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On Grundy Chromatic Number For Splitting Graph On Different Graphs

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Abstract

The Grundy coloring of a graph G is a proper vertex coloring in which every node colored with C_k ***is adjacent with all least colors of*** C_k***. The grundy number*** $\Gamma(G)$ ***is the maximum number of colors needed for proper grundy vertex coloring. In this paper, we find the accurate values of grundy chromatic number for splitting graph of cycle graph, path graph, pan graph, fan graph and double fan graph which are symbolised by*** $\Gamma[S(C_n)]$, $\Gamma[S(P_n)]$, $\Gamma[S(n-pan)]$, $\Gamma[S(F_{1,n})]$ ***&*** $\Gamma[S(F_{2,n})]$ ***respectively.***

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1 Introduction

In this, the Graph $G = \{V(G), E(G)\}$ we use is an undirected, simple, connected & finite graph. We follow [2, 6] for basic notations such that $V(G)$, $E(G)$, $\triangle(G)$ & $\delta(G)$ are the vertex set, edge set, maximum & minimum degree of G respectively. Throughout this paper, we derive grundy chromatic number for some splitting graphs. The notion of splitting graph was initiated by E.

Sampathkumar and H.B. Walikar in 1981 [1]. The main concept of splitting graph of G is to take a new vertex $V' \ \forall \ V \in G$ & join each V' to neighbors of V in G. And the grundy chromatic number was initially studied by P M Grundy regarding combinatorial games for directed version in 1939 but was properly introduced later by Claude A. Christen and Stanley M. Selkow in 1979 for undirected version. A Grundy k-coloring of G is a proper k-coloring of $V(G)$ such that $\forall \ v \in V(G)$ colored by smallest integer which has not appeared as color of any of its neighbors [3, 4, 7]. The grundy chromatic number $\Gamma(G)$ is the largest integer k for which there exists a grundy k-coloring of G [7]. This can also be predicted by using greedy coloring strategy which considers the vertices of graph in some sequence & color them first available color. It is evident that $\mu(G) \leq \chi(G) \leq \Gamma(G) \leq \triangle(G) + 1$ where $\mu(G)$ is the largest clique of G [5].

2 Preliminaries

[5] A Grundy n-coloring of G is an n-coloring of G such that $\forall$ color C_t, every node colored with C_t is adjacent to atleast one node colored with $C_s \ \forall \ C_s < C_t$. The Grundy chromatic number $\Gamma(G)$ is the maximum number n such that G is Grundy n-coloring.

[1, 8] For every vertex V of a graph G, take a new vertex V'. Join V' to all vertices of G adjacent to V. The graph $S(G)$ thus obtained is called the splitting graph of G.

The n-pan is obtained by connecting a cycle graph C_n with a singleton graph by an edge.

[9] The fan graph $F_{1,n}$ is obtained by joining every vertex in P_n with $\bar{K_1}$ where $\bar{K_1}$ is the complement of complete graph with one vertex and P_n is a path on n vertices.

[9] The double fan graph $F_{2,n}$ is obtained by joining every vertex in P_n with the vertices in $\bar{K_2}$ where $\bar{K_2}$ is the complement of complete graph with two vertices and P_n is a path on n vertices.

3 Main results

Here, we concentrate on exact values of Grundy chromatic number for splitting graph on cycle graphs, path graphs, pan graphs, fan graphs and double fan graphs which are symbolised by $\Gamma[S(C_n)]$, $\Gamma[S(P_n)]$, $\Gamma[S(n-pan)]$, $\Gamma[S(F_{1,n})]$ and $\Gamma[S(F_{2,n})]$ respectively.

Theorem 3.1. *For $n \geq 3$, the grundy chromatic number for splitting graph of cycle graph C_n is given by*

