Int. J. Open Problems Compt. Math., Vol. 17, No. 3, September 2024
Print ISSN: 1998-6262, Online ISSN: 2079-0376

Multiplicative generalized derivations acting on the semiprime ideals of the ring

M. Çelik, E. Koç Sögütcü

Department of Mathematics, Faculty of Science, Sivas Cumhuriyet University
Sivas, Türkiye
e-mail:mahmutcelik.6390@gmail.com
e-mail:eminekoc@cumhuriyet.edu.tr

Received 18 July 2024; Accepted 1 August 2024

Abstract

Let R be a ring, P a semiprime ideal of R. A map $F : R \to R$ is called a multiplicative generalized derivation if there exists a map $d : R \to R$ such that $F(xy) = F(x)y + xd(y)$, for all $x, y \in R$. Then, d is $P-$commuting map on R, if R admits a multiplicative generalized derivation F associated with a nonzero map d such that: (i) $[F(x), x] \in P$, (ii) $F(x) \circ x \in P$, (iii)$F([x, y]) \in P$,(iv) $F(xoy) \in P$,(v) $F([x, y]) \pm [G(x), y] \in P$, (vi)$F(xoy) \pm (G(x)oy) \in P$, (vii)$[F(x), y] \pm [x, G(y)] \in P$, (viii)$F([x, y]) \pm (G(x)oy) \in P$,(ix)$F(xoy) \pm [G(x), y] \in P$, (x) $F(x)oy \pm xoG(y) \in P$, (xi) $F(x)F(y) \pm [x, y] \in P$, (xii) $F(x)F(y) \pm (x \circ y) \in P$, $\forall x, y \in R$.

Keywords: *ring, semiprime ideal, generalized derivation, multiplicative generalized derivation.*

2010 Mathematics Subject Classification: 16N60, 16W25.

1 Introduction

Let R will be an associative ring with center Z. For any $x, y \in R$, as usual $[x, y] = xy - yx$ and $xoy = xy + yx$ will denote the well-known Lie and Jordan products respectively. Recall that a ring R is prime if for $x, y \in R$, $xRy = 0$ implies either $x = 0$ or $y = 0$ and R is semiprime if for $x \in R$, $xRx = 0$

implies $x = 0$. An additive mapping $d : R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. For a fixed $a \in R$, the mapping $I_a : R \to R$ given by $I_a(x) = [a, x]$ is a derivation which is said to be an inner derivation. An additive function $F : R \to R$ is called a generalized inner derivation if $F(x) = ax + xb$ for fixed $a, b \in R$. For such a mapping F, it is easy to see that

$$F(xy) = F(x)y + x[y, b] = f(x)y + xI_b(y) \text{ for all } x, y \in R.$$

This observation leads to the following definition given by M. Bresar in [6]: An additive mapping $F : R \to R$ is called a generalized derivation if there exists a derivation $d : R \to R$ such that

$$F(xy) = F(x)y + xd(y), \quad \text{for all } x, y \in R.$$

Familiar examples of generalized derivations are derivations and generalized inner derivations, and the later include left multipliers and right multipliers (i.e., $F(xy) = F(x)y$ for all $x, y \in R$).

The commutativity of prime or semiprime rings with derivation was initiated by Posner in [18]. Thereafter, several authors have proved commutativity theorems of prime or semiprime rings with derivations. In [7], the notion of multiplicative derivation was introduced by Daif motivated by Martindale in [17]. $d : R \to R$ is called a multiplicative derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. These maps are not additive. In [13], Goldman and Semrl gave the complete description of these maps. We have $R = C[0, 1]$, the ring of all continuous (real or complex valued) functions and define a map $d : R \to R$ such as

$$d(f)(x) = \left\{ \begin{array}{ll} f(x) \log |f(x)|, & f(x) \neq 0 \\ 0, & \text{otherwise} \end{array} \right\}.$$

It is clear that d is multiplicative derivation, but d is not additive. Inspired by the definition multiplicative derivation, the notion of multiplicative generalized derivation was extended by Daif and Tammam El-Sayiad in [9] as follows:

$F : R \to R$ is called a multiplicative generalized derivation if there exists a derivation $d : R \to R$ such that $F(xy) = F(x)y + xd(y)$, for all $x, y \in R$. Dhara and Ali gave a slight generalization of this definition taking g is any map (not necessarily an additive map or a derivation) in [10]. Every generalized derivation is a multiplicative generalized derivation. But the converse is not ture in general (see example [10, Example 1.1]). Hence, one may observe that the concept of multiplicative generalized derivations includes the concept of derivations, multiplicative derivation and the left multipliers. So, it should be interesting to extend some results concerning these notions to multiplicative generalized derivations. But there are only few papers about this subject. (see [9], [10], [12] for a partial bibliography).

