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Noetherian and Artinian Ternary Rings

Murhaf Alabdullah

Department of Mathematics, University of Aleppo, Syria
e-mail:murhaf.alabdullah@gmail.com

Bilal Al-Hasanat

Department of Mathematics, Faculty of Science, Al Hussein Bin Talal University, Ma'an, Jordan
e-mail:bilal_hasanat@yahoo.com

Jehad Jaraden

Department of Mathematics, Faculty of Science, Al Hussein Bin Talal University, Ma'an, Jordan
e-mail:jjaraden@mtu.edu

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Abstract

The main aim of this article is to study some special elements (zero divisors and units) in ternary rings. Then, the main properties and concepts of noetherian and artinian ternary rings have been studied. In addition, new results on noetherian and artinian ternary rings have been investigated.

Keywords: *Ternary ring, Noetherian ternary ring, Artinian ternary ring, Zero divisor, Unit.*

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1 Introduction

The concept of the ternary rings has been produced by W. G. Lister in 1971, where some special elements, ideals and regularity of these rings were also presented [8]. In [7] the author identified the noetherian ternary semirings and presented a result similar to what is known in the noetherian rings. The current research study some special elements in ternary rings, various known concepts in the noetherian and artinian rings in the ternary rings and then studying the properties of these rings.

2 Notations and basic concepts

In this section the notations and the necessary definitions of terms used in this paper have been introduced.

Definition 2.1. *[5] A nonempty set T together with binary operations (addition and a ternary multiplication) denoted by juxtaposition, is said to be a ternary ring, if T is an additive commutative group satisfying the following properties:*

(i) $(abc)de = a(bcd)e = ab(cde)$,

(ii) $(a + b)cd = acd + bcd$,

(iii) $a(b + c)d = abd + acd$ *and*

(iv) $ab(c + d) = abc + abd$, *for all* $a, b, c, d, e \in T$.

During this paper, we write T instead of $(T, +, \cdot)$ if there is nothing ambiguous. The next are examples of some ternary rings.

Example 2.2. *[8] The set T consisting of a single element 0 with binary operation defined by $0 + 0 = 0$ and ternary operation defined by $0 \cdot 0 \cdot 0 = 0$, is a ternary ring. This ternary ring is called the trivial ternary ring or the zero ternary ring.*

Example 2.3. *[6] The set $T = \{-2i, -i, 0, i, 2i, \cdots\}$ is a ternary ring with respect to addition and complex ternary multiplication.*

Example 2.4. *[6] The set $T = \{0, 1, 2, 3, 4\}$ is a ternary ring with respect to addition modulo 5 and ternary multiplication modulo 5.*

Definition 2.5. *[5] If T is a ternary ring, we call the unitary in the commutative group $(T, +)$ zero ternary ring, which is signified by 0 and meets the conditions that $x + 0 = x$ and $0xy = x0y = xy0 = 0$ for all $x, y \in T$.*

Through out this paper, T will always denote a ternary ring as zero.

Definition 2.6. *[5] In the ternary ring T. If there is an element $e \in T$ such that $eex = exe = xee = x$ for all $x \in T$, then e is called a identity element of the ternary ring T.*

It is obvious that $xye = (exe)ye = ex(eye) = exy$ and $xye = x(eye)e = xe(yee) = xey$ for all $x, y \in T$.

Proposition 2.7. *[5] If e is a identity element of a ternary ring T, then $exy = xey = xye$ for all $x, y \in T$.*

Definition 2.8. *[5] A ternary ring T is called commutative if it meets the following condition:*

$$abc = cba = acb = bca = cab = bac, \text{ for all } a, b, c \in T.$$

Definition 2.9. *[4] A nonempty subset S of a ternary ring T is called a ternary subring of T, if $(S, +)$ is a subgroup of $(T, +)$ and $abc \in S$ for all $a, b, c \in S$.*

Definition 2.10. *[2, 3] An element x of a ternary ring T is called idempotent, if $x^3 = x$.*

Definition 2.11. *[1, 5] An element a in a ternary ring T is said to be regular, if there is an element $x \in T$ such that $a = axa$.*

Definition 2.12. *[4] An element x of a ternary ring T is called nilpotent, if $x^n = 0$ for some odd positive integer n.*

The set of all nilpotent elements in T will be denoted by N_T.