$$\Gamma[S(C_n)] = \begin{cases} n-1, & n = 4 \\ 4, & n \neq 4 \end{cases}$$

Proof. Consider a cycle graph C_n with vertex set $V(C_n) = \{V_i : i \in [1,n]\}$ and edge set $E(C_n) = \{V_iV_{i+1} : i \in [1,n)\} \cup \{V_1V_n\}$ where $|V(C_n)| = |E(C_n)| = n$. Moreover, $\triangle(C_n) = \delta(C_n) = d(V_i) = 2 \; \forall \; i \in [1,n]$.
By the construction of splitting graph, we have $V[S(C_n)] = \{V_i : i \in [1,n]\} \cup \{V_i^{'} : i \in [1,n]\}$ and $E[S(C_n)] = \{V_iV_{i+1} : i \in [1,n)\} \cup \{V_1V_n\} \cup \{V_i^{'}V_{i+1} : i \in [1,n)\} \cup \{V_1^{'}V_n\} \cup \{V_1V_n^{'}\} \cup \{V_i^{'}V_{i-1} : i \in (1,n]\}$ along with $\triangle[S(C_n)] = d(V_i) = 4$ and $\delta[S(C_n)] = d(V_i^{'}) = 2 \; \forall \; 1 \leq i \leq n$. Consider the colors $C_1, C_2, C_3, ...$ and assign the colors as follows.

Case 1. *When $n = 4$*
Define a mapping $\alpha : V[S(C_n)] \to \{C_k : 1 \leq k \leq 3\}$ as follows:

- *For $1 \leq i \leq \frac{n}{2}$, $\alpha(V_{2i}) = C_2$ and $\alpha(V_{2i-1}) = C_3$*
- $\alpha(V_i^{'}) = C_1 \; \forall \; 1 \leq i \leq n$

Obviously, $\Gamma[S(C_n)] = 3$. Suppose $\Gamma[S(C_n)] > 3$, it leads to contradiction of grundy coloring and if $\Gamma[S(C_n)] < 3$, it leads to contradiction of proper coloring.

Case 2. *When $n \neq 4$*
Define a mapping $\beta : V[S(C_n)] \to \{C_k : 1 \leq k \leq 4\}$ such that

- *For $i \in [1,n]$, $\beta(V_i^{'}) = C_1$*
- *For $n \equiv$ 0 mod 3,* $\beta(V_i) = \begin{cases} C_4, i \equiv \textit{1 mod 3} \\ C_3, i \equiv \textit{2 mod 3} \\ C_2, i \equiv \textit{0 mod 3} \end{cases}$
- *For $n \equiv$ 1 mod 3,* $\beta(V_i) = \begin{cases} C_4, i \equiv \textit{1 mod 3} \\ C_3, i \equiv \textit{2 mod 3 \& i=n-3,n-1} \\ C_2, i \equiv \textit{0 mod 3 \& i=n-2,n} \end{cases}$
- *For $n \equiv$ 2 mod 3,* $\beta(V_i) = \begin{cases} C_4, i \equiv \textit{1 mod 3} \\ C_3, i \equiv \textit{2 mod 3 \& i=n-1} \\ C_2, i \equiv \textit{0 mod 3 \& i=n} \end{cases}$

$\therefore \Gamma[S(C_n)] = 4$ *for* $n \neq 4$. *Suppose* $\Gamma[S(C_n)] > 4$, *it contradicts the definition of grundy coloring. For instance,* $\Gamma[S(C_n)] = 5$, *the vertex* V_1 *colored with* C_5 *is not adjacent with* C_2 *for the mapping* $\beta(V_i) = \begin{cases} C_5, i = 1 \\ C_4, i \equiv 0 \text{ mod } 2 \\ C_3, i \equiv 1 \text{ mod } 2 \end{cases}$ *which is a contradiction. And suppose* $\Gamma[S(C_n)] < 4$, *eventhough it satisfies it is not maximum.*

Thus from the above cases, $\Gamma[S(C_n)] = \begin{cases} n-1, \ n = 4 \\ 4, \ n \neq 4 \end{cases}$ □

Theorem 3.2. *For* $n \geq 2$, *the grundy chromatic number for splitting graph of path graph* P_n *is given by*

$$\Gamma[S(P_n)] = \begin{cases} 3, \ n = 2, 3 \\ 4, \ n \geq 4 \end{cases}$$

Proof. Consider a path graph P_n with $V(P_n) = \{V_i : i \in [1, n]\}$ and $E(P_n) = \{V_iV_{i+1} : i \in [1, n)\}$ where $| V(P_n) |= n$ & $| E(P_n) |= n - 1$. Moreover, $\triangle(P_n) = 2$ & $\delta(P_n) = 1$.
By the construction of splitting graph, we have $V[S(P_n)] = \{V_i : i \in [1, n]\} \cup \{V_i' : i \in [1, n]\}$ and $E[S(P_n)] = \{V_iV_{i+1} : i \in [1, n)\} \cup \{V_i'V_{i+1} : i \in [1, n)\} \cup \{V_iV_{i+1}' : i \in [1, n)\}$ along with $\delta[S(P_n)] = d(V_1') = d(V_n') = 1$ and $\triangle[S(P_n)] = \begin{cases} 2, \ n = 2 \\ 4, \ n \neq 2 \end{cases}$
Consider the colors $C_1, C_2, C_3, ...$ and assign them as follows.