Let S be a nonempty subset of R. A mapping F from R to R is called centralizing on S if $[F(x), x] \in Z$, for all $x \in S$ and is called commuting on S if $[F(x), x] = 0$, for all $x \in S$. This definition has been generalized as: a map $F : R \to R$ is called a $D-$commuting map on S if $[F(x), x] \in D$, for all $x \in S$ and some $D \subseteq R$. In particular, if $D = 0$, then F is called a commuting map on S if $[F(x), x] = 0$. Note that every commuting map is a $D-$commuting map (put $0 = D$). But the converse is not true in general. Take D some a set of R has no zero such that $[F(x), x] \in D$; then F is a $D-$commuting map but it is not a commuting map.

Posner showed that if a prime ring has a nontrivial derivation which is centralizing on the entire ring, then the ring must be commutative in [18]. In [5], Awtar considered centralizing derivations on Lie and Jordan ideals. For prime rings Awtar showed that a nontrivial derivation which is centralizing on Lie ideal implies that the ideal is contained in the center if the ring is not of characteristic two or three. In [14] Lee and Lee obtained the same result while removing the restriction of characteristic not three. By Rehman, if R is a prime ring whose characteristic is different from two, U is a square-closed Lie ideal of the ring R and (f, d) is the generalized derivation of R, for every $u \in U$ $[f(u), u] = 0$, then $U \subset Z$ in [19] . This theorem was proven by Gölbaşı and Koç by removing the $u^2 \in U$ condition in [12] It was studied by Argaç and Albaş by taking generalized derivation in the prime ring in [4] . Rehman et.al. discussed this in prime ideals that affect the derivation [20]. In the present paper, we shall extend above results for semiprime ideals with multiplicative generalized derivation of R. Also, we will investigate these results for Jordan product.

In order to extend the standard theory of “derivations in rings” recently, Almahdi et al. initiated the study of derivations of an arbitrary ring R satisfying some $P-$valued conditions, where P is a prime ideal of R in [2]. Specifically, they improved the well-known Posner's Second Theorem as follows: If P is a prime ideal of a ring R and d a derivation of R such that $[[d(x), x], y] \in P, \forall x, y \in R$, then $d(R) \subseteq P$ or R/P is a commutative ring. Further Mamouni et al.investigated many $P-$valued differential identities such as: (i) $[d(x), g(y)] \in P$, (ii) $d(x) \circ g(y) \in P$, (iii) $[d(x), y] + [x, g(y)] \in P$, (iv) $[d(x), y] + [x, g(y)] - [x, y] \in P$, (v) $[d(x), y] + [x, g(y)] - [y, d(x)] \in P$ for all $x, y \in R$ in a prime ring R and d, g are the derivations of R in [16]. The authors also examined some particular cases of these identities in semi-prime rings. In the successive paper Mamouni et al. extended this theory to the class of generalized derivations and obtained the commutativity of the quotient rings in [15].

In [3], the authors explored the commutativity of prime ring R in which satisfies any one of the properties when f is a generalized derivation: (i)$f(x)f(y) - xy \in Z$, (ii) $f(x)f(y) + xy \in Z$, for all $x, y \in R$. In 2009, these conditions were discussed by Gölbaşı and Koç for generalized derivations on a Lie ideal U such that $u^2 \in U$ for each $u \in U$ of the prime ring [11]. Additionally, A. Ali, V. D. Filippis and F. Shujat examined these conditions in the semiprime ring in 2013 in [1].

In this paper we investigate the above-mentioned algebraic identities by multiplicative generalized derivation involving the semiprime ideal, without making any assumptions about semiprime on the ring under discussion.