Definition 2.13. *[1, 9] An additive subgroup I of a ternary ring T is called a left (right, lateral) ideal of T, if $bca \in I$ (respectively $abc \in I$, $bac \in I$) for all $b, c \in T$ and $a \in I$.*

If I is a left, right and lateral ideal of T, then I is called an ideal of T. If I is a left, and a right ideal of T, then I is called a two-sided ideal of T.

Definition 2.14. *[1, 6] A proper ideal P of a ternary ring T is called a semiprime of T, if this condition is met :$A^3 \subseteq P$ implies $A \subseteq P$, for any ideal A of T.*

- *A proper ideal P of a ternary ring T is called a completely semiprime of T, if there is an element a in T, such that: $a^3 \in P$ implies $a \in P$.*

- *A proper ideal P of a ternary ring T is called a prime ideal of T, if $ABC \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$, for A, B and C are ideals of T.*

- *A proper ideal P of a ternary ring T is called a completely prime ideal of T, if and only if for any elements $a, b, c \in T$, then $abc \in P$ implies that $a \in P$ or $b \in P$ or $c \in P$.*

- *A proper ideal P of a ternary ring T is called a idempotent ideal of T, if $P^3 = P$.*

Definition 2.15. *[1, 10] An ideal M of a ternary ring T is maximal, if it is not properly contained in other proper ideal of T i.e. $M \subseteq M' \subseteq T$ implies that $M = M'$ or $M' = T$.*

Definition 2.16. *[4] A proper ideal I of a ternary ring T is said to be irreducible, if $A \cap B \cap C = I$ implies that $A = I$ or $B = I$ or $C = I$, for A, B and C are ideals of T.*

Definition 2.17. *[5] If a is an element of a ternary ring T, then:*

$$aTT = \left\{ \sum_{fin} ax_iy_i \mid x_i \ , y_i \in T \right\}$$

$$TTa = \left\{ \sum_{fin} x_iy_ia \mid x_i \ , y_i \in T \right\}$$

$$TaT = \left\{ \sum_{fin} x_iay_i \mid x_i \ , \ y_i \in T \right\}$$

$$TTaTT = \left\{ \sum_{fin} x_iy_iax'_iy'_i \mid x_i \ , \ y_i \ , \ x'_i \ , \ y'_i \in T \right\}$$

Definition 2.18. *[5] Let T be a ternary ring and $a \in T$ and let $n \in \mathbb{Z}_0^+$ (the set of positive integers with zero). Then the following statements hold:*

(i) The left ideal generated by a, is given by $\langle a \rangle_l = TTa + na$.

(ii) The right ideal generated by a, is given by $\langle a \rangle_r = aTT + na$.

(iii) The two-sided ideal generated by a, is given by $\langle a \rangle_t = TTa + aTT + TTaTT + na$.

(iv) The lateral ideal generated by a, is given by $\langle a \rangle_m = TaT + TTaTT + na$.

(v) The ideal generated by a, is given by $\langle a \rangle = TTa + aTT + TaT + TTaTT + na$.

Proposition 2.19. *Let T be an unitary ternary ring and $a \in T$. Then:*
$\langle a \rangle_r = aTT$, $\langle a \rangle_r = aTT$, $\langle a \rangle_l = TTa$, $\langle a \rangle_t = TTa + aTT + TTaTT$, $\langle a \rangle_m = TaT + TTaTT$ and $\langle a \rangle = TTa + aTT + TaT + TTaTT$.