Case 1. *When* $n = 2, 3$
Define a mapping $\phi : V[S(P_n)] \to \{C_k : 1 \leq k \leq 3\}$ *as follows:*

- $\phi(V_{\lfloor \frac{n}{2} \rfloor}) = C_3$
- $\phi(V_{\lfloor \frac{n}{2} \rfloor + 1}) = C_2$
- $\phi(V_i') = C_1 \ \forall \ 1 \leq i \leq n$

and the remaining vertices are greedily colored. Obviously, $\Gamma[S(P_n)] = 3$ *for* $n = 2, 3$.

Case 2. *When* $n \geq 4$
Define a mapping $\psi : V[S(P_n)] \to \{C_k : 1 \leq k \leq 4\}$ *as follows:*

- *For* $1 \leq i \leq n$, $\psi(V_i') = C_1$

- *For $i = 2$, $\psi(V_i) = C_4$, $\psi(V_{i+1}) = C_3$, $\psi(V_{i-1}) = \psi(V_{i+2}) = C_2$*

and the remaining vertices are greedily colored.
$\therefore \Gamma[S(P_n)] = 4$ *for $n \geq 4$. Suppose $\Gamma[S(P_n)] > 4$, it leads to contradiction of grundy coloring. For instance, $\Gamma[S(P_n)] = 5$, the vertices $V_3^{'}$ & $V_4^{'}$ colored with C_2 is not adjacent with C_1 for the mapping $\psi(V_3) = C_5$, $\psi(V_4) = C_4$, $\psi(V_2) = \psi(V_5) = C_3$ & $\psi(V_1) = C_2$ and the remaining vertices $V_i^{'}$ & $V_{i+1}^{'}$ are colored by C_1 and C_2 $\forall$ odd 'i' such that $\psi(V_i^{'}) = \psi(V_{i+1}^{'})$ and then the remaining V_i vertices are greedily colored. And suppose $\Gamma[S(P_n)] < 4$, eventhough it satisfies it is not maximum.*

Thus from the above cases, $\Gamma[S(P_n)] = \begin{cases} 3, & n = 2, 3 \\ 4, & n \geq 4 \end{cases}$ □

Theorem 3.3. *For $n \geq 3$, the grundy chromatic number for splitting graph of pan graph $(n - pan)$ is given by*

$$\Gamma[S(n - pan)] = 4$$

Proof. Consider a pan graph with vertex set $V(n - pan) = \{V_i : 0 \leq i \leq n\}$ and edge set $E(n - pan) = \{V_iV_{i+1} : 0 \leq i \leq n - 1\} \cup \{V_1V_n\}$ where $| V(n-pan) |=| E(n-pan) |= n+1$. Moreover, $\triangle(n-pan) = d(V_1) = 3$ and $\delta(n - pan) = d(V_0) = 1$.
By the construction of splitting graph, we have $V[S(n - pan)] = \{V_i : 0 \leq i \leq n\} \cup \{V_i^{'} : 0 \leq i \leq n\}$ and $E[S(n - pan)] = \{V_1V_n\} \cup \{V_iV_{i+1} : 0 \leq i \leq n - 1\} \cup \{V_i^{'}V_{i+1} : 0 \leq i \leq n - 1\} \cup \{V_i^{'}V_{i-1} : 1 \leq i \leq n\} \cup \{V_1V_n^{'}\} \cup \{V_1^{'}V_n\}$ along with $\delta[S(n - pan)] = d(V_0^{'}) = 1$ and $\triangle[S(n - pan)] = d(V_1) = 6$.
Define a mapping $\lambda : V[S(n - pan)] \rightarrow \{C_k : 1 \leq k \leq 4\}$ and assign the colors as follows.

