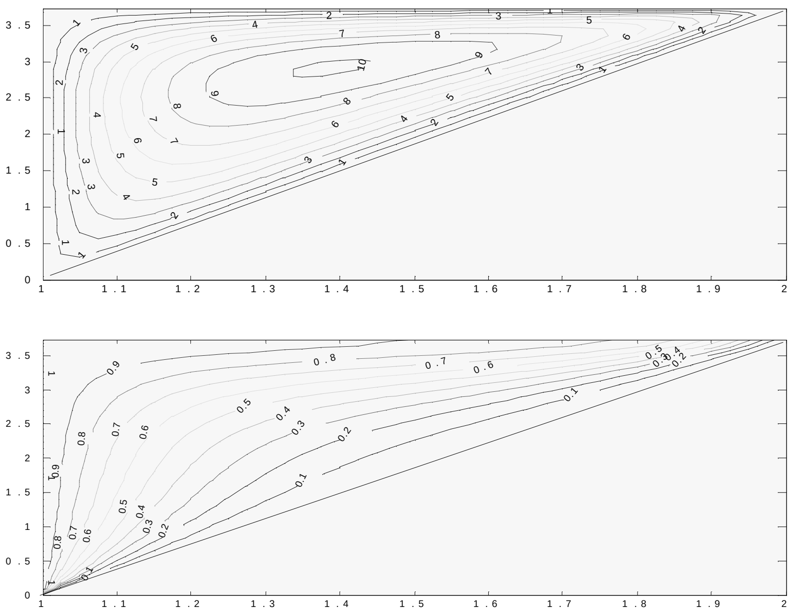


Fig:1.4.1: Streamlines(left) and Isotherms(Right) for Ra=50, R_r=1
a) C_A =15 b) C_A =45

Figure (1.4.1) shows the evaluation of streamlines and isothermal lines inside the porous medium for various values of Cone angle (C_A) at Ra = 50, R_r = 1. The magnitude of the streamlines decreases with the increase in Cone angles (C_A). The thermal bounded layer thickness decreases with the increase of Cone angles (C_A). It can be seen from streamlines and isothermal lines that the fluid movements shifts from lower portion of the hot wall to upper portion of the cold wall of the vertical annual cone with the increase of Cone angles (C_A). The circulation of the fluid covers almost whole domain at both lower and higher values of Cone angles (C_A) at 15^o.

Where the relation inversely proportion exists between streamlines and Cone angles (C_A). This trend is also observed with isothermal lines

a)



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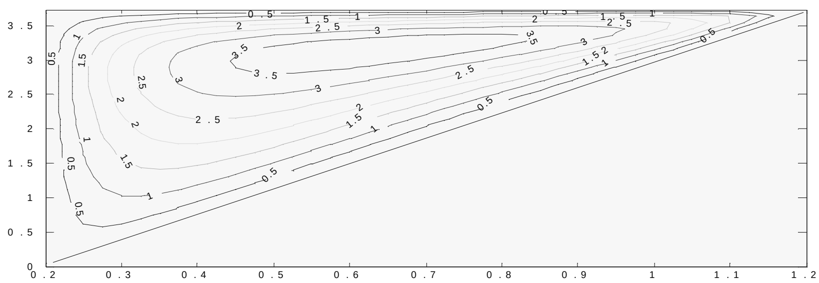

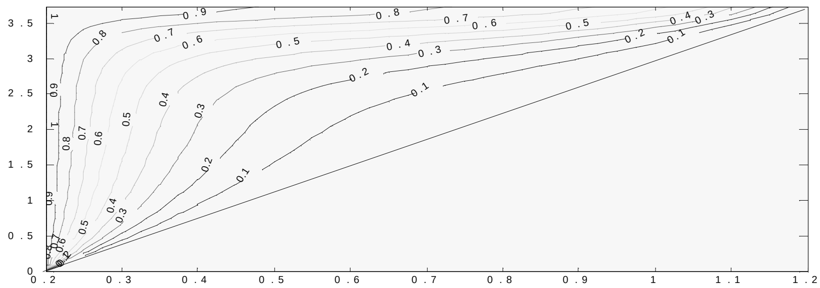

Fig:1.4.2: Streamlines(left) and Isotherms(Right) for Ra=100, C_A =15
a) R_r=1 b) R_r=5

Figure (1.4.5) shows the streamlines and isothermal lines inside the porous medium for various values of Radius ratio (R_r) at Ra = 100, and C_A = 15. The magnitude of the stream lines decreases with the increasing Radius ratio (R_r). The thermal boundary layer thickness decreases with the increase in Radius ratio (R_r). It can be seen from the stream lines and isothermal lines that the fluid movement shifts from the lower portion of the hot wall to the upper portion of the cold wall of the vertical annular cone with increasing Radius ratio (R_r).

5. Open Problem

In this paper we have studied Natural convection heat transfer in a vertical conical annular porous medium by finite element method. Instead of finite element method, one can adopt some other techniques to investigate the behavior the natural convection heat transfer in a vertical conical annular porous medium.

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