Proof. It is clear that $aTT \subseteq aTT + \mathbb{Z}_0^+ a = \langle a \rangle_r$. On the other hand, if e is the identity element of T then:

$$\begin{aligned} \text{Let } z \in \langle a \rangle_r. \text{ Then } z &= \sum_{fin} a x_i y_i + na \ , \ x_i \ , y_i \in T \ , \ n \in \mathbb{Z}_0^+ \\ &= \sum_{fin} a x_i y_i + a + a + \cdots + a \\ &= \sum_{fin} a x_i y_i + aee + aee + \cdots + aee \in aTT. \end{aligned}$$

Therefore, $\langle a \rangle_r \subseteq aTT$. Hence, $\langle a \rangle_r = aTT$.

In the same way, we prove the remaining cases. □

3 Some special elements in a ternary ring

Definition 3.1. *An element a of a ternary ring T is a zero divisor from right (left, lateral), if there are two nonzero elements b and c in T, such that $bca = 0$ ($abc = 0$, $bac = 0$ respectively).*

We say that a is zero divisor on both sides, if it is a zero divisor from both right and left. In addition, a is a zero divisor, if a is a zero divisor from all right, left and lateral.

Let Z_T be the group of zero divisors of T. Then, we have the next remarks:

Remark 3.2. *Let T be any ternary ring. Then, $0 \in Z_T$.*

Remark 3.3. *If $T = \{0\}$, then $Z_T = \emptyset$.*

Remark 3.4. *$Z_T = \{0\}$ for some ternary ring T. For example, $T = \mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$, which is a unitary ternary ring and $Z_T = \{0\}$.*

Remark 3.5. *$Z_T = T$ for some ternary ring T. For example, $T = \mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$, which is a unitary ternary ring and $Z_T = T$.*

Remark 3.6. *Let $(T, +)$ be an abelian group. Define a triple multiplication operation on T as $x.y.z = 0$, $\forall x, y, z \in T$. Then $(T, +, .)$ is a ternary ring for which $Z_T = T$.*

Definition 3.7. *Let T be a ternary ring with identity e, and let x be an element of T. We say that x is a unit from right (left, lateral), if there are two elements $y, z \in T$ where $xyz = e$ ($yzx = e$, $yxz = e$, respectively).*

Let T be a ternary ring, the element $x \in T$ is called a unit on both sides if it is a unit from both right and left. And, x is a unit if it is a unit from all right, left and lateral. Let U_T denotes the group of all units in a ternary ring T.

4 New results

Clearly, $\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$ is a unitary ternary ring whose identity element is 1, and we have $U_T = \{1, 2, 3\}$. Also, $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$ is an unitary ternary ring whose unit is 1, for which $U_T = \emptyset$. And, $\mathbb{Z}_7 = \{0, 1, 2, 3, 4, 5, 6\}$ is an unitary ternary ring whose identity element is 1, and we have $U_T = \{(1, 2, 4), (1, 3, 5), (2, 3, 6)\} = T - \{0\}$.

Lemma 4.1. *If T is a unitary ternary ring whose identity element is e, and let $x \in N_T$. Then $e - x \in U_T$.*

Proof. Let $x \in N_T$ and n is the nilpotency degree of x. Then

$$
\begin{aligned}
& (e-x)\, e \left(e + x + exx + xxx + exxxx + \cdots + x^{n-2} + exx^{n-2}\right) \\
= \; & ee \left(e + x + exx + xxx + exxxx + \cdots + x^{n-2} + exx^{n-2}\right) \\
& -xe \left(e + x + exx + xxx + exxxx + \cdots + x^{n-2} + exx^{n-2}\right) \\
= \; & e + x + exx + xxx + exxxx + \cdots + x^{n-2} + exx^{n-2} - xee - xex - xeexx \\
& -xexxx - xeexxxx - \cdots - xex^{n-2} - xeexx^{n-2} \\
= \; & e + x + exx + xxx + exxxx + \cdots + x^{n-2} + exx^{n-2} - x - exx - xxx - exxxx \\
& -xxxxx - \cdots - exx^{n-2} - xxx^{n-2} \\
= \; & e - x^n \\
= \; & e
\end{aligned}
$$

Thus $e - x$ is a unit from the right.