- $\lambda(V_1) = C_4$
- For $2 \leq i \leq n$, $\lambda(V_i) = \begin{cases} C_3, & \text{i} \equiv 0 \bmod 2 \\ C_2, & \text{i} \equiv 1 \bmod 2 \end{cases}$
- $\lambda(V_0) = C_2$
- $\lambda(V_i^{'}) = C_1 \; \forall \; 0 \leq i \leq n$

$\therefore \Gamma[S(n - pan)] = 4$. Suppose $\Gamma[S(n - pan)] > 4$, it leads to contradiction of grundy coloring. For instance, $\Gamma[S(n-pan)] = 5$, the vertices $\{V_i : 2 \leq i \leq n\}$ colored with C_4 & C_3 are not adjacent with C_2 for the mapping $\lambda(V_1) = C_5$ & $\lambda(V_i) = \begin{cases} C_4, & \text{i} \equiv 0 \bmod 2 \\ C_3, & \text{i} \equiv 1 \bmod 2 \end{cases}$ $\forall \; 2 \leq i \leq n$ which contradicts grundy coloring.

And suppose $\Gamma[S(n-pan)] < 4$, it contradicts the definition of proper coloring. Thus, $\Gamma[S(n-pan)] = 4$ for $n \geq 3$.

□

Theorem 3.4. *For $n \geq 1$, the grundy chromatic number for splitting graph of fan graph $F_{1,n}$ is given by*

$$\Gamma[S(F_{1,n})] = \begin{cases} 3, & n = 1 \\ 4, & n = 2, 3 \\ 5, & n \geq 4 \end{cases}$$

Proof. Consider a fan graph $F_{1,n}$ with vertex set $V(F_{1,n}) = \{V_i : 0 \leq i \leq n\}$ and edge set $E(F_{1,n}) = \{V_0V_i : 1 \leq i \leq n\} \cup \{V_iV_{i+1} : 1 \leq i \leq n-1\}$ where $\mid V(F_{1,n}) \mid = n+1$ & $\mid E(F_{1,n}) \mid = 2n-1$. Moreover, $\triangle(F_{1,n}) = d(V_0) = n$ & $\delta(F_{1,n}) = d(V_1) = d(V_n) = 2$.
By the construction of splitting graph, we have $V[S(F_{1,n})] = \{V_i : 0 \leq i \leq n\} \cup \{V_i' : 0 \leq i \leq n\}$ and $E[S(F_{1,n})] = \{V_0V_1\} \cup \{V_0V_1'\} \cup \{V_0'V_1\}$ for $n = 1$ otherwise $E[S(F_{1,n})] = \{V_0V_i : 1 \leq i \leq n\} \cup \{V_iV_{i+1} : 1 \leq i \leq n-1\} \cup \{V_0'V_i : 1 \leq i \leq n\} \cup \{V_i'V_{i+1} : 1 \leq i \leq n-1\} \cup \{V_n'V_0\} \cup \{V_iV_{i+1}' : 0 \leq i \leq n-1\} \cup \{V_0V_i' : 2 \leq i \leq n-1\}$ along with $\triangle[S(F_{1,n})] = d(V_0) = 2n$ and $\delta[S(F_{1,n})] = \begin{cases} 1, & n = 1 \\ 2, & n \neq 1 \end{cases}$
Consider the colors $C_1, C_2, C_3, ...$ and assign the colors as follows.

Case 1. *When $n = 1$*
Define a mapping $\mu : V[S(F_{1,n})] \rightarrow \{C_k : 1 \leq k \leq 3\}$ and assign the colors as follows:

- $\mu(V_0) = C_3$
- $\mu(V_1) = C_2$
- $\mu(V_i') = C_1 \ \forall \ i = 0, 1$

Obviously, $\Gamma[S(F_{1,n})] = 3$ *for* $n = 1$.

Case 2. *When $n = 2, 3$*
Consider a mapping $\rho : V[S(F_{1,n})] \rightarrow \{C_k : 1 \leq k \leq 4\}$ and assign the colors as follows:

- $\rho(V_0) = C_4$
- $\rho(V_{\lfloor \frac{n}{2} \rfloor}) = C_3$
- $\rho(V_{\lfloor \frac{n}{2} \rfloor + 1}) = C_2$

- $\rho(V_i^{'}) = C_1 \ \forall \ 0 \leq i \leq n$

and the remaining vertices are colored greedily. Thus, $\Gamma[S(F_{1,n})] = 4$ *for* $n = 2, 3$.

