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SAR Image Filtering in Wavelet Domain by Subband Depended Shrink

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Abstract

This paper proposes an adaptive threshold estimation method for denoising in wavelet domain merged with translation invariant denoising. The subband shrink is computationally more efficient and adaptive because the parameters required for estimating the threshold depend on subband data. A new probability density function is proposed to model the statistics of wavelet coefficients. The subband threshold is derived using Bayesian estimation theory and the new pdf. Different shifts are used and applied to the noisy image in order to attain different estimates to the unknown image an then linearly average the estimates. Synthetic aperture radar (SAR) images are inherently affected by multiplicative speckle noise, which is due to the coherent nature of the scattering phenomenon. We apply the proposed method for speckle SAR images by using logarithmic transformation. Experimental results on several test images are compared with various denoising techniques.

Keywords: *Wavelet Transform, Image Despeckling, Translation invariant, Bayesian estimation, and SAR images*

1 Introduction

Active radar sensing is often a prime source of inventory information about remote and cloud-covered areas of the world. Due to its high penetration power, Synthetic Aperture Radar (SAR) acquires high resolution images in almost all atmospheric conditions. However, the automatic interpretation of SAR images is often extremely difficult due to speckle noise. Appearing as a random granular pattern, speckle seriously degrades the image quality and hampers the interpretation of image content.

In recent years, wavelet-based denoising algorithm has been studied and applied successfully for speckle removal in SAR images [1]. Compact support of wavelet basis functions allows wavelet transformation to efficiently represents functions or signals, which have localized features. The representation of the signals is made available with the implementation of a simple windowing technique that is capable of providing different representation of signals than traditional filtering techniques. Denoising using wavelet-based algorithm is also known to be more computational efficient than standard speckle filters.

Several methods have been proposed for denoising the signals. Wavelets transform has been proved to be successful tool for analysis of signals because of its good localization properties in time and frequency domains [2] , [3]. In the recent years there has been a fair amount of research on wavelet thresholding and threshold selection for signal de-noising [4] , [5] ,[6] ,[7] , [8], because wavelet provides an appropriate basis for separating noisy signal from the image signal. The motivation is that as the wavelet transform is good at energy compaction, the small coefficients are more likely due to noise and large coefficient due to important signal features. These small coefficients can be thresholded without affecting the significant features of the image.

Thresholding is a simple non-linear technique, which operates on one wavelet coefficient at a time. In its most basis form, each coefficient is threshold by comparing against threshold, if the coefficient is smaller than threshold, set to zero; otherwise it is kept or modified. Replacing the small noisy coefficients by zero and inverse wavelet transforms on the result may lead to reconstruction with the essential signal characteristics and with less noise.

In this a paper, a near optimal threshold estimation technique for image denoising is proposed which is subband dependent i.e. the parameters for computing the threshold are estimated from the observed data, one set for each subband. A new probability density function (pdf) is proposed to model the statistics of wavelet coefficients, and the new threshold is derived using Bayesian theory. The proposed method is used to speckle filtering of SAR images. A logarithm is taken of the speckle image, then the speckle multiplicative corruption of the original image becomes additive.

The outline of this paper is as follows. The denoising process in the transform domain is discussed in section 2. In Section 3, we develop a new subband adaptive shrinkage function for natural images. Section 4 introduces the concept of translation – invariant denoising**.** The proposed algorithm for the denoising technique is presented in Section 5. The results of the proposed algorithm are presented in Section 6 . We close the paper with conclusions and open problem in Section7 and section 8 respectively.

2 Wavelet Thresholding

Let

$$f = \{ f_{ij} , i, j = 1,2,......, M \} \tag{1}$$

denote the $M \times M$ matrix of the original image to be recovered and M is some integer power of 2. During transmission to signal f is corrupted by independent and identically distributed (i.i.d) zero mean, white Gaussion noise n_{ij} with standard deviation σ i.e. $n_{ij} \sim N(0,\sigma^2)$ and at the receiver end, the noisy observation

$$g_{ij} = f_{ij} + \sigma n_{ij} \tag{2}$$

is obtained. The goal is to estimate an $\hat{f}$ which minimizes the mean squared error (MSE),

$$MSE = \frac{1}{M^2} \sum_{i,j=1}^{M} (\hat{f}_{ij} - f_{ij})^2 \tag{3}$$

Let W and W^{-1} denote the two dimensional orthogonal discrete wavelet transform (DWT) matrix and its inverse respectively. Then $Y = Wg$ represents the matrix of wavelet coefficients of g having four subbands (LL, LH, HL, HH). The sub-bands HH_k, HL_k, LH_k are called details, where k is the scale varying from 1,2,……,J and J is the total number of decomposition. The size of the subband at scale k is $M/2^k \times M/2^k$. The subband LL_J is the low-resolution residue.