2 Results

We will make some extensive use of the basic commutator identities:

$[x, yz] = y[x, z] + [x, y]z$
$[xy, z] = [x, z]y + x[y, z]$
$xo(yz) = (xoy)z - y[x, z] = y(xoz) + [x, y]z$
$(xy)oz = x(yoz) - [x, z]y = (xoz)y + x[y, z]$.

Moreover, we shall require the following lemmas.

Lemma 2.1 *[21, Lemma 1.3] Let R be a ring, P a prime ideal of R. If any of the following conditions is satisfied for all $x, y \in R$, then R/P is commutative integral domain.*
i) $[x, y] \in P$,
ii) $xoy \in P$.

Theorem 2.2 *Let R be a $2-$torsion free ring with P a semiprime ideal of R. Suppose that R admits a multiplicative generalized derivation F associated with a nonzero map d. If $[F(x), x] \in P$, for all $x \in R$, then d is $P-$commuting map on R.*

Proof. By the hypothesis, we have

$$F(x)x - xF(x) \in P, \text{ for all } x \in R.$$

By linearizing this expression, we have

$$F(x)y + F(y)x - xF(y) - yF(x) \in P, \text{ for all } x, y \in R. \tag{1}$$

Replacing y by yx in this expression, we get

$$F(x)yx + F(y)x^2 + yd(x)x - xF(y)x - xyd(x) - yxF(x) \in P, \text{ for all } x, y \in R. \tag{2}$$

Right multiplying (1) by x, we obtain

$$F(x)yx + F(y)x^2 - xF(y)x - yF(x)x \in P. \tag{3}$$

Subtracting (2) from (3), we get

$$yd(x)x - xyd(x) - yxF(x) + yF(x)x \in P. \tag{4}$$

Replacing y by $zy, z \in R$ in (3), we have

$$zyd(x)x - xzyd(x) - zyxF(x) + zyF(x)x \in P. \tag{5}$$

Left multiplying (4) by z, we get

$$zyd(x)x - zxyd(x) - zyxF(x) + zyF(x)x \in P. \tag{6}$$

Comparing (5) and (6), we obtain

$$[x, z]yd(x) \in P. \tag{7}$$

Replacing z by $d(x)$ in this expression, we have

$$[x, d(x)]yd(x) \in P. \tag{8}$$

Writing y by $xd(x)$ in the last expression, we get

$$[x, d(x)]yxd(x) \in P. \tag{9}$$

Left multiplying (8) by x, we get

$$[x, d(x)]yd(x)x \in P. \tag{10}$$

Subtracting (9) from (10), we get

$$[x, d(x)]y[x, d(x)] \in P.$$

Since P is a semiprime ideal of R, we conclude that

$$[x, d\,(x)] \in P, \text{ for all } x \in R$$

and so d is $P-$commuting map on R.

Theorem 2.3 *Let R be a $2-$torsion free ring with P a semiprime ideal of R. Suppose that R admits a multiplicative generalized derivation F associated with a nonzero map d. If $F(x) \circ x \in P$, for all $x \in R$, then d is $P-$commuting map on R.*

Proof. *B*y the hypothesis, we get

$$F(x) \circ x \in P, \text{ for all } x \in R.$$

Replacing x by $x + y$, we have

$$F(x+y)\circ(x+y) = (F(x)+F(y))\circ(x+y) = F(x)\circ x+F(x)\circ y+F(y)\circ x+F(y)\circ y \in P.$$

By the hypothesis, we have

$$F(x) \circ y + F(y) \circ x \in P, \text{ for all } x, y \in R. \tag{11}$$

Writing y by yx in the last expression, we have

$$\begin{aligned} F(x) \circ yx + F(yx) \circ x &= F(x) \circ yx + (F(y)x + yd(x)) \circ x \\ &= F(x) \circ yx + F(y)x \circ x + yd(x) \circ x \\ &= (F(x) \circ y)x - y[F(x), x] + (F(y) \circ x)x \\ &\quad +F(y)[x, x] + (y \circ x)d(x) + y[d(x), x] \in P \end{aligned}$$

Using (11), we have

$$-y[F(x), x] + (y \circ x)d(x) + y[d(x), x] \in P. \tag{12}$$

Replacing y by yz, $z \in R$ in this expression, we have

$$-yz[F(x), x] + y(z \circ x)d(x) + [y, x]zd(x) + yz[d(x), x] \in P.$$

Using (12), we have

$$[y, x]zd(x) \in P, \text{ for all } x, y \in R.$$

Replacing y by $d(x)$ in the last expression, we have

$$[d(x), x]zd(x) \in P, \text{ for all } x, y \in R.$$

Using the same arguments after (8) in the proof of Theorem 2.2, we get the required result.

