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A Temporal Logic For Reasoning About Actions

Fatiha Mamache

LRIA Laboratoire de Recherche en Intelligence Artificielle.
Faculté d'Electronique et d'Informatique.
Département d'Informatique, USTHB, BP 32 El Alia, Bab Ezzouar, Alger.
e-mail: amamache @usthb.dz

Abstract

In this paper, we propose a formalism to represent temporal relationship between effects/events and actions. We use equivalence classes to represent competitive actions and simultaneous actions. We define operators which allow:

- ***To enumerate all effects/events that happen in the future caused by an action/event and,***
- ***Preconditions/events that happened in the past and gave place to an action/event. We define an operator that gives the possibility of representing the continuous evolutions of the universe for varied futures (prediction) or past (diagnostic). These operators might allow representation of actions and their effects as well as the types of reasoning which are the prediction, explanation and planning.***

Keywords: *Artificial Intelligence, Description Logic, Knowledge Representation, Reasoning on Actions, Temporal Logic.*

1 Introduction

Logical formalism has been one of the first proposed formalisms to represent knowledge, and is still the basis of much research in AI. Knowledge that we are interested must be able to express themselves through language, with expressions of the representation. In other words, the system must be relatively complete with respect to the targeted area. The lack of completeness leads us to adopt representation systems more complex or use more than one system.

Causality plays a prominent role in the context of reasoning about actions. Causality is at the basis of the ability to predict effects/events. A representation is poor without causality. A causal notion, in addition to classical logic, is necessary.

The intervening order of actions in some events plays a significant role; like carrying out an action before another, reproduction of an action (process) or to carry out several actions at the same time. This led to introduce operators on actions. These operators define constraints over time.

The temporal reasoning is to formalize the notion of time and provide means to represent and reason about the temporal aspects of knowledge. To describe the properties of applications, temporal logic formalisms are well suited, notably through their ability to express the sequence of actions /events in time.

An event can be the cause of one or more events in the future as it is often due to one or more events which happened in the past. To represent this, we define two operators:

- The operator Fk allows to enumerate events that will happen in the future caused by an event (prediction) and it can used to represent effects of an action (ramification).

- The operator Pk allows to enumerate events happened in the past which gave place to an event (diagnostic) as we can use it to enumerate preconditions of an action. These operators give us the possibility of representing the continuous evolutions of the universe for varied futures (prediction) or past (diagnostic).

In this context, we propose a complete temporal logic to reason about actions and their effects, as well as the types of reasoning which are prediction, explanation and planning. The originality of this work lies in the proposed formalism based on equivalence classes to represent simultaneous actions and competitive actions.

The paper is organized as follows. In the next section, we establish the formal background that will be used throughout this paper. Sections 3 and 4 are the core of the work, in section 3 we define syntax and semantics of the temporal logic Lc. We also define the valuation in the following cases:

- Case of the effects/events which require the realization of several actions at the same time. In this case, we represent the set of the actions occurred at the same time by the equivalence class of an action which is the representative of the class.

- Case of an action which is repeated in different time-element (process). We represent the set of the time-elements by the equivalence class of a time-element which is the representative of the class.

- Case of the competitive actions. We have two possibilities for the choice of the actions.

  (i) The agent is interested by the first action carried out (temporal choice)

  (ii) The agent is interested by the simplest action to carry out. We can generalize this with several actions a_1,a_2, $\cdots$, a_m.

  Section 4 is devoted to completude and in section 5 we conclude with a general idea of researches on actions theory based on description logics.

2 Language, notation and terminology

Definition 2.1 *Actions $a_1, a_2, ..., a_m$ are said to be direct causes of an event e if as soon as one of these actions is not executed , the event is not carried out.*

To represent this, we need to introduce the following language which is a first order language with equality :

- Connectors:$\neg, \vee, \wedge, \supset$ *and* $\supset_c$ (causal implication)
- Two signs of quantification noted $\exists$ and $\forall$.
- A symbol of equality, which we will note $\equiv$ to distinguish it from the sign $=$.
- A countable infinite collection of propositional variable.
- A set of operational signs or symbols functional.
- Three unary temporal operators: P_k (past), F_k (future), and P_0 (present).
- The expressions are the symbol strings on this alphabet.
- The set of the formulas noted Φ is by definition the smallest set of expressions which checks the following conditions :
  - Φ contains the propositional variables.
  - A set of elements called symbols of individuals.
  - If A and B are elements of Φ it is the same for $\neg A$ and $A \supset_c B$.
  - If A is an element of Φit is the same for P_k A, $F_k A$ and $P_0 A$.