In the same way, we prove that $e - x$ is a left and lateral unit. Therefore, $e - x$ is unit. □

Lemma 4.2. *Let T is a commutative unitary ternary ring whose identity element is e. If $x \in N_T$ and $u \in U_T$, then $u - x \in U_T$.*

Proof. Let $x \in N_T$ and n is the nilpotency degree of x. Since $u \in U_T$, then there exist y and z in T, such that $yzu = yuz = uyz = e$. On the other hand, $u - x = uee - eex = uee - uyzex = ue(e - yzx)$. So $x^n = 0$. Then, $y^n z^n x^n = 0$, which implies that, $(yzx)^n = 0$. Therefore, $yzx \in N_T$. Using Lemma 4.1, we have $e - yzx \in U_T$. Thus, $u - x = ue(e - yzx) \in U_T$. □

Theorem 4.3. *If T is a regular unitary ternary ring, then*

$$(T \setminus U_T \subseteq Z_T) \text{ and } T \setminus Z_T \subseteq U_T.$$

Proof. Let T be a regular unitary ternary ring. Then, there is $y \in T$ such that $x = xyx$. So $x - xyx = 0$, implies that, $(e - exy)\, ex = 0$, since $x \notin Z_T$, then $e - exy = 0$. Therefore, $xey = e$.

On the other hand, we have $xe\,(e - eyx) = 0$, since $x \notin Z_T$, then $e - eyx = 0$ implies that, $eyx = e$. Subsequently, $exy = xey = eyx = e$. Then, $x \in U_T$. □

Theorem 4.4. *Each unit element in an unitary ternary ring is a regular element.*

Proof. Let T be a ternary ring with identity element e. If $u \in U_T$, then there exist y and z in T such that, $yzu = yuz = uyz = e$. On the other hand,

$$u = eeu = (uyz)\,eu = u\,(yze)\,u = uxu, \text{ where } x = yze \in T.$$

Thus, u is a regular element. □

5 Noetherian and artinian ternary rings

Definition 5.1. *Let be T a ternary ring. We say that T is left (right, lateral) artinian, if T meets the descending chain condition on left (right, lateral respectively) ideals. That is, for any chain of left (right, lateral respectively) ideals $I_1 \supseteq I_2 \supseteq \cdots$ there is a positive integer n such that $I_n = I_{n+1} = I_{n+2} = \cdots$.*

A ternary ring T is an artinian, if T is all a left, right and lateral artinian. A ternary ring T is a two-sided artinian, if T is both a left and right artinian.

Definition 5.2. *[1, 9, ?] A proper ideal I of a ternary ring T is said to be irreducible, if $J \cap K \cap L = I$ implies $J = I$ or $K = I$ or $L = I$, for J, K and L are ideals of T.*

Corollary 5.3. *Each prime ideal in a ternary ring is irreducible deal.*

Proof. Let P be a prime ideal of a ternary ring T, and let J, K and L be three ideals of T, such that $P = J \cap K \cap L$. Then $JKL \subseteq J \cap K \cap L \subseteq P$, and since P is prime ideal, then $J \subseteq P$ or $K \subseteq P$ or $L \subseteq P$. Therefore, $P = J \cap K \cap L$, implies that $P \subseteq J$ or $P \subseteq K$ or $P \subseteq K$. Hence,

$$P = J \text{ or } P = K \text{ or } P = L$$

□

Theorem 5.4. *Let T be a noetherian ternary ring. Then every ideal of T is a finite intersection of irreducible ideals.*