Theorem 2.4 *Let R be a $2-$torsion free ring with P a semiprime ideal of R. Suppose that R admits a multiplicative generalized derivation F associated with a nonzero map d. If any of the following conditions is satisfied for all $x, y \in R$, then d is $P-$commuting map on R.*

(i) $F([x, y]) \in P$,
(ii) $F(xoy) \in P$,
(iii) $F([x, y]) \pm [G(x), y] \in P$,
(iv) $F(xoy) \pm (G(x)oy) \in P$,
(v) $[F(x), y] \pm [x, G(y)] \in P$.
(vi) $F([x, y]) \pm (G(x)oy) \in P$,
(vii) $F(xoy) \pm [G(x), y] \in P$,
(viii) $F(x)oy \pm xoG(y) \in P$.

Proof. (i) By the hypothesis, we have

$$F([x,y]) \in P, \text{ for all } x,y \in R.$$

Replacing yx by y in the above expression and using this expression, we get

$$[x,y]d(x) \in P, \quad \text{for all } x,y \in R. \tag{13}$$

Writing $d(x)y$ for y in (13) and using (13), we obtain that

$$[x,d(x)]yd(x) \in P, \text{ for all } x,y \in R.$$

Using the same arguments after (8) in the proof of Theorem 2.2, we get the required result.

(ii) Assume that

$$F(xoy) \in P, \quad \text{for all } x,y \in R.$$

Writing yx for y in this expression and using this expression, we have

$$(xoy)d(x) \in P, \quad \text{for all } x,y \in R. \tag{14}$$

Taking $d(x)y$ for y in (14) and using (14), we obtain that

$$[x,d(x)]yd(x) \in P, \quad \text{for all } x,y \in R$$

Using the same arguments after (8) in the proof of Theorem 2.2, we get the required result.

(iii) By our hypothesis, we get

$$F([x,y]) \pm [G(x),y] \in P, \text{ for all } x,y \in R. \tag{15}$$

Replacing y by yx in (15) and using (15), we arrive that

$$[x,y]d(x) \pm y[G(x),x] \in P, \text{ for all } x,y \in R. \tag{16}$$

Writing $d(x)y$ instead of y in (16) and using (16), we have

$$[x,d(x)]yd(x) \in P, \text{ for all } x,y \in R. \tag{17}$$

Using the same arguments after expression (8) in the proof of Theorem 2.2, we get the required results.

(iv) We get

$$F(xoy) \pm (G(x)oy) \in P, \text{ for all } x,y \in R. \tag{18}$$

Writing yx for y in (18) and using (18), we obtain that

$$(xoy)d(x) \mp y[G(x), x] \in P, \text{ for all } x, y \in R. \tag{19}$$

Substituting $d(x)y$ for y in (19) and using this expression, we find that

$$[x, d(x)]yd(x) = 0, \text{ for all } x, y \in R.$$

Using the same arguments after expression (8) in the proof of Theorem 2.2, we complete the proof.

(v) By the hypothesis, we have

$$[F(x), y] \pm [x, G(y)] \in P, \text{ for all } x, y \in R.$$

Replacing y by yx in the hypothesis, we have

$$y[F(x), x] + [F(x), y]x \pm [x, G(y)]x \pm [x, y]g(y) \pm y[x, g(x)] \in P.$$

Application the hypothesis, we get

$$y[F(x), x] \pm [x, y]g(x) \pm y[x, g(x)] \in P. \tag{20}$$

Writing y by zy, $z \in R$ in (20) and using this expression, we get

$$[x, z]yg(x) \in P, \text{ for all } x, y, z \in R.$$

This expression is the same as in (6). The proof is completed using similar techniques.