The language, equally, contains :

- A set of elements called symbols of individuals.
- A set of operative signs or functional symbols.
- A set of relational signs or symbols of predicates.

To introduce causality J. Allen [4] uses the following formula: $Ecause(p_1, i_1, p_2, i_2)$.

It expresses, thus, the fact that p_1 which occurs in i_1 caused the event p_2 which occurs in i_2.

In the following e designate an effect of the action a or an event caused by the action a.

To express that an action a is the cause of an event e or an effect of a, we use the predicate $Ecause(a; e)$, that in the case of an atemporal expression of action type.
An action can be instantaneous as it can be carried out during in a certain interval of time [2], [3]. Consequently, the points and the intervals are necessary to express the execution time of an action.
We call time-element an interval or a point of time. Therefore, an action operates during a time-element [2], [3].
If a is a temporal expression of action type we use the following formulas :

- $t \cdot a$ if a is produced in the past at the element of time t.
- $a \cdot t$ if a it happens in the future at the element of time t.

We will keep the same notations in the case of an event (or effect) e:
- $e \cdot t$ for the future and,
- $t \cdot e$ for the past.

Example 2.2

(a) $Colloquium \cdot May$*, means: the colloquium will be held in May.*

(b) $May \cdot Colloquium$*, means: the colloquium was held in May.*

Let T a non empty set of time- elements, A a set of actions,
$A \cdot T$ (respectively $T \cdot A$) the set of elements $a \cdot t$ (respectively $t \cdot a$) and Dur; an application from $A \cdot T$ to $\mathbb{R}_+$ (respectively from $T \cdot A$ to $\mathbb{R}_+$) defined by [2], [3]:

$$\begin{cases} Dur(a \cdot t) = 0 & \text{if} \quad a \text{ is an instantaneous action, thus, } t \text{ is a point of time .} \\ Dur(a \cdot t) > 0 & \text{if} \quad a \text{ is a durative action, thus, } t \text{ is an interval.} \end{cases}$$

T is, thus, the union of two sets P and I, I is a set which elements are intervals and P a set which elements are points of time [5].
If a is an action carried out in t' then the predicate $Ecause(a.t'; e.t)$ expresses the fact that a carried out in t' is the cause of e true in t.

The actions seem first argument of the *Ecause* predicate.

The case where several actions $a_1, a_2, ..., a_m$ are the cause of the same effect or a single event is expressed by the formula: $Ecause(a_1, a_2, ..., a_m; e)$ defined by $Ecause(a_1, a_2, ..., a_m; e) \equiv Ecause(a_1; e) \wedge ... \wedge Ecause(a_m; e)$ where $a_1, a_2, ..., a_m$ are the atemporal expressions of actions type. This formula can be expressed as : $Ecause(a_1, a_2, ..., a_m; e) \equiv ((\exists k)(\neg a_k \supset_c \neg e))$, if $a_1, a_2, ..., a_m$ are direct causes of e.

Example 2.3 *Ecause(prepare one's paper, travelling, ..., communicate)* $\equiv$ *(*$\neg$ *travelling)* $\supset_c$ *(*$\neg$ *communicate)* $\vee$*(*$\neg$ *no prepare paper)* $\supset_c$ *(*$\neg$ *communicate)*$\vee$*...*

If $a_1, a_2, ..., a_m$ are the temporal expressions of actions type carried out respectively in $t_1, t_2, .., t_m$, we use the formula : $Ecause(a_1.t_1, a_2.t_2, ..., a_m.t_m; e.t) \equiv Ecause(a_1.t_1; e.t) \wedge ... \wedge Ecause(a_m.t_m; e.t)$.

Example 2.4 *Ecause (January* 2010. *prepare one's paper, send paper. April* 2010, *..., travelling.*15 *July* 2010 *; Communicate.*20 *July* 2010*)* $\equiv$ *Ecause(January* 2010 *. prepare one's paper; communicate.* 20*July* 2010*)*$\wedge$*...*$\wedge$ *Ecause (travelling.*15 *July* 2010*;communicate.*20 *July* 2010*).*

Example 2.5 *The fact of travelling on* 15 *July* 2010 *to communicate on* 20 *July* 2010 *can be expressed as follows :*

*(a) Ecause(travelling .*15 *July* 2010 *; communicate .*20 *July* 2010*) expresses: the agent will travel on* 15 *July* 2010 *and will communicate on* 20 *July* 2010*.*

