Int. J. Open Problems Compt. Math., Vol. 4, No. 1, March 2011
ISSN 1998-6262; Copyright ©ICSRS Publication, 2011
www.i-csrs.org

Nearest Symmetric Trapezoidal Approximation of Fuzzy Numbers

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Abstract

Many authors analyzed triangular and trapezoidal approximation of fuzzy numbers. But, to best of our knowledge, there is no method for symmetric trapezoidal fuzzy number approximation of fuzzy numbers. So, in this paper, we try to convert any fuzzy number into symmetric trapezoidal fuzzy number by using metric distance. This approximation helps us to avoid the computational complexity in the process of decision making problems. Moreover, we investigate some reasonable properties of this approximation. An application of this new method is also provided.

Keywords: Fuzzy number, metric distance, nearest symmetric trapezoidal approximation, fuzzy partition

1 Introduction

Fuzzy modelling has been used to model the phenomena arising in many branches of science and industry. The semantic rule of any linguistic variable associates a fuzzy set which is characterized by means of a membership function. The necessary point in fuzzy modeling is to assign membership functions corresponding to fuzzy numbers that represent vague concepts and imprecise terms expressed often in a normal language. This representation not only depends on the concept but also on the environment in which it is used. Hence, the problem of constructing meaningful membership functions is a difficult one. There are numerous methods for their constructions have been described in the literature.

The main simplification is realized via Defuzzification in [3, 10, 11, 18, 22, 23, 24] has been proposed as a method for summarizing a fuzzy number. The major idea was to obtain a typical value from a given fuzzy set according to some specific characteristics such as central gravity, median, etc. In other words, each defuzzification method provides a correspondence from the set of all fuzzy sets into the set of real numbers. Obviously, in a defuzzification method every fuzzy set is converted to a real number, therefore we generally lose too much important information.

Recently, there have been many research papers investigating on approximation of fuzzy numbers [9, 14, 29]. In 2001, Chanas [9] have introduced the notion of an approximation interval of a fuzzy number. In 2002, Grzegorzewski [14], have suggested a new approach to interval approximation of fuzzy numbers. The proposed operator, called the nearest interval approximation operator, leads to the interval which is the best one with respect to a certain measure of distances between fuzzy numbers. An interval approximation for fuzzy number is considered in which a problem in the fuzzy area is converted into one in the interval arithmetic area, which reduces the information loss.

To avoid the above problems, triangular and trapezoidal approximation is used. There have been many papers investigatingr triangular and trapezoidal approximation of fuzzy numbers [1, 4, 12, 25, 15, 16, 33, 35, 37]. In 1998 Delgado et al. [12] proposed two parameters (value and ambiguity) to obtain canonical representations of fuzzy number. In 2000, Ma et. al. [25] have used the concept of the symmetric triangular fuzzy number, and they have introduced a new method to defuzzy a general fuzzy quantity. The basic idea of their method was obtaining the nearest symmetric triangular fuzzy number for each fuzzy quantity. In 2004, Abbasbandy and Asady [1] introduced a fuzzy trapezoidal approximation using the metric (distance) between two fuzzy numbers.

In 2005, Grzegorzewski et al., [15] have suggested a new approach to trapezoidal approximation of fuzzy numbers, called the nearest trapezoidal approximation operator preserving expected interval and possesses many desired properties. Unfortunately, they have not noticed that in some situations their operator may fail to lead a trapezoidal fuzzy number. In 2007, Allahviranloo and Adabitabar Firozja [5] showed two examples illustrating such situations. Later, in 2007, Grzegorzewski, [16] proposed a corrected version of the trapezoidal approximation operator. In 2007, Yeh [33] studied an improvement of the nearest trapezoidal approximation operator preserving the expected interval. Also he discussed some properties about distance between a fuzzy number and its trapezoidal approximation. In 2007, Zeng and Li [37] proposed a weighted triangular approximation of a fuzzy number. Unfortunately, his approximation may fail to be a fuzzy number. In 2008, Yeh [35] have improved this approximation and propose a generalization by the name of weighted trapezoidal approximation and their algorithms are presented, also some examples and relevant properties have discussed. In 2009, Sridevi and Nadarajan [30] studied fuzzy similarity measure for generalized guzzy numbers. In 2010, Permana and Hashim [27] discussed generating fuzzy membership Function by using Particle swarm optimization technique. Recently, Abbasbandy and Hajjari [4] have proposed a new weighted trapezoidal approximation of an arbitrary fuzzy number, which preserves its cores.

Why do we need symmetric trapezoidal fuzzy number? The linguistic assessments are just approximate assessments, given by experts and accepted by decision maker whenever obtaining more accurate value is impossible or unnecessary. Whatever method is used one should be aware that in an environment of uncertainty it is reasonable to leave some area for variations in estimating membership functions. This is correct to our expectations that small variations in membership functions must not, as a result of our calculations, turn into major differences. When operating with fuzzy numbers, the result of our calculations strongly depend on the shape of the membership functions of these numbers. Less regular membership functions lead to more complex calculations. Moreover, fuzzy numbers with simpler shape of membership functions often have more perceptive and more natural interpretation. All these reasons as a natural need of simple approximations of fuzzy numbers those are easy to handle and have a natural interpretation. For the sake of simplicity, symmetric trapezoidal fuzzy numbers are good enough to capture and handle the fuzziness inherent in fuzzy numbers and to avoid the computational complexity. Hence, we propose symmetric trapezoidal membership function for fuzzy number and labels of linguistic variables.In this paper, we use metric distance between fuzzy numbers, to investigate nearest symmetric trapezoidal approximation of fuzzy number(NSTFNA).

In section 2, we recall some notions of fuzzy numbers. In section 3, we propose the nearest symmetric trapezoidal approximation of a fuzzy number with respect to metric distance and give its general calculating formula. In section 4, we give the relation between y-coordinate of the centroid point of fuzzy number and that its nearest symmetric trapezoidal approximation. In section 5, We discuss some properties of the approximation including translation invariant, scale invariant, identity, continuity, monotonic, expected interval, correlation coefficient and linear property. In section 6, we investigate an application to generation of fuzzy partitions. In section 7, we discuss the comparative example of our method and existing method and conclusion is given in section 8. Finally, we arise the open problem in section 9.

2 Preliminaries

Throughout this paper $\mathbb{R}$ stands for set of all real numbers, $F(\mathbb{R})$ stands for the set of all fuzzy numbers on $\mathbb{R}$. Let A is a fuzzy number and μ_A is its membership function, $T(A)$ is its nearest symmetric trapezoidal fuzzy number approximation.