(vi) By the hypothesis, we get

$$F([x, y]) \pm (G(x)oy) \in P, \text{ for all } x, y \in I.$$

Replacing y by yx in the hypothesis, we have

$$F([x, y])x + [x, y]d(x) \pm (G(x)oy)x \pm y[G(x), x] \in P.$$

Application the hypothesis, we get

$$[x, y]d(x) \pm y[G(x), x] \in P.$$

Writing y by zy in this expression and using this expression, we get

$$z[x, y]d(x) + [x, z]yd(x) \pm zy[G(x), x] \in P, \text{ for all } x, y, z \in R. \tag{21}$$

Using (21) in this expression, we get

$$[x, z]yd(x) \in P, \text{ for all } x, y, z \in R.$$

This expression is the same as in (6). The proof is completed using similar techniques.

(vii) By the hypothesis, we have

$$F(xoy) \pm [G(x), y] \in P.$$

Replacing y by yx in the hypothesis, we have

$$F(xoy)x + (xoy)d(x) \pm [G(x), y]x \pm y[G(x), x] \in P.$$

Application the hypothesis, we get

$$(xoy)d(x) \pm y[G(x), x] \in P. \tag{22}$$

Writing y by zy in this expression and using this expression, we get

$$z(xoy)d(x) + [x, z]yd(x) \pm zy[G(x), x] \in P, \text{ for all } x, y, z \in R.$$

Using (22) in this expression, we get

$$[x, z]yd(x) \in P, \text{ for all } x, y, z \in R.$$

This expression is the same as in (7). The proof is completed using similar techniques.

(viii) By the hypothesis, we have

$$F(x)oy \pm xoG(y) \in P.$$

Replacing y by yx in the hypothesis, we have

$$y[F(x), x] + (F(x)oy)x \pm (xoG(y))x \pm [x, y]g(y) \pm y[x, g(x)] \in P.$$

Application the hypothesis, we get

$$y[F(x), x] \pm [x, y]g(x) \pm y[x, g(x)] \in P.$$

Writing y by zy in this expression and using this expression, we get

$$[x, z]yg(x) \in P, \text{ for all } x, y, z \in R.$$

This expression is the same as in (7). The proof is completed using similar techniques.

Theorem 2.5 *Let R be a $2-$torsion free ring with P a semiprime ideal of R. Suppose that R admits a multiplicative generalized derivation F associated with a nonzero map d. If any of the following conditions is satisfied for all $x, y \in R$, then d is $P-$commxting map on R.*

i) $F(x)F(y) \pm [x, y] \in P$,

ii) $F(x)F(y) \pm (x \circ y) \in P$

Proof. i) By the hypothesis, we have

$$F(x)F(y) \pm [x, y] \in P, \text{ for all } x, y \in R. \tag{23}$$

Replacing y by yz, $z \in I$ in (23), we get

$$(F(x)F(y) \pm [x, y])z \pm y[x, z] + F(x)yd(z) \in P.$$

Using (2.1), we obtain that

$$\pm y[x, z] + F(x)yd(z) \in P. \tag{24}$$

Taking x by xz in the last expression, we have

$$F(x)zyd(z) + xd(z)yd(z) \pm y[x, z]z \in P. \tag{25}$$

Replacing y by zy in (24), we have

$$F(x)zyd(z) \pm zy[x, z] \in P. \tag{26}$$

If expression (26) subtracts (25) from expression, we find that

$$xd(z)yd(z) \pm [y[x, z], z] \in P. \tag{27}$$

Replacing x by xz in the above expression, we have

$$xzd(z)yd(z) \pm [y[x, z], z]z \in P. \tag{28}$$

Right multiplying by z in (27) and if this expression is subtracted from expression (28), we obtain that

$$x[d(z)yd(z), z] \in P$$

Replacing x by $[d(z)yd(z), z]x$ in this expression and using this expression, we get