Case 3. *When* $n \geq 4$
Define a mapping $\omega : V[S(F_{1,n})] \rightarrow \{C_k : 1 \leq k \leq 5\}$ *and assign the colors as follows:*

- $\omega(V_0) = C_5$
- *For* $i = 2$, $\omega(V_i) = C_4$, $\omega(V_{i+1}) = C_3$, $\omega(V_{i-1}) = \omega(V_{i+2}) = C_2$
- $\omega(V_i^{'}) = C_1 \ \forall \ 0 \leq i \leq n$

and the remaining vertices are colored greedily.
$\therefore \Gamma[S(F_{1,n})] = 5$ *for* $n \geq 4$. *Suppose* $\Gamma[S(F_{1,n})] > 5$, *it leads to contradiction of grundy coloring. For instance,* $\Gamma[S(F_{1,n})] = 6$, *the vertices colored with* $\{C_k : 3 \leq k \leq 5\}$ *is not adjacent with* C_2 *for the mapping* $\omega(V_0) = C_6$, $\omega(V_i^{'}) = C_1 \ \forall \ 0 \leq i \leq n$ *and for* $i = 3$, $\omega(V_i) = C_5$, $\omega(V_{i-1}) = C_4$, $\omega(V_{i+1}) = \omega(V_{i-2}) = C_3$, $\omega(V_{i+2}) = C_2$*and then the remaining are greedily colored. Similarly,* $7 \leq \Gamma[S(F_{1,n})] \leq 2n + 1$ *arrives at contradiction. And suppose* $\Gamma[S(F_{1,n})] < 5$, *eventhough it satisfies it is not maximum.*

Hence, from the above cases, $\Gamma[S(F_{1,n})] = \begin{cases} 3, \ n = 1 \\ 4, \ n = 2, 3 \\ 5, \ n \geq 4 \end{cases}$ □

Theorem 3.5. *For* $n \geq 1$, *the grundy chromatic number for splitting graph of double fan graph* $F_{2,n}$ *is given by*

$$\Gamma[S(F_{2,n})] = \begin{cases} 3, \ n = 1 \\ 4, \ n = 2, 3 \\ 5, \ n \geq 4 \end{cases}$$

Proof. Consider a double fan graph $F_{2,n}$ with vertex set $V(F_{2,n}) = \{V_i : 1 \leq i \leq n\} \cup \{u_1, u_2\}$ and edge set $E(F_{2,n}) = \{V_1u_1\} \cup \{V_1u_2\}$ for $n = 1$ otherwise $E(F_{2,n}) = \{V_iV_{i+1} : 1 \leq i \leq n-1\} \cup \{u_1V_i : 1 \leq i \leq n\} \cup \{u_2V_i : 1 \leq i \leq n\}$ where $\mid V(F_{2,n}) \mid = n + 2$ & $\mid E(F_{2,n}) \mid = 3n - 1$. Moreover, $\triangle(F_{2,n}) = \begin{cases} n+1, \ 1 \leq n \leq 3 \\ n, \ n \geq 4 \end{cases}$ and $\delta(F_{2,n}) = \begin{cases} n, \ n = 1, 2 \\ 3, \ n \geq 3 \end{cases}$
By the construction of splitting graph, we have $V[S(F_{2,n})] = \{V_i : 1 \leq i \leq n\} \cup \{u_1, u_2\} \cup \{V_i^{'} : 1 \leq i \leq n\} \cup \{u_1^{'}, u_2^{'}\}$ and $E(F_{2,n}) = \{V_1u_1\} \cup \{V_1u_2\} \cup \{V_1^{'}u_1\} \cup \{V_1^{'}u_2\} \cup \{V_1u_1^{'}\} \cup \{V_1u_2^{'}\}$ for $n = 1$ otherwise $E[S(F_{2,n})] = \{u_1V_i :$

$1 \leq i \leq n\} \cup \{u_2V_i : 1 \leq i \leq n\} \cup \{V_iV_{i+1} : 1 \leq i \leq n-1\} \cup \{u_1^{'}V_i : 1 \leq i \leq n-1\} \cup \{u_2^{'}V_i : 1 \leq i \leq n-1\} \cup \{V_i^{'}u_1 : 1 \leq i \leq n\} \cup \{V_i^{'}u_2 : 1 \leq i \leq n\} \cup \{V_i^{'}V_{i+1} : 1 \leq i \leq n-1\} \cup \{V_iV_{i+1}^{'} : 1 \leq i \leq n-1\}$ along with $\triangle[S(F_{2,n})] = \begin{cases} 2(n+1), & 1 \leq n \leq 3 \\ 2n, & n \geq 4 \end{cases}$ and $\delta[S(F_{2,n})] = \begin{cases} n, & n = 1,2 \\ 3, & n \geq 3 \end{cases}$
Consider the colors $C_1, C_2, C_3, ...$ and assign the colors as follows.