Definition 2.1 *A fuzzy subset A of the real line $\mathbb{R}$ with membership function $\mu_A : \mathbb{R} \to [0, 1]$ is called fuzzy number if*

(i) *A is normal, i.e., there exist an element x_0 such that $\mu_A(x_0) = 1$.*

(ii) *A is fuzzy convex, i.e., $\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2)$, for all $x_1, x_2 \in \mathbb{R}$, for all $\lambda \in [0, 1]$.*

(iii) *μ_A is upper semi continuous.*

(iv) *Support of A is bounded, where $suppA = cl(\{x \in \mathbb{R} : \mu_A(x) > 0\})$ and cl is closure operator.*

$$\mu_A(x) = \begin{cases} f_A^L(x), & a \le x \le b, \\ 1, & b \le x \le c, \\ f_A^R(x), & c \le x \le d, \\ 0 & otherwise, \end{cases} \tag{1}$$

where f_A^L and f_A^R are nondecreasing and nonincreasing functions, respectively. The function f_A^L and f_A^R are also called the left side and right side of fuzzy number A, respectively.
A needful tool for dealing fuzzy numbers are their $r-cuts$. The $r-cut$ of a fuzzy number A is a nonfuzzy set defined as

$$A_r = \{x \in \mathbb{R} : \mu_A(x) \ge r\}. \tag{2}$$

A family $\{A_r : r \in (0,1]\}$ is a set representation of the fuzzy number A. According to the definition of fuzzy number it is seen at once that every $r-cut$ of a fuzzy number is a closed interval. Hence, we have

$$A_r = \left[\ \underline{A(r)},\ \overline{A(r)}\ \right] \tag{3}$$

where $\underline{A(r)} = inf\,\{x \in \mathbb{R} : \mu_A(x) \ge r\}$, $\overline{A(r)} = sup\,\{x \in \mathbb{R} : \mu_A(x) \ge r\}$. Also, Eq. (3) is called parametric form of fuzzy number A. If the sides of the fuzzy number A are strictly monotone then by Defn. (1) $\underline{A(r)}$ and $\overline{A(r)}$ are inverse function of f_A^L and f_A^R respectively.

Definition 2.2 *A fuzzy number A in parametric form is a pair $[\underline{A(r)}, \overline{A(r)}]$ of functions $\underline{A(r)}, \overline{A(r)}$, $0 < r \le 1$, which satisfies the following requirements:*
(i). $\underline{A(r)}$ is a bounded monotonic increasing left continuous function,
(ii). $\overline{A(r)}$ is a bounded monotonic decreasing left continuous function
(iii). $\underline{A(r)} \le \overline{A(r)}$, $0 < r \le 1$.

The trapezoidal fuzzy number $A = (x_0, y_0, \sigma, \beta)$, with two defuzzifier x_0, y_0 and left fuzziness $\sigma > 0$ and right fuzziness $\beta > 0$ is a fuzzy set where the membership function is as

$$\mu_A(x) = \begin{cases} \frac{x - x_0 + \sigma}{\sigma}, & x_0 - \sigma \le x \le x_0, \\ 1, & x_0 \le x \le y_0, \\ \frac{y_0 - x + \beta}{\beta}, & y_0 \le x \le y_0 + \beta, \\ 0 & otherwise, \end{cases} \tag{4}$$

and its parametric form is

$$A = [\ \ x_0 - \sigma + \sigma r, \quad y_0 + \beta - \beta r\ \ ] \tag{5}$$

If $\sigma = \beta$, then the trapezoidal fuzzy number A is called symmetric trapezoidal fuzzy number and it is denoted as $A = (x_0, y_0, \sigma)$.
If $x_0 = y_0$, then the trapezoidal fuzzy number A is called triangular fuzzy number and it is denoted by $A = (x_0, \sigma, \beta)$, also $\sigma = \beta$, then triangular fuzzy number is callded symmetric triangular fuzzy number and it is denoted by $A = (x_0, \sigma)$
The addition and scalar multiplication of fuzzy numbers are defined by the extension principle as follows. For arbitrary $A = [\underline{A(r)}, \overline{A(r)}]$, $B = [\underline{B(r)}, \overline{B(r)}]$ we defined (A + B) and multiplication by scalar $k > 0$ as

$$(i).\ \underline{(A+B)(r)} = \underline{A(r)} + \underline{B(r)}, \quad \overline{(A+B)(r)} = \overline{A(r)} + \overline{B(r)}$$
$$(ii).\ \underline{(kA)(r)} = k\underline{A(r)}, \quad \overline{(kA)(r)} = k\overline{A(r)}$$

Definition 2.3 *For arbitrary fuzzy numbers A and B their parametric form is $[\underline{A(r)}, \overline{A(r)}]$, $[\underline{B(r)}, \overline{B(r)}]$ respectively, then the distance between A and B is defined as*

$$d(A, B) = \sqrt{\int_0^1 (\underline{A(r)} - \underline{B(r)})^2 dr + \int_0^1 (\overline{A(r)} - \overline{B(r)})^2 dr} \tag{6}$$

simply,

$$D(A, B) = d^2(A, B) = \int_0^1 (\underline{A(r)} - \underline{B(r)})^2 dr + \int_0^1 (\overline{A(r)} - \overline{B(r)})^2 dr \tag{7}$$

Another important notion of connected with fuzzy numbers is an expected interval EI(A)of a fuzzy number A, introduced by Dubois and Prade[13] and Heilpern [19]. It is given by

$$EI(A) = \left[\int_0^1 \underline{A(r)} dr, \int_0^1 \overline{A(r)} dr\right]. \tag{8}$$

3 Nearest symmetric trapezoidal fuzzy number approximation

In this section, we propose the nearest symmetric trapezoidal fuzzy number approximation(NSTFNA) of a arbitrary fuzzy number with respect to our distance D is defined Eqn. (7).

Given a fuzzy number A and its parametric form $A = [\underline{A(r)}, \overline{A(r)}]$, for $r \in (0, 1]$. Our aim is to find a NSTFNA, $T(A) = (x_0, y_0, \sigma)$ that approximate fuzzy number of A, where $[x_0, y_0]$ is core of T(A) and σ is a left and right width of T(A). Also, T(A) is called NSTFNA, of A with respect to the distance D defined by eqn. (7). Also, the parametric form of T(A) is $[x_0 - \sigma + \sigma r, y_0 + \sigma - \sigma r]$.

Hence we have to minimize

$$\begin{aligned} D(x_0, y_0, \sigma) &= D(A, T(A)) \\ &= \int_0^1 (\underline{A(r)} - x_0 + \sigma - \sigma r)^2 dr + \int_0^1 (\overline{A(r)} - y_0 - \sigma + \sigma r)^2 dr \end{aligned}$$

In order to minimize $D(x_0, y_0, \sigma)$, we consider

$$\begin{aligned} \frac{\partial}{\partial x_0} D(x_0, y_0, \sigma) &= -2 \int_0^1 (\underline{A(r)} - x_0 + \sigma - \sigma r) dr \\ \frac{\partial}{\partial y_0} D(x_0, y_0, \sigma) &= -2 \int_0^1 (\overline{A(r)} - y_0 - \sigma + \sigma r) dr \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial \sigma} D(x_0, y_0, \sigma) &= 2 \int_0^1 [(\underline{A(r)} - x_0 + \sigma - \sigma r)](1 - r) dr \\ &\quad +2 \int_0^1 [(\overline{A(r)} - y_0 - \sigma + \sigma r)](1 - r) dr \\ &= 2 \int_0^1 (\underline{A(r)} - \overline{A(r)})(1 - r) dr - x_0 + y_0 + 4\sigma/3 \end{aligned}$$

and solve

$$\frac{\partial}{\partial x_0} D(x_0, y_0, \sigma) = 0, \ \frac{\partial}{\partial y_0} D(x_0, y_0, \sigma) = 0 \ and \ \frac{\partial}{\partial \sigma} D(x_0, y_0, \sigma) = 0.$$