The different methods for denoising differ only in the selection of the threshold. The basic procedure remains the same.

- Calculate the discrete wavelet transform of the image.
- Threshold the wavelet coefficients. (Threshold may be universal or subband adaptive)
- Compute the inverse wavelet transforms to get the denoised estimate $\hat{f}$.

Soft thresholding has been used over hard thresholding to the following reasons: Soft thresholding has been shown to achieve near minimax rate over a large number of Besov spaces [4]. Moreover, it is also found to yield visually more pleasing images. Hard thresholding is found to introduce artifacts in the recovered images. The soft thresholding with threshold λ is defined as follows

$$D(U,\lambda) = \operatorname{sgn}(U) \bullet \max(0, |U| - \lambda) \qquad (4)$$

A disadvantage of the DWT is that, in contrast of the CWT, this decimated representation is not invariant under translation. The lack of shift invariance makes it unsuitable for pattern recognition and also limits the performance in denoising. The latter is perhaps more clear from the viewpoint of the lack of redundancy. The redundancy of a representation, in general, helps to better estimate a signal from its noisy observation. In this respect, Translation – invariant was proposed as: one averages denoising results of several cyclically shifted image versions.

3 Estimation of the threshold using Bayesian denoising model

In, the wavelet domain, if we use an orthogonal wavelet transform, the problem can be formulated as

$$Y = X + V \qquad (5)$$

where $Y = Wg$ denote the matrix of wavelet coefficients of g, and similarly $X = Wf$ and $V = Wn$.

Our aim in this section is to estimate the desired signal f from the noisy observation. The maximum a-posteriori (MAP) estimator will be used for this purpose. The classical MAP estimator for (5)

$$\hat{X}(Y) = \arg\max_{X} P_{X|Y}(X|Y) \qquad (6)$$

Using Bayes rule, one gets

$$\begin{aligned}\hat{X}(Y) &= \arg\max_{X}\{P_{Y|X}(Y|X) \bullet P_X(X)\} \\ &= \arg\max_{X}\{P_V(Y-X) \bullet P_X(X)\}\end{aligned} \qquad (7)$$

Therefore, these equations allow us to write this estimation in terms of the pdf of the noise (P_V) and pdf of the signal coefficient (P_X) . From the assumption on the noise, P_V is zero mean Gaussian with variance σ_n ,i.e.,

$$P_V(V) = \frac{1}{\sigma_n\sqrt{2\pi}} e^{-\frac{V^2}{2\sigma_n^2}} \qquad (8)$$

It has been observed that wavelet coefficients of natural images have highly non-Gaussian statistics [9], [10]. The pdf for wavelet coefficients is often modeled a generalized (heavy-tailed) Gaussian [11]

$$P_X(X) = K(s,p)e^{-\left|\frac{X}{s}\right|^p} \quad (9)$$

where s, p are the parameters for this model, and $K(s,p)$ is the parameter-dependent normalization constant. In practice, generally, two problems arise with the Baysian approach when an accurate but complicated $P_X(X)$ is used:

(1) It can be difficult to estimate the parameters of P_X for a specific image, especially from noisy data,

(2) the estimators for these models may not have simple closed form solution and can be difficult to obtain.