Case 1. *When $n = 1$*
Define a mapping $\xi : V[S(F_{2,n})] \rightarrow \{C_k : 1 \leq k \leq 3\}$ and assign the colors as follows:

- $\xi(u_j) = C_3 \ \forall \ 1 \leq j \leq 2$
- $\xi(V_1) = C_2$
- $\xi(V_i^{'}) = \xi(u_j^{'}) = C_1 \ \forall \ i = 1$ *&* $1 \leq j \leq 2$

Obviously, $\Gamma[S(F_{2,n})] = 3$ for $n = 1$.

Case 2. *When $n = 2, 3$*
Define a mapping $\pi : V[S(F_{2,n})] \rightarrow \{C_k : 1 \leq k \leq 4\}$ as follows:

- $\pi(u_j) = C_4 \ \forall \ 1 \leq j \leq 2$
- $\pi(V_{\lfloor \frac{n}{2} \rfloor}) = C_3$
- $\pi(V_{\lfloor \frac{n}{2} \rfloor + 1}) = C_2$
- $\pi(V_i^{'}) = \pi(u_j^{'}) = C_1 \ \forall \ 1 \leq i \leq n$ *&* $1 \leq j \leq 2$

and the remaining vertices are colored greedily. Thus, $\Gamma[S(F_{2,n})] = 4$ for $n = 2, 3$.

Case 3. *When $n \geq 4$*
Define a mapping $\sigma : V[S(F_{2,n})] \rightarrow \{C_k : 1 \leq k \leq 5\}$ and assign the colors as follows:

- $\sigma(u_j) = C_5 \ \forall \ 1 \leq j \leq 2$
- *For $i = 2$, $\sigma(V_i) = C_4$, $\sigma(V_{i+1}) = C_3$, $\sigma(V_{i-1}) = \sigma(V_{i+2}) = C_2$*
- $\sigma(V_i^{'}) = \sigma(u_j^{'}) = C_1 \ \forall \ 1 \leq i \leq n$ *&* $1 \leq j \leq 2$

and the remaining vertices are colored greedily.
$\therefore$ $\Gamma[S(F_{2,n})] = 5$ *for $n \geq 4$. Suppose $\Gamma[S(F_{2,n})] > 5$, it leads to contradiction of grundy coloring. For instance, $\Gamma[S(F_{2,n})] = 6$, the vertices $\{V_i^{'} : 2 \leq i \leq n-1\}$ colored with C_2 is not adjacent with C_1 for the mapping $\sigma(u_j) = C_6$*

$\forall\ 1 \leq j \leq 2$, $\sigma(V_2) = C_5$, $\sigma(V_i^{'}) = C_2\ \forall\ 2 \leq i \leq n-1$, $\sigma(u_j^{'}) = \sigma(V_i^{'}) = C_1\ \forall$ $i = 1, n$ *&* $1 \leq j \leq 2$ *and then the remaining are colored by* C_3 *&* C_4 *simultaneously which contradicts grundy coloring. Similarly* $7 \leq \Gamma[S(F_{2,n})] \leq 2n+1$ *leads to contradiction. And suppose* $\Gamma[S(F_{2,n})] < 5$*, eventhough it satisfies it is not maximum.*

Hence, from the above cases, $\Gamma[S(F_{2,n})] = \begin{cases} 3, & n = 1 \\ 4, & n = 2,3 \\ 5, & n \geq 4 \end{cases}$ □

4 Conclusion

Atlast, we derived the exact grundy chromatic number for splitting graph on cycle graph, path graph, pan graph, fan graph and double fan graph.

5 Open Problem

However, the above results can be the foundation step to develop the general bound to find grundy chromatic number of splitting graph of any graph G, which is still an open problem.

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