The solution is

$$x_0 = \int_0^1 \underline{A(r)} dr + \sigma/2 \tag{9}$$

$$y_0 = \int_0^1 \overline{A(r)} dr - \sigma/2 \tag{10}$$

$$\sigma = 6 \int_0^1 (\overline{A(r)} - \underline{A(r)})(1 - r) dr - 3 \int_0^1 (\overline{A(r)} - \underline{A(r)}) dr \tag{11}$$

Moreover, we can get the Hessian matrix

$$det \left[\left(\frac{\partial^2 D(x_0, y_0, \sigma)}{\partial x_i \partial x_j} \right)_{x_i \, = \, x_0, \, y_0, \, \sigma, \quad x_j \, = \, x_0, \, y_0, \, \sigma} \right] = \frac{7}{3} > 0. \tag{12}$$

Eqn. (12) shows that x_0, y_0 and σ given by Eqs. (9), (10) and (11) minimize $D(A, T(A))$ and actually $d(A, T(A))$ simultaneously.
Therefore, we obtain the NSTFNA, $T(A) = (x_0, y_0, \sigma)$ for any fuzzy number A, where x_0, y_0 and σ are given by Eqs. (9), (10) and (11). Also, T is called NSTFNA operator.

Example 3.1 *Consider the Gaussian membership function*

$$\mu_A(x) = e^{-(x-\mu)^2/\sigma^2} \tag{13}$$

its parametric form is

$$\underline{A(r)} = \mu - \sigma\sqrt{-log\ r}, \qquad \overline{A(r)} = \mu + \sigma\sqrt{-log\ r} \tag{14}$$

Applying Eqs. (9),(10) and (11), then its NSTFNA T(A) is characterized by

$$\begin{aligned}
\sigma &= 6\int_0^1 2\sigma\sqrt{-log\ r}(1-r)dr - 3\int_0^1 2\sigma\sqrt{-log\ r}dr] \\
&= 3\frac{\sigma\sqrt{\pi}}{2}(4-\sqrt{2}) - 3\sigma\sqrt{\pi} \\
&= 3\sigma\sqrt{\pi}(1-\frac{1}{\sqrt{2}}) \\
x_0 &= \int_0^1 \mu - \sigma\sqrt{-log\ r}dr + \frac{3}{2}\sigma\sqrt{\pi}(1-\frac{1}{\sqrt{2}}) \\
&= \mu - \frac{\sigma\sqrt{\pi}}{2} + \frac{3}{2}\sigma\sqrt{\pi} - \frac{3}{2}\sqrt{2}\sigma\sqrt{\pi} \\
&= \mu - \sigma\sqrt{\pi}(\frac{3}{2\sqrt{2}} - 1) \\
y_0 &= \int_0^1 \mu + \sigma\sqrt{-log\ r}dr - \frac{3}{2}\sigma\sqrt{\pi}(1-\frac{1}{\sqrt{2}}) \\
&= \mu + \sigma\sqrt{\pi}(\frac{3}{2\sqrt{2}} - 1)
\end{aligned}$$

Hence, the NSTFNA is $(\mu - \sigma\sqrt{\pi}(\frac{3}{2\sqrt{2}}-1), \mu + \sigma\sqrt{\pi}(\frac{3}{2\sqrt{2}}-1), 3\sigma\sqrt{\pi}(1-\frac{1}{\sqrt{2}}))$.

In needs to point out Fig.1 shows that the NSTFNA for Gaussian membership function with mean is 2.5 and standard deviation is 1.

Example 3.2 *Consider the trapezoidal fuzzy number* $A = (t_1, t_2, t_3, t_4)$, *where* $[t_2, t_3]$ *is core of* A, t_1 *is left width and* t_4 *is right width, then* $\underline{A(r)} =$

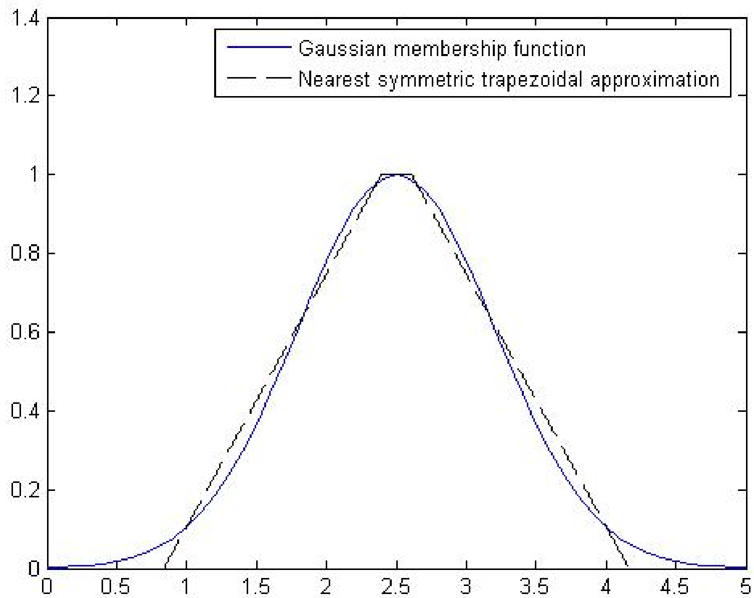

Figure 1: NSTFNA for Gaussion membership function .

$t_2 - t_1 + t_1 r$, $\overline{A(r)} = t_3 + t_4 - t_4 r$.

$$\begin{aligned}
\sigma &= 6\int_0^1 [(t_1 - t_2 + t_3 + t_4) - r(t_1 + t_4)](1-r)dr \\
&\qquad -3\int_0^1 [(t_1 - t_2 + t_3 + t_4) - r(t_1 + t_4)]dr \\
&= \frac{t_1 + t_4}{2} \\
x_0 &= \int_0^1 [t_2 - t_1(1-r)]dr + \frac{t_1 + t_4}{4} \\
&= t_2 + \frac{t_4 - t_1}{4} \\
y_0 &= \int_0^1 [t_3 + t_4(1-r)]dr - \frac{t_1 + t_4}{4} \\
&= t_3 + \frac{t_4 - t_1}{4}
\end{aligned}$$

Hence, the NSTFNA is $(t_2 + \frac{t_4 - t_1}{4},\ t_3 + \frac{t_4 - t_1}{4},\ \frac{t_1 + t_4}{2})$.