The solution for these problems usually requires numerical techniques. Equation (7) is also equivalent to

$$\hat{X}(Y) = \arg\max_X [\log(P_V(Y - X) + \log(P_X(X)] \quad (10)$$

As in [8], [10], let us define

$$f(X) = \log(P_X(X)) \quad (11)$$

By using (8), (11), (10) becomes

$$\hat{X}(y) = \arg\max_X [-\frac{(Y-X)^2}{2\sigma_n^2} + f(X)] \quad (12)$$

This is equivalent to solving the following for $\hat{W}$ if $P_X(X)$ is assumed to strictly convex and differentiable

$$\frac{Y - \hat{X}}{\sigma_n^2} + f'(\hat{X}) = 0 \quad (13)$$

$$P_X(X) = \frac{1}{\beta\sigma} e^{-\frac{\beta|X|}{\sigma}} \quad (14)$$

Where, the scale parameter β is computed once for each scale using the following equation as in [4],

$$\beta = \sqrt{\log(\frac{L_k}{J})} \quad (15)$$

Where L_k is the length of the subband at k th scale, then

$$f(X) = -\log(\beta\sigma) - \frac{\beta|X|}{\sigma} \quad (16)$$

And the estimator will be

$$\hat{X}(Y) = sign(Y) \bullet [|Y| - \frac{\beta\sigma_n^2}{\sigma}]_+ \quad (17)$$

Here, $(h)_+$ is defined as

$$(h)_+ = \begin{cases} 0\,; \text{if } h < 0 \\ g\,; \text{other wise} \end{cases} \tag{18}$$

Equation (18) is the soft shrinkage function.

4 Translation – invariant denoising

Thresholding in the orthogonal wavelet domain has been observed to produce significantly, noticeable artifacts such as Gibbs-like ringing around edges and specks in smooth regions. To ameliorate this unpleasant phenomenon, Coifman and Donoho [12] proposed the translation invariant (TI) denoising. The discussion in [12] is one – dimensional (1-D), but Ismail and Nabil proposed TI in 2-D [13]. Let $Shif_{k,l}[g]$ denote the operation of circularly shifting the input image g by k indices in the vertical direction and l indices in the horizontal, and let $Unshif_{k,l}[g]$ be a similar operation but in the opposite direction. Also, let $Denoise[g,T]$ denote the operation of taking the DWT of the input image g, threshold it with a threshold T according to equation (17), the transform it back to the space domain. Then TI denoising yields an output which is the average of the threshold copies over all possible shifts:

$$\hat{f} = \frac{1}{M^2} \sum_{k,l}^{M-1} Unshift_{k,l}[Denoise[Shift_{k,l}[g],T]] \tag{19}$$

The rationale is that since the orthogonal wavelet transform is a time-varying transform and thresholding the coefficients produces ringing-like phenomena, thresholding a shifted input would produce ringing at different locations, and averaging over all different shifts would yield an output with more attenuated artifacts than a signal copy alone.

5 Proposed Algorithm

Let us first introduce the global description of the method for computing the subband threshold and speckle removing.

5.1 Estimation of parameters for the threshold

This section describes the method for computing the various parameters used to calculate the threshold value (T), which is adaptive to different subband characteristics:

$$T = \frac{\beta \sigma_n^2}{\hat{\sigma}_y} \tag{20}$$

Where, the scale parameter β is computed one for each scale using the following equation:

$$\beta = \sqrt{\log(\frac{L_k}{J})} \qquad (21)$$

where L_k is the length of the subband k th scale.

$\hat{\sigma}_n^2$ is the noise variance, which is estimated for the subband HH_1, using formula [14]

$$\hat{\sigma}_n^2 = [\frac{median(|Y_{ij}|)}{0.6745}]^2, Y_{ij} \in \text{subband } HH_1 \qquad (22)$$

and σ_y is the standard deviation of the subband consideration.

5.2 Image denoising algorithm

This section describes the proposed image denoising algorithm in the wavelet domain for recovering original from the noisy one. The algorithm is very simple to implement and computationally more efficient. It has following steps:

(1) for $k = 1,\ldots\ldots,M$, $l = 1,\ldots\ldots,M$ do $unshift_{k,l}[Denoise[Shift_{k,l}[g],T]]$

Where the steps of $Denoise[I,T]$ are:

- perform multiscale decomposition of the image corrupted by Gaussian noise using wavelet transform(let the total number of subbands of different scales is J).
- estimate $\hat{\sigma}_n^2$ using equation (22)
- for each level, compute the scale parameter β using equation (21)
- for each subband (except the lowpass residual):
  - (a) compute the standard deviation σ_y
  - (b) compute threshold T using equation (20)
  - (c) apply soft thresholding to the noisy coefficients

(2) compute the mean average to reconstruct the denoised image $\hat{f}$ using equation (19).