(b) Ecause(15 *July* 2010*.traveling; communicate.*20 *July* 2010*) expresses: the agent travelled on* 15 *July* 2010 *and will communicate on* 20 *July* 2010*.*

(c) Ecause(15 *July* 2010*. travelling;* 20 *July* 2010*. communicate) expresses: the agent travelled on* 15 *July* 2010 *and communicated on* 20 *July* 2010*.*

As effects do not precede action so we cannot have:
The agent will travel on 15 *July* 2010 *and communicated on* 20 *July* 2010*.*
Ecause(travelling. 15 *July* 2010*;* 20 *July* 2010 *.communicate)*

An action a can be primitive as it can be complex. In the case of a complex action, to express that s actions $a_{i_1}, ..., a_{i_s}$ carried out in $t_{i_1}, ..., t_{i_s}$ (precondition) are the cause of a_i realized in t_i and this one will cause the effect (or event) e carried out in t we use the following expression

$$\begin{aligned} Ecause(a_i.t_i; t.e). &\equiv Ecause(a_{i_1}.t_{i_1}, a_{i_2}.t_{i_2}, ..., a_{i_s}.t_{i_s}; e.t) \\ &\equiv Ecause(a_{i_1}.t_{i_1}) \wedge Ecause(a_{i_2}.t_{i_2}) \wedge ... \wedge Ecause(a_{i_s}.t_{i_s}; e.t) \\ &\equiv \bigwedge_{j=1}^{j=s} Ecause(a_{ij}.t_{ij}; e.t). \end{aligned}$$

To represent the connection which link actions to its effects/events, we define the following application :

$$\begin{aligned} \Psi : A \times A \times ... \times A &\longrightarrow E \\ (a_1, a_2, \ldots, a_m) &\longmapsto a_1 \wedge a_1 \wedge \ldots \wedge a_m \equiv e. \end{aligned}$$

where E is the set of the events or effects, A: the set of the actions and $a_1, a_2, \cdot \cdot \cdot, a_m$ are the actions which are the cause of the achievement of e.

J. A. Pinto [6] established in his thesis a relation between events, actions and situations but he finds it more convenient to establish a relation between events, actions which occur for the realization of these events and the time when they are carried out. Indeed, there is a relationship between time, actions and effects/events [7]. We can see that in the following diagram:

$$\begin{tikzcd} A \times A \times ... \times A \arrow[r, "\Psi"] \arrow[d, "\varphi"'] & E \\ T \times T \times ... \times T \arrow[r] & T \arrow[u, "\phi"'] \end{tikzcd}$$

T is the set of the time-elements t_i where an action a_i is carried out so that an effect/event e occurs or is true in a time-element t and h is an application defined as follows :

$$\begin{aligned} h : T \times T \times ... \times T &\longrightarrow T \\ (t_1, t_2, \cdot, \cdot, \cdot, t_m) &\longmapsto h(t_1, t_2, \cdot, \cdot, \cdot, t_m) = t_1 \oplus t_2 \oplus \cdots \oplus t_m \equiv t. \end{aligned}$$

If E represents the set of effects of actions, the above diagram represents the relationship between time, actions and effects of these actions and if E represents the set of events caused by an action a, the above diagram represents the relationship between time, actions and events.

$\oplus$ is an operator defined on T as follows :

$t_1 \oplus t_2$ is defined if there are two actions a_1 and a_2 taking place in t_1 and t_2 respectively and which are the cause of an event (or effect) e carried out in a point of time t.

This operator has the following characteristics:

$\star$ The operator $\oplus$ is internal if $t \in T$ (the agent must act so that the event or effect takes place in time-element t belonging to T).

$\star$ The operator is commutative if the order of the actions does not intervene (the agent is free to start with any action). We denote: $t_1 \oplus t_2 \equiv t_2 \oplus t_1$.

The intervening order of the actions in some events plays a significant role; like carrying out an action before another, reproduction of an action (process)or to carry out several actions at the same time. This led us to introduce operators on the actions. These operators define constraints over time.

We define on T a relation of precedence noted R_c as follow : $t_1\ R_c\ t_2$ or rather t_1 precedes t_2 if the action a_1 must occur before the action a_2 (a_1 and a_2 being the actions which are the cause of e). The relation (T, R_c) is a strict order temporal framework. (T, R_c) and has the property of discretion, than (T, R_c) is a discrete temporal framework provided with a strict order.