4 The y-coordinate of the centroid point

There are many distance based method for ranking fuzzy numbers by using the centroid points of fuzzy numbers [3, 10, 11]. In this section, we propose the relationship between centroid points of a fuzzy number and its NSTFNA. Let A be a fuzzy number and its parametric form is $A = [\underline{A(r)}, \overline{A(r)}]$, whose centroid point $(\overline{x}(A), \overline{y}(A))$ can be determined by the following formulas [7]:

$$\overline{x}(A) = \frac{\int_0^1 (1/2)((\overline{A(r)})^2 - (\underline{A(r)})^2)dr}{\int_0^1 (\overline{A(r)} - \underline{A(r)})dr}, \tag{15}$$

$$\overline{y}(A) = \frac{\int_0^1 (\overline{A(r)} - \underline{A(r)})rdr}{\int_0^1 (\overline{A(r)} - \underline{A(r)})dr} \tag{16}$$

If $T(A) = (x_0, y_0, \sigma)$ is NSTFNA of A, with parametric form $[x_0 - \sigma + \sigma r, y_0 + \sigma - \sigma r]$, then by Eqn.(16) the y-coordinate of the centroid point is simplified as follows:

$$\overline{y}(T(A)) = \frac{1}{3}(1 + \frac{y_0 - x_0}{2(y_0 - x_0 + \sigma)}) \tag{17}$$

We now claim that y-coordinate of a fuzzy number coincides with of its NSTFNA.

Lemma 4.1 $\overline{y}(A) = \overline{y}(T(A))$.

Proof: Eqs. (10)-(9) implies that

$$y_0 - x_0 = 2\int_0^1 (\overline{A(r)} - \underline{A(r)})(3r - 1)dr \tag{18}$$

$$y_0 - x_0 + \sigma = \int_0^1 (\overline{A(r)} - \underline{A(r)})dr \tag{19}$$

Therefore, by Eq. (17) is

$$\begin{aligned} \overline{y}(T(A)) &= \frac{1}{3}(1 + \frac{\int_0^1 (\overline{A(r)} - \underline{A(r)})(3r - 1)dr}{(\int_0^1 (\overline{A(r)} - \underline{A(r)})dr)}) \\ &= \frac{\int_0^1 (\overline{A(r)} - \underline{A(r)})rdr}{\int_0^1 (\overline{A(r)} - \underline{A(r)})dr} \\ &= \overline{y}(A) \end{aligned}$$

Hence $\overline{y}(T(A)) = \overline{y}(A)$.
The following lemma provides a criterion to determine the type of NSTFNA of fuzzy numbers interms of the y-coordinate of the centroid points.

Lemma 4.2 *Let A be a fuzzy number.*
(i). If $\overline{y}(A) > \frac{1}{3}$, then $T(A)$ is an exactly nearest symmetric trapezoidal fuzzy number.
(ii). If $\overline{y}(A) = \frac{1}{3}$, then $T(A)$ is an exactly nearest symmetric triangular fuzzy number.
(iii). If $\overline{y}(A) < \frac{1}{3}$, then $T(A)$ is not fuzzy number.

Proof: Let A be a fuzzy number with $\overline{y}(A) > \frac{1}{3}$. Then, by Eqn. (16), this is equaivalent to

$$2\int_0^1 (\overline{A(r)} - \underline{A(r)})(3r-1)dr > 0.$$

Also, by Eqn. (18), this is equivalent to $y_0 - x_0 > 0$. Hence, $T(A)$ is an exactly nearest symmetric trapezoidal fuzzy number.
If $\overline{y}(A) < \frac{1}{3}$ then $y_0 < x_0$. Hence $T(A)$ is not a fuzzy number. On the other hand, it is easy to show that $y_0 = x_0$ holds if and only if $\overline{y}(A) = \frac{1}{3}$. Hence, $T(A)$ is an exactly nearest symmetric triangular fuzzy number. This is complete the proof.
The following example shows the the NSTFNA of a fuzzy number may fail to lead the fuzzy number.

Example 4.1 *Let us conider a fuzzy number A whose membership function is*

$$\mu_A(x) = \begin{cases} \frac{x+2}{2}, & -2 \le x \le 0, \\ (1-x)^2, & 0 \le x \le 1, \\ 0 & otherwise, \end{cases} \tag{20}$$

Thus the parametric form is $A = [2r-2, 1-\sqrt{r}]$ for $r \in (0,1]$. By Eqn. (16), we get, $y(A) = \frac{10}{3} < \frac{1}{3}$. Hence, $T(A)$ is not a fuzzy number by lemma(4.2). In fact, by eqs.(9) and (10) we may get $x_0 = \frac{-3}{10}$ and $y_0 = \frac{-11}{30}$. So, $x_0 > y_0$. This is also show that $T(A)$ is not a fuzzy number.

Lemma 4.3 $\sigma \ge 0$ *for all fuzzy numbers.*

Proof: Let $A = [\underline{A(r)}, \overline{A(r)}]$, then

$$\begin{aligned}
\sigma &= 6\int_0^1 (\overline{A(r)} - \underline{A(r)})(1-r)dr - 3\int_0^1 (\overline{A(r)} - \overline{A(r)})dr] \\
&= 3\int_0^1 (1-2r)(\overline{A(r)} - \underline{A(r)})dr \\
&= 3\int_0^{1/2} (1-2r)(\overline{A(r)} - \underline{A(r)})dr + 3\int_{1/2}^1 (1-2r)(\overline{A(r)} - \underline{A(r)})dr
\end{aligned}$$

Let $r = \alpha$, $r = 1-\alpha$. Substituting into the first and second intergrals respectively, we get

$$\begin{aligned}
\sigma &= 3\int_0^{1/2} (1-2\alpha)(\overline{A(\alpha)} - \underline{A(\alpha)})dr \\
&\quad +3\int_{1/2}^1 (2\alpha-1)(\overline{A(1-\alpha)} - \underline{A(1-\alpha)})d\alpha \\
&= 3\int_0^{1/2} (1-2\alpha)[(\overline{A(\alpha)} - \overline{A(1-\alpha)}) + (\underline{A(1-\alpha)} - \underline{A(\alpha)})]d\alpha
\end{aligned}$$

As $\overline{A(\alpha)} - \overline{A(1-\alpha)} \geq 0 \; forall \; \alpha \leq 1/2$ and
$\underline{A(1-\alpha)} - \underline{A(\alpha)} \geq 0 \; forall \; \alpha \leq 1/2$.
Therefore the integral value is greater than or equal to zero.
Hence $\sigma \geq 0$.

5 Properties of NSTFNA

In this section, we discuss some properties NSTFNA. Also, If A is a fuzzy number, T(A) is its NSTFNA then Core(A) is $[x_0, y_0]$ and σ is left and right width of A and Core(T(A)) is $[T_{x_0}, T_{y_0}]$ and T_σ is left and right width of T(A).

Proposition 5.1 *The NSTFNA operator T is Translation invariance. i.e., $T(A+z) = T(A) + z$, for all $z \in \mathbb{R}$, and $A \in F(\mathbb{R})$*

Proof: For any fuzzy number $A = [\underline{A(r)}, \overline{A(r)}]$, for all $r \in (0,1]$ and arbitrary real number z, then we can get $A + z = [\underline{A(r)} + z, \overline{A(r)} + z]$, hence we have,

$$T_\sigma(A+z) = 6\int_0^1 [(\overline{A(r)} + z) - (\underline{A(r)} + z)](1-r)dr$$

$$
\begin{aligned}
& \quad -3\int_0^1 [(\overline{A(r)}+z)-(\underline{A(r)}+z)]dr \\
&= 6\int_0^1 (\overline{A(r)}-\underline{A(r)})(1-r)dr \\
& \quad -3\int_0^1 (\overline{A(r)}-\underline{A(r)})dr \\
&= \sigma \\
T_{x_0}(A+z) &= \int_0^1 (\underline{A(r)}+z)dr+\frac{\sigma}{2}=x_0+z \\
T_{y_0}(A+z) &= \int_0^1 (\overline{A(r)}+z)dr-\frac{\sigma}{2}=y_0+z
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
T(A+z) &= (x_0+z,\ y_0+z,\ \sigma) \\
&= (x_0,\ y_0,\ \sigma)+z \\
&= T(A)+z.
\end{aligned}
$$

This is complete the proof.