$$[d(z)yd(z), z]R[d(z)yd(z), z] \subseteq P.$$

Since P is a semiprime ideal of R, we get

$$[d(z)yd(z), z] \in P, \forall y, z \in R,$$

and so

$$d(z)yd(z)z - zd(z)yd(z) \in P.$$

Replacing y by $yd(z)x$ in last expression and using this expression, we have

$$d(z)y[d(z), z]xd(z) \in P. \tag{29}$$

Taking y by zy in (29) , we get

$$d(z)zy[d(z),z]xd(z) \in P. \tag{30}$$

Left multiplying by z this (29), we find that

$$zd(z)y[d(z),z]xd(z) \in P. \tag{31}$$

If expressions (30) and (31) are brought to the side, we see that

$$[d(z),z]y[d(z),z]xd(z) \in P. \tag{32}$$

Replacing x by xz in this expression, we find that

$$[d(z),z]y[d(z),z]xzd(z) \in P.$$

Right multiplying by z in (32), we find that

$$[d(z),z]y[d(z),z]xd(z)z \in P.$$

If the last two expressions hand side of the extraction is done, we see that

$$[d(z),z]y[d(z),z]x[d(z),z] \in P.$$

Right multiplying by $y[d(z),z]$ last expression, we find that

$$[d(z),z]y[d(z),z]R[d(z),z]y[d(z),z] \in P.$$

Since P is a semiprime ideal of R, we get

$$[d(z),z]y[d(z),z] \in P.$$

That is,

$$[d(z),z]R[d(z),z] \in P.$$

Since P is a semiprime ideal of R, we have d is $P-$commuting map.

ii) By the hypothesis, we have

$$F(x)F(y) \pm (x \circ y) \in P. \tag{33}$$

Replacing y by yz, $z \in I$ in the last expression, we get

$$(F(x)F(y) \pm (x \circ y))z \pm y[x,z] + F(x)yd(z) \in P.$$

Commxting this expression with z and using the expression (33), we obtain that

$$\pm y[x,z] + F(x)yd(z) \in P.$$

This expression is the same as expression (24), in Theorem 2.5. Therefore, using the methods in Theorem 2.5 (i), we get d is $P-$commuting map.

Corollary 2.6 *Let R be a $2-$torsion free ring with P a prime ideal of R. Suppose that R admits a multiplicative generalized derivation F associated with a nonzero map d. If any of the following conditions is satisfied for all $x, y \in R$, then $d(R) \subseteq P$ or R/P is commutative integral domain.*
(i) $[F(x), x] \in P$, *(ii)* $F(x) \circ x \in P$, *(iii)*$F([x, y]) \in P$,*(iv)* $F(xoy) \in P$,*(v)* $F([x, y]) \pm [G(x), y] \in P$, *(vi)*$F(xoy) \pm (G(x)oy) \in P$, *(vii)*$[F(x), y] \pm [x, G(y)] \in P$, *(viii)*$F([x, y]) \pm (G(x)oy) \in P$,*(ix)*$F(xoy) \pm [G(x), y] \in P$, *(x)* $F(x)oy \pm xoG(y) \in P$, *(xi)* $F(x)F(y) \pm [x, y] \in P$, *(xii)* $F(x)F(y) \pm (x \circ y) \in P$.

Proof. By the same techniques in the proof of above Theorems, we obtain that

$$[x, y]zd(x) \in P, \text{ for all } x, y, z \in R.$$

That is,

$$[x, y]Rd(x) \subseteq P, \text{ for all } x, y \in R.$$

Since P is prime ideal, we get either $[x, y] \in P$ or $d(x) \in P$, for each $x \in R$. We set $K = \{x \in R \mid [x, y] \in P, \text{ for all } y \in R\}$ and $L = \{x \in R \mid d(x) \in P\}$. Clearly each of K and L is additive subgroup of R. Morever, R is the set-theoretic union of K and L. But a group can not be the set-theoretic union of two proper subgroups, hence $K = R$ or $L = R$. In the first case, we have $[x, y] \in P$, for all $x, y \in R$. By Lemma 2.1, we obtain that R/P is an integral domain. In the latter case, we have $d(x) \subseteq P$, for all $x \in R$. We conclude that $d(R) \subseteq P$. This completes the proof.

Data Availability Statement: My manuscript has no associate data.

Conflict of Interest : The authors have no conflicts of interest to declare that are relevant to the content of this article.