An event can be the cause of one or more events in the future as it is often due to one or more events which proceeded in the past.

To represent this, we define the following operator which can be used to represent the effects, post and pre conditions for an action

$$\begin{array}{rccl} \otimes : & \mathbb{Z} \times T & \rightarrow & T \\ & (k, t) & \mapsto & \otimes(k, t) \equiv k \otimes t \end{array}$$

- If $k = 0$, then $k \otimes t = {}_0t$ where ${}_0t = t_1 \oplus t_2 \oplus \cdots \oplus t_m$ is time-element where e occurs at the present and where m is the number of actions which are the cause of e true in ${}_0t$. We denote $e = P_0 e$.

- If $k > 0$ then $k \otimes t = {}_kt$ where ${}_kt$ is time-element where the event $F_k e$ will occur in the future and which is due to e carried in ${}_0t = t_1 \oplus t_2 \oplus \cdots \oplus t_m$.

- If $k < 0$ then $k \oplus t = {}^kt$ where ${}^kt$ is time-element where the event denoted $P_k e$ which occurred in the past and gave place to e in ${}_0t = t_1 \oplus t_2 \oplus \cdots \oplus t_m$. Here, m is the intervening number of actions so that e is true in ${}_0t$, consequently, $F_k e$ (respectively $P_k e$) is true in ${}_kt$ (respectively in ${}^kt$).

  $|k|$ is the number of events $F_k e$ (respectively $P_k e$). The number of events $F_k e$ and $P_k e$ is not necessary the same.

The operator F_k will allow us to enumerate all effects/events that will happen in the future whereby e is the cause (ramification) and the operator $P_k e$ will allow us to enumerate all precondition/ events which happened in the past and which gave place to e. The operator $\otimes$ may give us the possibility of representing the continuous evolutions of the universe for varied futures (prediction) or past (diagnostic). It may allow the representation of the actions and their effects as well as the types of reasoning which are the prediction, the explanation and planning.