Proposition 5.2 *The NSTFNA operator T is scale of invariance. i.e., $T(\alpha A)=\alpha T(A)$, for all $\alpha \in \mathbb{R}-\{0\}$, $A \in F(\mathbb{R})$.*

Proof: Proof is similar to Proposition 4.1.

Proposition 5.3 *The NSTFNA operator T statisfies the identity property. i.e., $T(A)=A$, for all $A \in F(\mathbb{R})$ where, A is symmetric trapezoidal fuzzy number.*

Proof : Proof is obtained by straight forward from Example (3.2).

Proposition 5.4 *The NSTFNA operator T is continuous.i.e., for all $\epsilon > 0$, $\exists\ \delta > 0$, $d(A,B) < \delta \Rightarrow d(T(A),T(B)) < \epsilon$, for all $A,B \in F(\mathbb{R})$.*

Proof : For $A,B \in F(\mathbb{R})$ and its parametric form is $A=[\underline{A(r)},\overline{A(r)}]$ and $B=[\underline{B(r)},\overline{B(r)}]$. Also, $T(A)=(x_0,y_0,\sigma_0)$ and $T(B)=(x_1,y_1,\sigma_1)$ is NSTFNA of A and B respectively. Then,

$$
\begin{aligned}
& D(T(A),T(B)) \\
& \quad = (\int_0^1 ((x_0-x_1)+(1-r)(\sigma_1-\sigma_0))dr)^2
\end{aligned}
$$

$$
\begin{aligned}
&\quad +(\int_0^1((y_0-y_1)+(1-r)(\sigma_0-\sigma_1))dr)^2 \\
&= (x_0-x_1)^2+(y_0-y_1)^2+\frac{2}{3}(\sigma_0-\sigma_1)^2 \\
&\quad +(\sigma_0-\sigma_1)(y_0-y_1-x_0+x_1) \\
&= (\int_0^1(\underline{A(r)}-\underline{B(r)})dr)^2+(\int_0^1(\overline{A(r)}-\overline{B(r)})dr)^2+\frac{1}{6}(\sigma_0-\sigma_1)^2 \\
&= \frac{5}{2}(\int_0^1(\underline{A(r)}-\underline{B(r)})dr)^2+\frac{5}{2}(\int_0^1(\overline{A(r)}-\overline{B(r)})dr)^2 \\
&\quad +6(\int_0^1(r(\underline{A(r)}-\underline{B(r)})dr)^2+6(\int_0^1(r(\overline{A(r)}-\overline{B(r)})dr)^2 \\
&\quad -3\int_0^1(\underline{A(r)}-\underline{B(r)})dr\int_0^1(\overline{A(r)}-\overline{B(r)})dr \\
&\quad +6\int_0^1(\overline{A(r)}-\overline{B(r)})dr\int_0^1(r(\underline{A(r)}-\underline{B(r)}))dr \\
&\quad -12\int_0^1 r(\overline{A(r)}-\overline{B(r)})dr\int_0^1 r(\underline{A(r)}-\underline{B(r)})dr \\
&\quad +6\int_0^1(\underline{A(r)}-\underline{B(r)})dr\int_0^1 r(\overline{A(r)}-\overline{B(r)})dr \\
&\leq \frac{5}{2}(\int_0^1(\underline{A(r)}-\underline{B(r)})dr)^2+\frac{5}{2}(\int_0^1(\overline{A(r)}-\overline{B(r)})dr)^2 \\
&\quad +6(\int_0^1(r(\underline{A(r)}-\underline{B(r)})dr)^2+6(\int_0^1(r(\overline{A(r)}-\overline{B(r)})dr)^2 \\
&\leq \frac{5}{2}\int_0^1(\underline{A(r)}-\underline{B(r)})^2dr+\frac{5}{2}\int_0^1(\overline{A(r)}-\overline{B(r)})^2dr \\
&\quad +6\int_0^1 r^2(\underline{A(r)}-\underline{B(r)})^2dr+6\int_0^1 r^2(\overline{A(r)}-\overline{B(r)})^2dr \\
&= \int_0^1(\underline{A(r)}-\underline{B(r)})^2(\frac{5}{2}+6r^2)dr+\int_0^1(\overline{A(r)}-\overline{B(r)})^2(\frac{5}{2}+6r^2)dr \\
&\leq \frac{17}{2}\int_0^1(\underline{A(r)}-\underline{B(r)})^2+\frac{17}{2}\int_0^1(\overline{A(r)}-\overline{B(r)})^2 \\
&= \frac{17}{2}D(A,B).
\end{aligned}
$$

Hence $D(T(A),T(B))\leq D(A,B)$. This is complete the proof.

Proposition 5.5 *The NSTFNA operator T preserving monotonic.i.e., $A\subseteq B\Rightarrow T(A)\subseteq T(B),\ \ forall\ A,B\in F(\mathbb{R})$.*

Proof : since $A \subseteq B$, there exist a function $k(r)$ *and* $h(r) \geq 0$, such that $\underline{A(r)} = \underline{B(r)} + k(r)$ and $\overline{A(r)} = \overline{B(r)} - h(r)$ for all $r \in (0, 1]$.

$$
\begin{aligned}
T_\sigma(A) &= 6\int_0^1 (\overline{B(r)} - h(r) - \underline{B(r)} - k(r))(1-r)dr \\
&\quad -3\int_0^1 (\overline{B(r)} - h(r) - \underline{B(r)} - k(r))dr \\
&= 6\int_0^1 (\overline{B(r)} - \underline{B(r)})(1-r)dr - 3\int_0^1 (\overline{B(r)} - \underline{B(r)})dr \\
&\quad -6\int_0^1 h(r)(1-r)dr - 6\int_0^1 k(r)(1-r)dr \\
&\quad +3\int_0^1 h(r)dr + 3\int_0^1 h(r)dr \\
&= T_\sigma(B) + 3\int_0^1 h(r)(2r-1)dr + 3\int_0^1 k(r)(2r-1)dr \\
&\geq T_\sigma(B) \\
T_{x_0}(A) &= \int_0^1 \underline{A(r)}dr + T_\sigma(A)/2 \\
&= \int_0^1 (\underline{B(r)} + k(r))dr + T_\sigma(A)/2 \\
&\geq \int_0^1 \underline{B(r)}dr + T_\sigma(B)/2 + k(\varsigma) \\
&\geq T_{x_0}(B) \\
T_{y_0}(A) &= \int_0^1 \overline{A(r)}dr - T_\sigma(A)/2 \\
&= \int_0^1 (\overline{B(r)} - h(r))dr - T_\sigma(A)/2 \\
&\leq \int_0^1 \overline{B(r)}dr + T_\sigma(B)/2 - h(\varsigma) \\
&\leq T_{y_0}(B)
\end{aligned}
$$

Hence $T(A) \subseteq T(B)$. This is complete the proof.