5.3 Speckle model

The denoising framework described is based on the assumption that the distribution of the noise is additive zero mean Gaussian. In speckle image, the noise content is multiplicative and non- Gaussian. Such noise is generally more difficult to remove than additive noise because the intensity of the noise varies with the image intensity. A model of multiplicative noise is given by

$$g(i,j) = f(i,j)n(i,j) \qquad (23)$$

Where the speckle *g(i,j)* is the product of the original image *f(i,j)* and the non-Gaussian noise *n(i,j)*.

In most application involving multiplicative noise, the noise content ia assumed to be stationary with unitary mean and unknown noise variance σ^2 .to obtain an additive noise model, we have to apply a logarithmic transformation on the special image *g(i,j)*. The noise component in *n(i,j)* can be approximated as on additive zero mean Gaussian process as shown in the following equation.

$$\ln g(i, j) = \ln f(i, j) + \ln n(i, j) \quad (24)$$

The *DWT* is then applied to ln *g(i,j)*. After the inverse *DWT*, the processed image is subject to an exponential transformation to reverse the logarithmic operation.

6 Experimental Results

Here we present the results of the proposed algorithm and compare our results with popular threshold based denoising methods. We have performed our experiments on simulated 256×256 SAR images. As a quantitative performance measure we have used the signal to noise ratio as

$$SNR = 10\log_{10}(P_{signal} / P_{noise}) \quad (25)$$

where the power is estimated by calculating the variance. We have also compared the methods by calculating peak-signal-to-noise ratio

$$PSNR = 10\log_{10}((255)^2 / MSE) \quad (26)$$

The Daubechies D_1, D_2, D_3, D_4 filter pairs are used in wavelet transform. Here, we report the results only for the Haar wavelet D_1 .

For performance computation of the proposed method and comparison with other methods, we have taken a noise-free SAR image and added a multiplicative speckle noise. We have compared our result with those of other similar methods. Such as Multilevel soft-threshold based method [15], Universal threshold based denoising and Visushrink. The computation of results of the other methods was done to compare the results on the same image and on similar scale. In all the multilevel experiments we have done wavelet decomposition up to 4 levels.

Figure 1 show the expanded view of the same image of denoised image by Multilevel thresholding, Universial thresholding, and our proposed algorithms.

The result in Fig.1 show that, the proposed method preserves the detail features and sharp information to a great extend compared to other methods, and the superiority of the proposed method over the other methods for better noise removal as well as better preservation of sharp features. The Multilevel thresholding method also preserves sharp features but is poor in noise removal, while the Universal threshold is very poor for preserving sharp features.

(a) Noisy image (SNR=49.8432, PSNR=34.1659)

(b) denoised image by proposed method (SNR=63.451, PSNR=44.4660)

(c)Mltilevel threshold denoised image (SNR=54.0672, PSNR=36.763)

(d)Universal threshold denoised image (SNR=56.4763, PSNR=40.935)

Figure 1. Simulation Results using the proposed algorithm and several other methods

7 Conclusion

In this paper, a simple and subband adaptive threshold with TI algorithm is proposed to address the issue of image recovery from its noisy counterpart. It is based on the generalized Guassian distribution modeling of subband coefficients. The method is computationally efficient and significantly reduces the speckle while preserving the sharp features in the original images. The proposed method has a large potential in real time SAR imaging enhancement.

8 Open Problem

With sensors becoming ubiquitous and computers becoming more powerful, scientists are collecting and analyzing data at ever increasing pace. In many fields such as astronomy, medical imaging, and computer vision, the data that is collected is often noisy, either as a result of the data acquisition process or due to natural phenomena such as atmospheric disturbances. This noise must be removed from the data before it can be analyzed. In the last decade, several new techniques have been developed for removing the noise. The main problem in image denoising is removing the noise more effectively while preserving the edges in the data. For this reason , techniques have made image denoising a very active research area. In this paper, we formulate novel solution to the image processing SAR denoising problem. There are a lot of types of noise such as deconvolution noise. Deconvolution aims to extract crisp images from blurry observations. Deconvolution is extremely important in applications such as satellite imaging and seismic imaging. Decnvolution would also be very interesting open problem in applied and computational mathematics.

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