Proof. Let T be a noetherian ternary ring, and S be the collection of all ideals none is a finite intersections of irreducible ideals from T. Since T is a noetherian, then S has maximal element I. So, I is not irreducible, $I = J \cap K \cap L$ for ideals J, K and L all are quite larger than I. Maximality implies that J, K and L are finite intersections of irreducible ideals. Therefore I is also a finite intersection of irreducible ideals, which is a contradiction. Hence, $S = \emptyset$. □

Definition 5.5. *Let T be a ternary ring. Define the nil radical of T as:*

$$\begin{aligned} N(T) &= \{a \in T \mid a^n = 0 \ , \quad n \text{ is a positive odd integer}\} \\ &= \left\{ \bigcap_{P \in \wp} P \mid \wp \text{ set of prime ideals} \right\}. \end{aligned}$$

For an ideal I of T, define the radical ideal of I as:

$$\begin{aligned} \sqrt{I} &= \{a \in T \mid a^n \in T \ , \ n \text{ is a positive odd integer}\} \\ &= \left\{ \bigcap_{I \in P} P \mid P \text{ is a prime ideal} \right\}. \end{aligned}$$

Then $N(T) = \sqrt{\langle 0 \rangle}$.

Theorem 5.6. *Let T be an artinian ternary ring. Then there exists a positive odd integer n, such that $N(T)^n = 0$ (we write $N(T) = N$ for simplicity).*

Proof. Let T be an artinian ternary ring. Then, there is a positive odd integer n with $N^n = N^{n+2} = N^{n+4} = \cdots$. Suppose that $I \neq 0$ and let S be the set of all ideals J with $IIJ \neq 0$. Then S is nonempty because $I \in S$. Since T is an artinian ternary ring, then S has a minimal element say K. Since $IIK \neq 0$, then there exists $a \in K$ for which $IIa \neq 0$. So $a = K$ by minimality of K. Therefore, $IIa \in S$ and $IIa \subseteq a = K$. Thus, there is $x \in I$ where $xxa = a$. Multiplying both sides by xx, we get $xx(xxa) = xxa = a$. Repeating this ,multiplication to get $x^n xa = a$. However, $x \in I$, which means that $x \in N = N(T)$, so x is nilpotent. Thus, $x^n = 0$ for some values of positive odd integers n. Consequently, $0 = x^n xa = a$, which is a contradiction. Hence $I = N^n = 0$. □

Theorem 5.7. *Let T be an artinian ternary ring. Then any prime ideal is maximal.*

Proof. Let T be an artinian ternary ring, and let P be a prime ideal. we need to show that, given $f \in T/P$, then $TTf + P = T$. Since T is artinian ternary ring, then the chain $TTf + P \supseteq TTf^3 + P \supseteq TTf^5 + P \supseteq \cdots$ must stabilize. This implies that, there is a positive odd integer n for which $TTf^n + P = TTf^{n+2} + P$. Then, we conclude that:

$$f^n \in TTf^n + P = TTf^{n+2} + P \text{ implies that } f^n = xyf^{n+2} + h \ , \ h \in P \ , \ x.y \in T.$$

Then $wf^n - xyf^{n+2} = h \in P$, therefore $f^n \left(e - xyf^2 e\right) e \in P$.

Since $f \notin P$, then $f^n \notin P$ and P is prime, so $e \in P$ or $e - xyf^2 e \in P$. If $e \in P$ then P is maximal, and if $e - xyf^2 e \in P$, then there is $g \in P$ such that $e - xyf^2 e = g$, therefore $xyf^2 e + g = e$. Hence, the ideal $TTf + P$ contains the element $xyeff + g = e$. □

Definition 5.8. *Let T be a commutative ternary ring with an identity element e. Then T is called a ternary field, if every element non-zero element a in T is unit.*

Proposition 5.9. *A ternary field does not contain a proper zero divisor.*

Definition 5.10. *Let T be a commutative unitary ternary ring. Then T is called a ternary integral domain, if it does not contain proper zero divisor.*