Proposition 5.6 *The NSTFNA operator T is order of invariant with respect to expected interval is defined by (8). i.e., $EI(T(A)) = EI(A)$ for all $A, B \in F(\mathbb{R})$.*

Proof : If $A = [\underline{A(r)}, \overline{A(r)}]$ be any fuzzy number and its NSTFNA is $T(A) = (x_0, y_0, \sigma)$ with parametric form is $[x_0 - \sigma + \sigma r, y_0 + \sigma - \sigma r]$, then

$$\begin{aligned} EI(T(A)) &= \left[\int_0^1 (x_0 - \sigma + \sigma r)dr, \int_0^1 (y_0 + \sigma - \sigma r)dr\right] \\ &= [x_0 - \sigma/2, y_0 + \sigma/2] \\ &= \left[\int_0^1 \underline{A(r)}dr, \int_0^1 \overline{A(r)}dr\right] \\ &= EI(A). \end{aligned}$$

This is complete the proof.

We can rank fuzzy numbers in many ways. Recently, Jimenez [21] proposed a ranking method based on the comparsion of expected intervals. Let $EI(A) = [E^L(A), E^R(A)]$ and $EI(B) = [E^L(B), E^R(B)]$ denote the expected intervals of fuzzy numbers A and B, respectively. Then we get the preference fuzzy relation M with the following membership function

$$M(A,B) = \begin{cases} 0, & E^R(A) - E^L(B) \le 0, \\ \frac{E^R(A)-E^L(B)}{E^R(A)-E^L(B)-(E^L(A)-E^R(B))}, & 0 \in [E^L(A) - E^R(B), \\ & \qquad E^R(A) - E^L(B)] \\ 1 & E^L(A) - E^R(B) > 0, \end{cases} \tag{21}$$

showing the degree of preference of A over B.

Proposition 5.7 *The NSTFNA operator T is order of invariant with respect to the preference relation M is defined by (21),i.e., $M(T(A), T(B)) = M(A, B)$, for all $A, B \in F(\mathbb{R})$*

Proof : Straight forward.

Let us consider another ranking method for fuzzy numbers is A is greater then B ($A \succ B$) if the expected value $EV(A)$ of fuzzy number A exceeds the expected value $EV(B)$ of a fuzzy number B, i.e.,

$$A \succ B \Leftrightarrow EV(A) > EV(B), \quad \forall A, B \in F(\mathbb{R}) \tag{22}$$

where $EV(A) = (1/2)[E^L(A) + E^R(A)]$

Proposition 5.8 *The NSTFNA operator T is order of invariant with respect to preference relation $\succ$ defined by (22), i.e., $A \succ B \Leftrightarrow T(A) \succ T(B)$ for all $A, B \in F(\mathbb{R})$.*

Proof : Straight forward.

In many application correlation between fuzzy number is of interest. Several authors have proposed different measures of correlation between membership functions. Hung [19] defined a correlation coefficient by means of expected interval is

$$\rho(A,B)=\frac{E^L(A)E^L(B)+E^R(A)E^R(B)}{\sqrt{(E^L(A)+E^R(A))^2}\sqrt{(E^L(B)+E^R(B))^2}} \tag{23}$$

Proposition 5.9 *The NSTFNA operator T is order of invariant with respect to correlation coefficient is defined by (23), i.e., $\rho(T(A),T(B))=\rho(A,B)$ for all $A,B\in F(\mathbb{R})$.*

Proof : Stight forward.

Proposition 5.10 *If A and B are any two fuzzy numbers, then*
(i). $T(A+B)=T(A)+T(B)$
(ii). $T(-A)=-T(A)$
(iii). $T(A-B)=T(A)-T(B)$

Proof : (i) If A and B are any two fuzzy numbers, its parametric form is $A=[\underline{A(r)},\overline{A(r)}]$ and $B=[\underline{B(r)},\overline{B(r)}]$ $\forall r\in(0,1]$. $T(A)=(x_1,y_1,\sigma_1)$ and $T(B)=(x_2,y_2,\sigma_2)$ are their NSTFNA.
Now,

$$\begin{aligned}
T_\sigma(A+B) &= 6\int_0^1[(\overline{A(r)}+\overline{B(r)})-(\underline{A(r)}+\underline{B(r)})](1-r)dr\\
&\quad -3\int_0^1[(\overline{A(r)}+\overline{B(r)})-(\underline{A(r)}+\underline{B(r)})]dr\\
&= 6\int_0^1(\overline{A(r)}-\underline{A(r)})(1-r)dr-3\int_0^1(\overline{A(r)}-\underline{A(r)})dr\\
&\quad +6\int_0^1(\overline{B(r)}-\underline{B(r)})(1-r)dr-3\int_0^1(\overline{B(r)}-\underline{B(r)})dr\\
&= \sigma_1+\sigma_2
\end{aligned}$$

similary,
$T_{x_0}(A+B)=x_1+x_2$, and $T_{y_0}(A+B)=y_1+y_2$.
Hence,

$$\begin{aligned}
T(A+B) &= (x_1+x_2,\ y_1+y_2,\ \sigma_1+\sigma_2)\\
&= (x_1,\ y_1,\ \sigma_1)+(x_2,\ y_2,\ \sigma_2)\\
&= T(A)+T(B).
\end{aligned}$$

(ii). If $A = [\underline{A(r)}, \overline{A(r)}]$, then $-A = [-\overline{A(r)}, -\underline{A(r)}]$
Now,

$$\begin{aligned}
T_\sigma(-A) &= 6\int_0^1 (-\underline{A(r)} + \overline{A(r)}(1-r)dr) - 3\int_0^1 (-\underline{A(r)} + \overline{A(r)})dr \\
&= \sigma \\
T_{x_0}(-A) &= \int_0^1 -\overline{A(r)}dr + \frac{T_\sigma(-A)}{2} \\
&= -[\int_0^1 \overline{A(r)}dr - \frac{\sigma}{2}] \\
&= -y_0 \\
T_{y_0}(-A) &= \int_0^1 -\underline{A(r)}dr - \frac{T_\sigma(-A)}{2} \\
&= -[\int_0^1 \underline{A(r)}dr + \frac{\sigma}{2}] \\
&= -x_0
\end{aligned}$$

Therefore,
$T(-A) = (-y_0, -x_0, \sigma) = -T(A)$.
(iii). $T(A - B) = T(A + (-B)) = T(A) + T(-B) = T(A) - T(B)$. This is complete the proof.