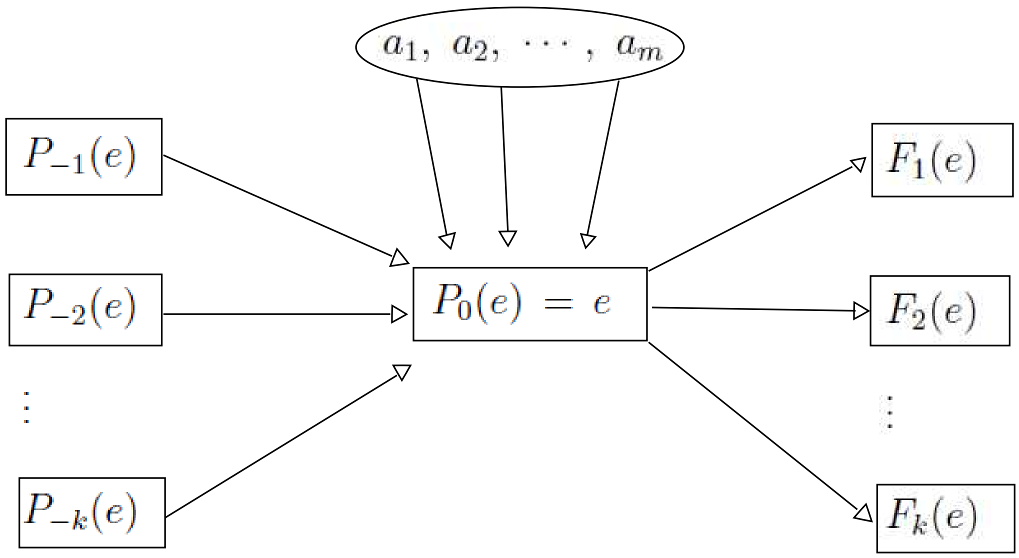

Fig 1: graph of representing relationships between actions and effects/events

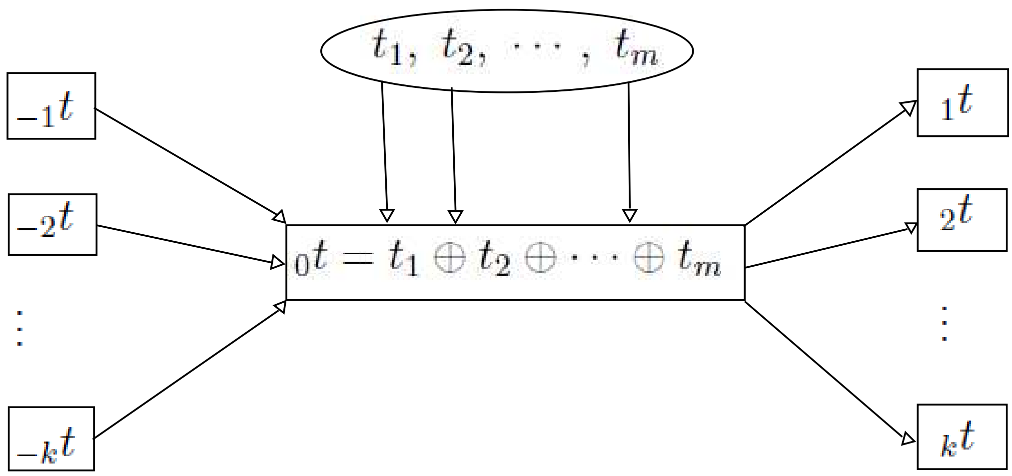

Fig 2: graph of representing temporal relationships between actions and effects/events.

3 Temporal Logic L_c

We propose a temporal logic for reasoning about actions.

3.1 deductive system

The axioms of the temporal logic L_c are:

(i) Axioms of propositional logic [8].

(ii) (a) $F_k(A \supset_c B) = (F_kA) \supset_c (F_kB)$ where $F_k(A \supset_c B)$ is the effect/event which will occur in the future and which will take place only if $A \supset_c B$ takes place.

($A \supset_c B$ is due to m actions $a_1, a_2, \cdots, a_m$)

(b) $P_k(A \supset_c B) = (P_kA) \supset_c (PkB)$ where P_kA is an event/precondition which occurred in the past and which gave place to $(A \supset_c B)$

(c) $P_0(A \supset_c B) = (P_0A) \supset_c (P_0B)$.

The axioms (ii) : (a), (b) and (c) express the distributivity of the operators F_k, P_k and P_0 with regard to the causal implication.

The rules of deductions are :

(i) The modus ponens [8].

(ii) Temporal generalization: If A is a theorem, F_kA, P_kA and P_0A are equally theorems.

The theorems of L_c are by definition all the formulas deductible from the axioms by using the rules of deductions. In particular all the theorems of propositional calculus are theorems.

3.2 semantic of L_c

In the semantic of propositional calculus, an assignment of values of truth V is an application, that each propositional variable associates a value of truth. An assignment of value of truth describes a state of the world.

In the case of L_c, we choose as variable propositional the actions whose effect occurs in a time-element t or actions which are the cause so that an event e is true in a time-element t.

Let V_c the valuation defined on the framework temporal (T, R_c) :

$$\begin{array}{rccl} V_c : & A & \rightarrow & P(T) \\ & ai & \mapsto & V_c(ai) = T_i = \{t_i / a_i trueint_i\} \end{array}$$

t_i is the time-element when the action a_i occurs so that the event e is true in ${}_0t = t_1 \oplus t_2 \oplus \cdots \oplus t_m$ or the effect e occurs in ${}_0t$.

The action a_i occurs only once in T, then $T_i = \{t_i\}$. The case of an action which reproduces in T will be studied later on.

If T_i is empty then, a_i is not true in t_i, consequently e will not be carried out in ${}_0t$.

Notation: $V_c(e)$= $V_c(a)$, e is an event caused by a.

Definition 3.1

1. *$V_c(P_0e) = V_c(e) = V_c(a_1 \wedge \cdots \wedge a_m) =_{def} V_c(a_1) \oplus \cdots \oplus V_c(a_m) \equiv \{t_1\} \oplus \{t_2\} \oplus \cdots \oplus \{t_m\} \equiv \{{}_0t\}$*

2. *$V_c\{\neg a_i\} = T - V_c\{a_i\} = T - T_i$*

3. *if $a_1, a_2, \cdots, a_m$ are direct causes of an event e than if there is k such that an action a_k does not take place in t_k, this would inevitably involve non-achievement of e (or e will not be true in $\{{}_0t\}$ accordingly :*

   $V_c\{e\} = V_c\{a_1 \wedge \cdots \wedge \neg a_k \wedge \cdots \wedge a_m\}$= $V_c\{a_1\} \oplus \cdots \oplus V_c\{\neg a_k\} \oplus \cdots \oplus V_c\{a_m\}$= $T_1 \oplus \cdots \oplus T - T_k \oplus \cdots \oplus T_m \equiv T - V_c(e)$.