Proposition 5.11. *Each commutative ternary field is a ternary integral domain.*

Definition 5.12. *[10] Let T be a ternary ring and I be an ideal of T. Define $+I = \{a+x \mid x \in I\}$ for each $a \in T$ and $T/I = \{a+I \mid a \in T\}$. Then T/I is a ternary ring with addition and multiplication, which defined by $(a+I)+(b+I) = (a+b)+I$ and $(a+I)(b+I)(c+I) = abc+I$ for all $a, b, c \in T$. This ternary ring T/I is called the ternary division ring of T by I.*

Corollary 5.13.

1. *If T is a commutative ternary ring, then the ternary division ring T/I is commutative.*
2. *If T is a unitary ternary ring with identity element e, then the ternary division ring T/I is unitary with identity element $e+I$.*

Theorem 5.14. *Let T be a ternary ring and I be an ideal of T. If T is an artinian (noetherian) ternary ring, then the ternary division ring T/I is an artinian (a noetherian) respectively.*

Proof. The proof is similar to the proof of the same theory in rings. □

Theorem 5.15. *[10] Let T be a commutative ternary ring with identity element e. Then, an ideal I of T is maximal if and only if T/I is ternary field.*

Theorem 5.16. *Let T be an artinian unitary ternary ring. Then:*

1. *Every non-zero divisor in T is a unit.*
2. *If T is commutative, then every prime ideal in T is maximal.*

Proof. Let e be the identity element of T.

1. If $x \in T$ be a non-zero divisor. Then x^n is also a non-zero divisor for any positive odd integer n. Let $\langle x^n \rangle$ be the left ideal of T generated by x^n of some values of n. Then the descending chain $\langle x \rangle \supseteq \langle x^3 \rangle \supseteq \cdots \supseteq \langle x^n \rangle \supseteq \cdots$ must stabilize, as T is artinian. We conclude that $\langle x^m \rangle = \langle x^{m+2} \rangle = \cdots$ for some values of the positive odd integer m. So, $x^m \in \langle x^{m+2} \rangle$, that is $x^m = qsx^{m+2}$ for some $q, s \in T$. Thus $x^m - qsx^{m+2} = 0$ implies

$e(e - eqsxx)x^m = 0$, which leads to $e - e(qsx)x = 0$, as x^m is a non-zero divisor, this means that $e(qsx)x = e$. Hence x has a left unit.

Since $x - xe(e(qsx)x) = 0$ and $x = xee = xe(e(qsx)x)$. Thus $x - (xee)(qsx)x = 0$, this leads to $x - x(qsx)x = 0$, so $e\,[e - x(qsx)e]\,x = 0$ as x is a non-zero divisor, this implies that $e - x(qsx)e = 0$, so $x(qsx)e = e$. Hence x has a right unit.

On the other hand, we have $x = exe$, implies that $x = [x(qsx)e]\,xe$, then $x - [x(qsx)e]\,xe = 0$, so $x - x(qsx)(exe) = 0$, which gives $x - x(qsx)x = 0$, therefore $x\,[e - (qsx)xe]\,e = 0$. Thus, $e - (qsx)xe = 0$ implise that $(qsx)xe = e$ as x is a non-zero divisor. Thus, x has a lateral unit. Hence x is a unit.

2. Let P be a prime ideal of T. Then T/P is an Artinian ternary integral domain. From part (1), follows that every non-zero element of T/P is a unit. Hence T/P is a ternary field and thus P is maximal.

□

Proposition 5.17. *Every Artinian ternary integral domain is a ternary field.*

Corollary 5.18. *Let T be an artinian ternary ring. Then $dim(T) = 0$.*

Proof. Let P be a prime ideal of T. Then T/P is a ternary integral domain. Since T is artinian, then T/P is artinian. Thus, T/P is a ternary field according to Proposition 5.17. Thus, P is maximal and $dim(T) = 0$. □

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