6 Applications

Fuzzy partition is vividly used in fuzzy control, bioinformatics, data mining, image processing and pattern recognition etc.,. So, in this section, we apply the NSTFNA to obtain a fuzzy partition from two extreme values. In order to obtain fuzzy partition, first by using our NSTFNA to merge the two fuzzy numbers into a single fuzzy quantity.
Let A and B are fuzzy numbers with parametric forms are$[\underline{A(r)}, \overline{A(r)}]$ and $[\underline{B(r)}, \overline{B(r)}]$ respectively. We try to find the NSTFNA, $T(x_0, y_0, \sigma)$ near both A and B, we minimize

$$\begin{aligned}
&K(x_0, y_0, \sigma) \\
&\quad = D(A, T(x_0, y_0, \sigma)) + D(B, T(x_0, y_0, \sigma)) \\
&\quad = \int_0^1 (\underline{A(r)} - (x_0 - \sigma + \sigma r))^2 dr + \int_0^1 (\overline{A(r)} - (y_0 + \sigma - \sigma r))^2 dr \\
&\qquad + \int_0^1 (\underline{B(r)} - (x_0 - \sigma + \sigma r))^2 dr + \int_0^1 (\overline{B(r)} - (y_0 + \sigma - \sigma r))^2 dr
\end{aligned}$$

In order to minimize, we consider

$$\frac{\partial K}{\partial x_0} = 0, \ \frac{\partial K}{\partial y_0} = 0 \text{ and } \frac{\partial K}{\partial \sigma} = 0.$$

Thus,

$$\begin{aligned}
& \frac{\partial K}{\partial x_0} = 0 \\
\Rightarrow \quad & -2\int_0^1 [\underline{A(r)} - (x_0 - \sigma(1-r))]dr \\
& \qquad -2\int_0^1 [\underline{B(r)} - (x_0 - \sigma(1-r))]dr = 0 \\
\Rightarrow \quad & x_0 = \frac{1}{2}[\int_0^1 (\underline{A(r)} + \underline{B(r)})dr + \sigma]
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial K}{\partial y_0} = 0 \\
\Rightarrow \quad & -2\int_0^1 [\overline{A(r)} - (y_0 + \sigma(1-r))]dr \\
& \qquad -2\int_0^1 [\overline{B(r)} - (y_0 + \sigma(1-r))]dr = 0 \\
\Rightarrow \quad & y_0 = \frac{1}{2}[\int_0^1 (\overline{A(r)} + \overline{B(r)})dr - \sigma]
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial K}{\partial \sigma} = 0 \\
\Rightarrow \quad & 2\int_0^1 (\underline{A(r)} - \overline{A(r)})(1-r)dr - x_0 + y_0 + \frac{4\sigma}{3} \\
& \qquad +2\int_0^1 (\underline{B(r)} - \overline{B(r)})(1-r)dr - x_0 + y_0 + \frac{4\sigma}{3} = 0 \\
\Rightarrow \quad & 2x_0 - 2y_0 - 2\int_0^1 [(\overline{A(r)} - \underline{A(r)}) + (\overline{B(r)} - \underline{B(r)})](1-r)dr = \frac{8\sigma}{3} \\
\Rightarrow \quad & 2\int_0^1 [(\overline{A(r)} - \underline{A(r)}) + (\overline{B(r)} - \underline{B(r)})](1-r)dr \\
& \qquad - \int_0^1 [(\overline{A(r)} - \underline{A(r)}) + (\overline{B(r)} - \underline{B(r)})]dr = \frac{2\sigma}{3} \\
& \sigma = 3\int_0^1 [(\overline{A(r)} - \underline{A(r)}) + (\overline{B(r)} - \underline{B(r)})](1-r)dr \\
& \qquad -\frac{3}{2}\int_0^1 [(\overline{A(r)} - \underline{A(r)}) + (\overline{B(r)} - \underline{B(r)})]dr.
\end{aligned}$$

Therefore,

$$x_0 = \frac{1}{2}[\int_0^1 (\underline{A(r)} + \underline{B(r)})dr + \sigma] \tag{24}$$

$$y_0 = \frac{1}{2}[\int_0^1 (\overline{A(r)} + \overline{B(r)})dr - \sigma] \tag{25}$$

$$\sigma = 3\int_0^1 [(\overline{A(r)} - \underline{A(r)}) + (\overline{B(r)} - \underline{B(r)})](1-r)dr - \frac{3}{2}\int_0^1 [(\overline{A(r)} - \underline{A(r)}) + (\overline{B(r)} - \underline{B(r)})]dr. \tag{26}$$

Now, using above Eqs. (24), (25) and (26) we can find fuzzy partition. To obtain fuzzy partition, we follow the following process

Step 1: Given the extreme values 0 and 1, we define the fuzzy number 'medium' A_1 as $\underline{A_1(r)} = \frac{r}{3}$ and $\overline{A_1(r)} = 1 - \frac{r}{3}$

Step 2: Merge $\overline{0}$ and A_1, then we can obtain 'lower medium' A_{21} for which

$$\begin{aligned} \sigma &= 3\int_0^1 (1-2r/3)(1-r)dr - \frac{3}{2}\int_0^1 (1-2r/3)dr = \frac{1}{6} \\ x_0 &= \frac{1}{2}[\int_0^1 r/3dr + 1/6] = \frac{1}{6} \\ y_0 &= \frac{1}{2}[\int_0^1 (1-r/3)dr - 1/6] = \frac{1}{3} \end{aligned}$$

Thus, $\underline{A_{21}(r)} = \frac{r}{6}$, $\overline{A_{21}(r)} = \frac{1}{2} - \frac{r}{6}$,

with the same reason, we can merge A_1 and 1 and get the 'upper medium' A_{23} with $\sigma = \frac{1}{6}$, $x_0 = \frac{2}{3}$ and $y_0 = \frac{5}{6}$.

That is, $\underline{A_{23}(r)} = \frac{3}{6} + \frac{r}{6}$, $\overline{A_{23}(r)} = 1 - \frac{r}{6}$

Step 3: Update the 'medium' by merging the 'lower medium'A_{21} and 'upper medium' A_{23}. The result is the 'medium' A_{22} with $\sigma = \frac{1}{6}$, $x_0 = \frac{5}{12}$ and $y_0 = \frac{7}{12}$.

That is, $\underline{A_{22}(r)} = \frac{1}{6} + \frac{r}{6}$, $\overline{A_{22}(r)} = \frac{3}{4} - \frac{r}{6}$.

Therefore, we obtained a fuzzy partition with five elements for two extreme values 0 & 1. shown in Fig.2.

$$P = \{0, A_{21}, A_{22}, A_{23}, 1\}.$$

The finer partition of nine elements

$$P' = \{0, A_{31}, A_{32}, A_{33}, A_{34}, A_{35}, A_{36}, A_{37}, 1\}$$

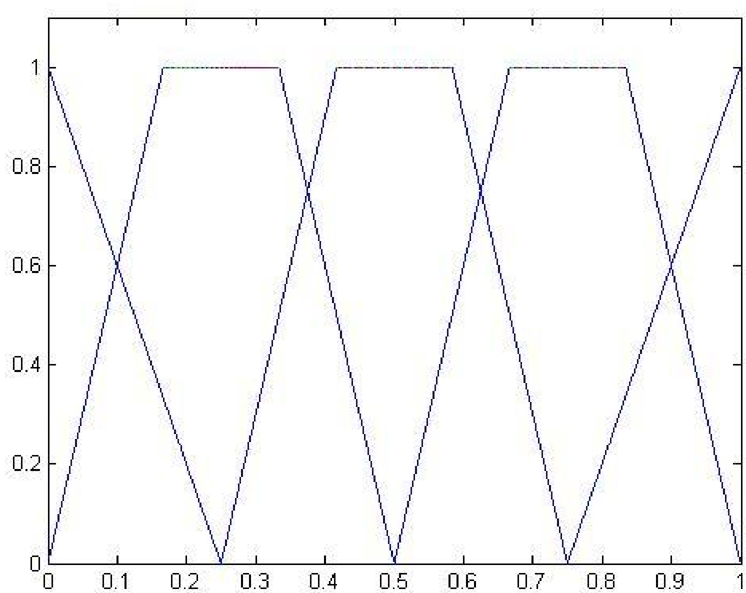

Figure 2: Fuzzy partition.

can be calculated using the same technique.
In fact, the process is applicable for two arbitrary fuzzy events as well as linguistic expression 'seldom', 'medium' and 'almost all'. It is easy to find that the fuzziness of its elements decreases when the fuzzy partition becomes finer.