4. *The effect/event e can give place to several effect/events in the future (ramification) noted F_ke, $k \geq 1$, and each effect/event will occur in a time-element ${}_kt$ with the following condition:*

   $t_i\ R_c\ {}_0t\ R_c\ {}_kt$ and ${}_0t\ = t_1 \oplus t_2 \oplus \cdots \oplus t_m$ then

   $V_c(F_ke)\ = \{{}_kt\ /t_i\ R_c\ {}_0t\ R_c\ {}_kt,\ {}_0t\ = t_1 \oplus t_2 \oplus \cdots \oplus t_m\}$.

5. *the event e can be due to several events P_ke which occurred in the past and each event P_ke occurred in a time-element ${}_kt$ with the following condition:*

   $t_i\ R_c\ {}_0t\ R_c\ {}_kt\ {}_0t\ = t_1 \oplus t_2 \oplus \cdots \oplus t_m$ and therefore :

   $V_c(P_ke)\ = \{{}^kt\ /t_i\ R_c\ {}_0t\ R_c\ {}^kt,\ {}_0t\ = t_1 \oplus t_2 \oplus \cdots \oplus t_m\}$.

6. *$Vc(A \supset_c B)\ = \{t/t_A\ R_c\ t_B\ R_c\ t,\ t = t_1 \oplus t_2 \oplus \cdots \oplus t_m\}$,*

   indeed $(A \supset_c B)$ is true in a certain time-element t pertaining to T only if A is true in one time-element t_A of T; but A true in t_A is the cause of B true in t_B, thus, to have B in t_B it is enough to have A in t_A and this will give $A \supset_c B$ true in t.

7. *$V_c\{A \cap B\}\ = V_c(A) \wedge V_c(B)$.*

8. *$V_c(A \sqcup B) = V_c(A) \vee V_c(B)$.*

We also define the valuation in the following cases :

- Case of the effects/events require the realization of several actions at the same time. For that we define on A a relation defined as follows :

  $$a_1 R a_2 \Leftrightarrow V(a_1) = V(a_2) \Leftrightarrow t_1 = t_2.$$

  It will ,thus, be said that a_1 and a_2 are in relation if they occur in even time. R is a relation of equivalence.

  We have the following diagram [9]:

  $$\begin{array}{ccc} A & \xrightarrow{V} & P(T) \\ s\downarrow & & \uparrow i \\ A/R & \xrightarrow{\overline{V_c}} & ImV \end{array}$$

  $\overline{V}_c(\overline{a}) = V(a)$, $i(t_1) = \{t_1\}$ and $s(a) = \overline{a} = \{a' \in A/a'Ra\}$ is the class of equivalence of a, it contains all the actions which occur at the same time as a, $ImV = \{s(a), a \in A\}$is a subset of $P(T)$ and A/R is the set of the classes of equivalence of the elements of A, it, thus ,contains the sets of the actions which occur in even-time.

  We can, thus, represent the set of the actions occurred at the same time by the class of equivalence of an action that is the representative of the class.

- Case of an action which is repeated in different time-element (process). Let

  $$\begin{array}{cccc} f: & T & \rightarrow & A \\ & t_i & \mapsto & a_i \end{array}$$

  We define on T a relation :

  $$t_1 R_p t_2 \Leftrightarrow a_1 = a_2$$

  it will ,thus, be said that t_1 and t_2 are in relation if the same action (a_1=a_2) occurs in t_1 and t_2, $t_1 \neq t_2$. R_p is a relation of equivalence. We have the following diagram [9] :

  $$\begin{array}{ccc} T & \xrightarrow{f} & A \\ s\downarrow & & \uparrow i \\ T/R_p & \xrightarrow{\overline{f}} & Imf \end{array}$$

  $T/R_p = \{\overline{t}/t \in T\}$, is the set of the classes of equivalence, $Imf \subset A$is the set of images of the elements of T, $\overline{t} = \{t_i \in T/tR_pt_i\}$ is the class of equivalence of t, it contains all the time-elements t_i where an action a produced in t and is reproduced in other time-element t_i (process).

Therefore, we represent the set of the time-elements when an action is repeated by the class of equivalence of a first time-element, it's the representative of the class. For this case one defines a valuation

$$\begin{array}{rcl} V_p : A & \rightarrow & P(T) \\ a & \mapsto & V_p(a) = \{t_i / \text{ a true in } t_i\} \end{array}$$

- Case of the competitive actions. Let a and a' two competitive actions for the realization of an effet/event e. We have several possibilities for the choice of the actions. For example:

  (i) The agent is interested by the first action carried out (temporal choice)

  (ii) The agent is interested by the simplest action to carry out.