Acknowledgement : This paper is studied as a masters thesis at Sivas Cumhuriyet University, Institute of Science.

3 Open Problem

Our hypotheses are considered for the multiplicative generalized derivation on a semiprime ideal of the ring. Considering all hypotheses on the ring with semiprime ideals gives more general results. Also, if different derivations and conditions are considered on prime or semiprime ideals of the ring, many papers on this topic turn out to have different results.

References

[1] A. Asma, V. De Filippis and F. Shujat, *On one sided ideals of a semiprime ring with generalized derivations*, Aequat. Math., 85 (3) (2013), 529-537

[2] F.A.A. Almahdi, A. Mamouni and M. Tamekkante, *A generalization of Posner's theorem on derivations in rings,* Indian J. Pure Appl. Math., 51(1) (2020), 187–194.

[3] M. Ashraf, A. Ali and S. Ali, *Some commutativity theorems for rings with generazlized derivations*, *Southeast Asian Bull. Math.,* 31 (2007), 415-421.

[4] N. Argaç, E. Albaş, *Generalized derivations of prime rings*, Algebra Coll., 11 (3) (2004), 399-410.

[5] R. Awtar, *Lie structure in prime rings with derivations,* Publ. Math. Debrecen, 31 (1984), 209-215.

[6] M. Bresar, *On the distance of the compositions of two derivations to the generalized derivations*, Glasgow Math. J., 33 (1) (1991), 89-93.

[7] M. N. Daif, *When is a multiplicative derivation additive,* Int. J. Math. Math. Sci., 14(3) (1991), 615-618.

[8] M. N. Daif, H. E. Bell, *Remarks on derivations on semiprime rings*, Int. J. Math. Math. Sci., 15(1) (1992), 205-206.

[9] M. N. Daif, M. S. Tamman El-Sayiad, *Multiplicative generalized derivation which are additive*, East-West J. Math., 9(1) (1997), 31-37.

[10] B. Dhara, S. Ali, *On multiplicative (generalized) derivation in prime and semiprime rings,* Aequat. Math., 86 (2013) 65-79.

[11] Ö. Gölbaşı, Ö., E. Koç, *Notes on commutativity of prime rings with generalized derivations*, Commun. Fac. Sci. Ank. Series A1 58, (2009), 39-46.

[12] Ö. Gölbaşı, E. Koç, *Generalized derivations on Lie ideals in prime rings*, Turk. J. Math., 35 (2011), 23-28.

[13] H. Goldman, P. Semrl, *Multiplicative derivations on $C(X)$, Monatsh Math.*, 121(3) (1969), 189-197.

[14] P.H. Lee, T.K. Lee, *Lie ideals of prime rings with derivations*, Bull. Institute of Math. Acedemia Sinica, 11, (1983), 75-79.

[15] A. Mamouni, B. Nejjar, L. Oukhtite, *Differential identities on prime rings with involution*, J. Algebra & Appl., 17(9), 11, 2011.

[16] A. Mamouni, L. Oukhtite, M. Zerra, *Prime ideals and generalized derivations with central values on rings*, Rendiconti del Circolo Matematico di Palermo Series 2. doi: 10.1007/s12215-020-00578-3.

[17] W. S. Martindale III, *When are multiplicative maps additive,* Proc. Amer. Math. Soc., 21 (1969), 695-698.

[18] E. C. Posner, *Derivations in prime rings*, Proc Amer.Math.Soc., 8, 1093-1100, 1957.

[19] N. Rehman, *On commutativity of rings with generalized derivations*, Math. J. of Okayama Univ., 44 (2002), 43-49.

[20] N. Rehman, H. Alnoghashi and A. Boua, *Identities in a Prime Ideal of a Ring Involving Generalized Derivations*, Kyungpook Math. J., 61 (2021), 727-735.

[21] N. Rehman, H. Alnoghashi, *Action of prime ideals on generalized derivations-I*, arXiv preprint arXiv:2107.06769, 2021.

[22] M. A. Quadri, M. S. Khan and N. Rehman, *Generalized derivations and commxtativity of prime rings,* Indian J. Pure Appl. Math., 34(9) (2003), 1393-1396.