7 Comparative example

In this section, we compare our method NSTFNA and previous (trapezoidal approximation) methods [4] and [15].

Example 7.1 *Suppose a fuzzy number A has membership function μ_A is given below*

$$\mu_A(x) = \begin{cases} 1 - (\frac{x-7}{17})^2, & -10 \le x \le 7, \\ 1, & 7 \le x \le 40 - 15\sqrt{2}, \\ 2(\frac{x-40}{10-40})^2, & 40 - 15\sqrt{2} \le x \le 40, \\ 0 & otherwise, \end{cases}$$

Then its trapezoidal approximation [15] $T(A)$ is $t_1(A) = -\frac{167}{15}$, $t_2(A) = \frac{37}{15}$, $t_3(A) = 40 - 16\sqrt{2}$ and $t_4(A) = 40 - 4\sqrt{2}$.
Our approach $T(A)$ is $x_0 = 3.3107$, $y_0 = 18.2142$ and $\sigma = 15.2874$. The comparative example of its membership functions shows in Fig. (3)

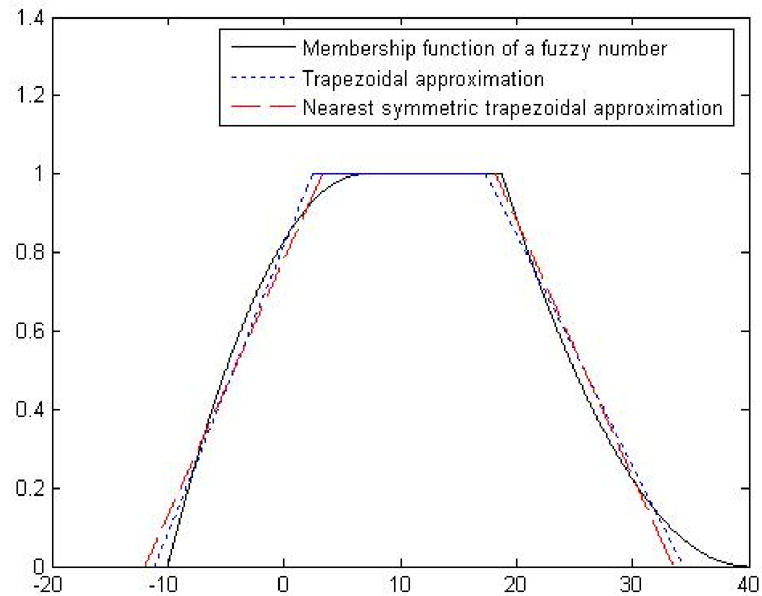

Figure 3: Comparative membership function for Example 7.1.

Example 7.2 *Suppose that a fuzzy number A has membership function μ_A is*

$$\mu_A(x) = \begin{cases} (\frac{x-1}{18})^2, & 1 \le x \le 19, \\ 1, & 19 \le x \le 24, \\ (\frac{30-x}{6})^2, & 24 \le x \le 30, \\ 0 & otherwise, \end{cases}$$

Then, in [15], its trapezoidal approximation $T(A)$ is $t_1(A) = -\frac{29}{5}$, $t_2(A) = \frac{101}{5}$, $t_3(A) = \frac{118}{5}$ and $t_4(A) = \frac{142}{5}$ and in [4], weighted trapezoidal approximation for its weight $f(\alpha) = \alpha$ $T(A)$ is $t_1(A) = 7.6857$, $t_2(A) = 19$, $t_3(A) = 24$ and $t_4(A) = 27.7714$.
In our approach, symmetric trapezoidal approximation$T(A)$ is $x_0 = \frac{89}{5}$, $y_0 =$

$\frac{106}{5}$ *and* $\sigma = \frac{48}{5}$. *The comparative example of its membership functions shows in Fig. (4). When we compare our result to other results, our result is only symmetric and other results are non-symmetric(see Fig.3 and Fig.4).*

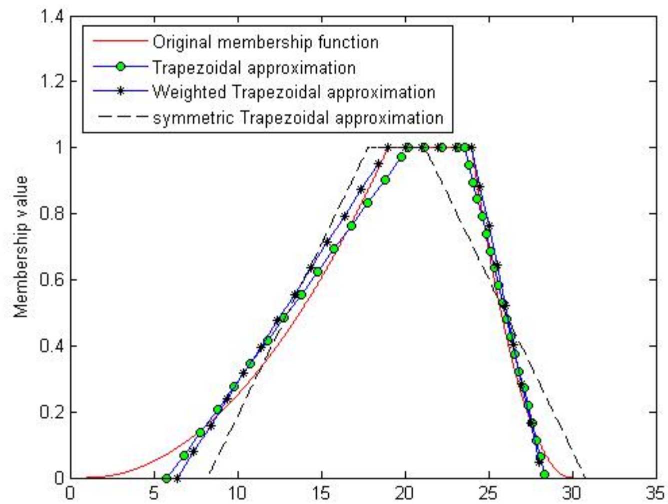

Figure 4: Comparative membership function for Example 7.2.

Remark 7.1 *In the symmetric fuzzy number case, our method coincides to the method in [15](see Ex. 3.1, Gaussian membership function). But non-symmetric case (Ex. 7.1 and 7.2), using our method NSTFNA we get new fuzzy membership function (see Fig. 3 and Fig. 4) which is different from [15].*

8 Conclusion

In this paper, we used a metric distance between fuzzy numbers to investigate symmetric trapezoidal approximation of arbitrary fuzzy numbers. The proposed operator, called nearest symmetric trapezoidal fuzzy number approximation operator which preserving many desired properties, i.e., translation invariant, scale invariant, identity, continuity, monotonic, expected interval, correlation coefficient and linear property. Also, we discussed the relationship between y-coordinate of centeroid point of fuzzy numbers and that its NSTFNA. Moreover, we provided an application to generation of fuzzy partitions.

9 Open Problem

In lemma (3.2) shows that, if the y-coordinate of the centroid point of a fuzzy number is less than $\frac{1}{3}$, then its NSTFNA is not a fuzzy number. In future, one may impose some conditions to form a NSTFNA eventhough the centroid point of a fuzzy number less than $\frac{1}{3}$.

Acknowledgements

The authors thanks anonymous referee for their valuable suggestions which greatly improved this paper.

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