For that we define on A the following relation : $a \ R_c \ a' \Leftrightarrow a'$ is negligible in front of a for the realization of e. In the first case a' negligible in front of a , it expresses the fact that action a is the first carried out. So, it is the action chosen by the agent. On the other hand, the agent is interested by the simplest action to carry out, a' negligible in front of a will express the fact that a is simpler action to realize than a'.

We define a valuation:

$$\begin{array}{rcl} V_c : \ A & \rightarrow & P(T) \\ a & \mapsto & V_c(a) = \{t_a / \text{a true in} t_a\} \end{array}$$

$V_c(a) = \{ta\}$ if a' is negligible in front of a if not $V_c(a) = \emptyset$.

We can generalize this with several actions a_1,a_2, $\cdots$, a_m.
$V_c(a_i) = \{t_{a_i}\}$ if a_j is negligible in front of a_i for any $j \neq i$ if not $V_c(a_i) = set$.

4 Completude

Is Axiomatic L_c complete for the class K of the temporal framework. For that, we must show the validity : Are the theorems valid formulas ?

Theorem 4.1 *(validity of L_c) Any theorem of L_c is a valid formula in the class K of the temporal framework. It should be checked that:*

(1) The axioms of L_c are valid formulas in K.

(2) The rules of deductions preserve the validity of the formulas : if their arguments are valid, their result is true.

***Proof**: Recall that the axioms of the temporal logic L_c are:*

(i) Axioms of propositional logic.

(ii) (a) $F_k(A \supset_c B) = (F_k A) \supset_(c) (F_k B)$ *where* $F_k(A \supset_c B)$ *is the effects/event which will occur in the future and which will take place only if* $A \supset_c B$ *takes place (*$A \supset_c B$ *is due to* m *actions* $a_1, a_2, \cdots, a_m$*).*

(b) $P_k(A \supset_c B) = (P_k A) \supset_c (P_k B)$ *where* $P_k A$ *is a precondition/event which occurred in the past and which gave place to* $(A \supset_c B)$

(c) $P_0(A \supset_c B) = (P_0 A) \supset_c (P_0 B)$.

Let us show that the axiom (a) is valid in K. Let us suppose that there is a temporal framework (T, R_c) *and a valuation* V_c *such as the value associated to* $F_k(A \supset_c B) =$

$(F_k A) \supset_c (F_k B)$ *is false at* ${}_k t \in T$.

Then, the formula $F_k(A \supset_c B)$ *is true and the formula* $(F_k(A) \supset_c F_k(B))$ *which express* $F_k(B)$ *is the cause of* $F_k(A))$ *is false .Then, we have* $F_k(A)$ *true and* $F_k(B)$ *false according to the definition of the semantic of* F_k, $V_c(F_k(B))$ $= \{{}_k t \ / \ F_k(B)$ *true in* ${}_k t$, $t_i \ R_c \ {}_0 t \ R_c \ {}_k t$ *and* ${}_0 t \equiv t_1 \oplus \cdots \oplus t_m\}$.

$F_k(B)$ *false in* ${}_k t$ *means that* B *is false in* ${}_0 t$. $F_k(A)$ *true in* ${}_k t$ *means that* A *is true in* ${}_0 t$*, but if* A *is true in* ${}_0 t$ *this would involve necessarily* B *true in* ${}_0 t$ *. So,* $F_k(A)$ *true and* $F_k(B)$ *in* ${}_k t$*, contradiction*

It results that it is impossible to build a model where the axiom (a) would be false. Consequently, this axiom is valid within any temporal framework of K.

Let us show that the axiom (b) is valid in K*. Let us suppose that there is a temporal framework* (T, R_c) *and a valuation* V_c *such as the value associated to* $P_k(A \supset B) = (P_k A) \supset_c (P_k B)$ *is false in* ${}_k t \in T$.

Then, the formula $P_k(A \supset_c B)$ *is true and the formula* $(P_k(A) \supset_c P_k(B))$ *which express* $P_k(B)$ *is the cause of* $P_k(A))$ *is false .Then, we have* $P_k(A)$ *true and* $P_k(B)$ *false according to the definition of the semantic of* F_k, $V_c(F_k(B))$ $= \{{}_k t \ / \ F_k(B)$ *true in* ${}_k t$, $t_i \ R_c \ {}_0 t \ R_c \ {}_k t$ *and* ${}_0 t \equiv t_1 \oplus \cdots \oplus t_m\}$.

$P_k(B)$ *false in* ${}_k t$ *means that* B *is false in* ${}_0 t$. $P_k(A)$ *true in* ${}_k t$ *means that* A *is true in* ${}_0 t$*, but if* A *is true in* ${}_0 t$ *this would involve necessarily* B *true in* ${}_0 t$ *. So,* $P_k(A)$ *true and* $P_k(B)$ *true in* ${}_k t$*, contradiction.*

It results that it is impossible to build a model where the axiom (b) would be false. Consequently, this axiom is valid within any temporal framework of K.

Recall *The rules of deductions of* L_c *are :*

(i) the modus ponens

(ii) Temporal generalization: if A *is a theorem, it is the same for the Formulas* $F_k(A)$, $P_k(A)$ *and* $P_0(A)$.

If A *is a theorem, thus a valid formula, then* $V(A) = T$

- $V_c(F_k(A)) = \{{}_k t \ / \ F_k(A)$ *true in* ${}_k t$, $t_1 \ R_c \ t_0 \ R_c \ {}_k t$ *and* ${}_0 t \equiv t_1 \oplus \cdots \oplus t_m\}$.

As $F_k(A)$ *is an event due to* A *then if* A *is true,* $F_k(A)$ *is true.*

Therefore, if $V_c(A) = T$ *then,* $V_c(F_k(A)) = T$*, thus,* $F_k(A)$ *valid.*

- $V_c(P_k(A)) = \{_k t \;/\; P_k(A)$ *true in* $_k t$, $_k t\ R_c\ _O t\ R_c\ t_i$ *and* $_0 t \equiv t_1 \oplus \cdots \oplus t_m\}$.

$P_k(A)$ *gave place to* A. *Since* $V_c(A) = T$ *then* $V_c(Pk(A)) = T$, *thus,* $P_k(A)$ *is valid*

5 Conclusion and open problems

In this paper, we have introduced a formalism to represent the temporal causal relationship between actions and their effects and relationship between events and actions. We used equivalence classes to represent the time of a process and the time of competitive actions. We defined operators that allow enumerate all effects of an action or all events that will happen in the future caused by actions/events (ramification/prediction), all precondition of an action or all events that happened in the past which gave place to an event. These operators allow us the representation of the actions and their effects as well as the types of reasoning which are prediction, explanation and planning.

The Information Extraction (EI) is a important subject of research in Automatic Processing of Natural language. Actually, it knows an increasing interest, it answers a need become major in the information society [10]. It aims at extracting and at structuring automatically a set of information appearing in one or several textual documents written in natural language. This information is intended to create or to feed a data warehouse (data bank). The task of extraction is realized thanks to the filling of predefined forms (Template).

These forms, said extraction forms, describe a set of entities, the relations between these entities and the events implying these entities. For example, a form concerning road accidents will have to specify fields as 'Cause of the accident ", " Place of the accident ", " Number of victims ", " Identity of the victims "...

The analysis of named entities (NE) focuses generally on the classic notions of place, organization, person or date. The events are rarely considered. When the entities of the same event are scattered in the same document or distributed on several documents, it is difficult to extract them. The entities extraction was the object of several works [11], but few of them were interested by events.

Our formalism could be used to develop a system of entities extraction of event type to use it in a search engine.

F.Baader and al propose a formalism of action based on description logics [12]. They make a first proposal for an action formalism in which the states of the world, the pre and post-conditions can be described by using Dl-concepts. The idea to investigate action formalisms based on description logics was inspired by the expressivity space between existing action formalisms. To represent the temporal dimension, classical Description Logics are extended with temporal constructors; thus a uniform representation for states, actions and plans is provided.

H.Strass and M.Thielscher study the integration of two prominent fields of logic- based AI: action formalisms and non-monotonic reasoning. The resulting framework allows an agent employing an action theory as internal world model to make useful default assumptions. They show that the mechanism behaves properly in the sense that all intuitively possible conclusions can be drawn and no implausible inferences arise. In particular, it suffices to make default assumptions only once (in the initial state) to solve projection problems [13].

H. Liu, have investigated updates of *ABoxes* in *DLs* and analyzed their computational behavior. The main motivation for this endeavor is to establish the theoretical foundations of progression in action theory based on *DLs* and to provide support for reasoning about action in *DLs* [14].

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