Int. J. Open Problems Compt. Math., Vol. 12, No. 3, November 2019
ISSN 1998-6262;
www.i-csrs.org

Numerical codes coupling for solving vibro-acoustic problems in harmonic mode

R. El Khaoulani

Laboratoire Modlisation et Calcul Scientifique, Facult des Sciences et Techniques,
Universit Sidi Mohamed Ben Abdellah
Fs, Maroc
e-mail:relkhaoulani@gmail.com

Received 8 January 2019; Accepted 22 June 2019

(Communicated by A. Taakili)

Abstract

The coupling of two finite element parallel softwares is a technique that has many interests in modeling fluid-structure interaction problems. The most important challenge however is to achieve an accurate and efficient coupling. This study makes a contribution to deal with this issue in vibro-acoustic problems in harmonic mode. The fluid-structure interaction in this case is described as a term that depends on the pressure in the fluid and the vibrations of the structure. Finite element method was used to approximate this term on a refined mesh to ensure a better accuracy of this approximation. An implementation method has been detailed to optimize the computing cost of this term as well as the data exchanges that is required for this computing.

Keywords: *Vibro-acoustic coupling, harmonic mode, finite element method, parallel computing.*

2010 Mathematics Subject Classification: 65M60, 65Y05, 65M50, 65K05.

1 Introduction

The apprehension of fluid-structure interaction (FSI) is an important challenge for research and industry. Nowadays numerical simulation, particularly the

finite element method, has become reliable and widely employed to model such issues. This is made possible especially by the coupling codes approach, which consists in the development of an interface between two solvers, each one dealing with only a part of the phenomena involved, to numerically model a multi-physical problem. A general overview of FSI problems can be found in [1, 2, 3, 4].

The reduction of noise and vibrations in the passenger compartment in automobiles can be further improved, despite the level of perfection held in this area. Optimization by numerical modeling of this application is an FSI problem, which requires modeling both the propagation of sound waves in the air and structural vibrations resulting from sound waves.

FSI problems can be solved by the monolithic or partitioned approaches. The monolithic approach [5, 6, 7] treats the fluid and the structure dynamics in the same mathematical framework to form a single system equation, including interface conditions, for the entire problem.

The monolithic approach can potentially achieve better accuracy and it is advantageous in terms of stability [8, 9]. However, it requires a multidisciplinary expertise and needs developing a code that incorporats both physical domains, which results in equations of large sizes difficult to solve numerically and need substantially more resources.

Conversely, the equations governing fluid flow and structural vibrations in the coupling approach are solved separately using two different solvers, the FSI is taken into account in each solver by the addition of a coupling module. The later approach allows also to take advantage of the numerical algorithms that have been validated and used for solving many complicated fluid or structural problems. The coupling approach preserves the independence of the two solvers, however the most important challenge of this approach is how to achieve accurate and efficient coupling module. This study makes a contribution to deal with this issue in the case of a vibro-acoustic problem in harmonic mode.

The coupling module involves fields derived from the two domains. Therfore data exchanges on the intreface is essential. Moreover, the precision of the approximations and the computation cost in the coupling module must be optimized so that this does not negatively affect the accuracy or the cost of this approach. Besides, meshes of the shared border in the two domains are non-matching, generally the fluid mesh is finer than the mesh associated with the structure solver [10]. Accordingly, it is necessary to establish the neighborhood relations between the two meshes in the interface level in order to extrapolate fields and carry out data exchanges.

In this study, we develop a coupling interface in vibro-acoustic mode of two parallel finite element softwares, a fluid software and a structure software. In the next section we briefly expose mathematical models and numerical ap-

proximations of the solvers. In section 3, at first we describe the coupling as a term that depends on the pressure in the fluid and the vibrations of the structure, then we focus on the numerical approximation of this term by the finite element method. We propose a method that consist in approximating this term on a refined mesh to have a better accuracy of this approximation. Afterwards, we focus in the implementation aspects, we detail some implementation techniques in order to optimize the computing cost of this term as well as the data exchanges that is required for this computing. Then we expose the geometric processing on the interface, which is necessary on handling spatially non-matching meshes. In the last part, we give a conclusion of this work.

2 FSI formulation and approximation

This study concerns the coupling of two finite element parallel codes. Separate computer codes for the fluid and the structure are coupled to enable numerical modeling of vibro-acoustic problems in harmonic mode. It is important to explain the mathematical model i.e. the theoretical background, the numerical aspects in each code and coupling boundary conditions. The FSI problem in this study involves a flexible structure excited an acoustic fluid. In the literature, this type of problems are usually referred to as vibro-acoustic problems or structural-acoustic problems with fluid interaction.

In the structural acoustics the time harmonic vibration of an elastic structure are studied in response to incident acoustic waves. The motion of the structure is modeled by the elasto-dynamic equations for the structural displacements, and the acoustic waves in the fluid are modeled by the Helmholtz equation for the fluid pressure.

Let Ω_s and Ω_f denote polygonal or polyhedral bounded domains in $\mathbb{R}^n$ ($n = 2$ or 3) occupied by the fluid and the solid, respectively, as schematized in figure 1 in a 2D case. Let Σ the interface between the two media, $\partial\Omega_s = \Sigma \cup \Sigma_s$ where Σ_s the part of the structure border that is not in contact with the fluid and $\partial\Omega_f = \Sigma \cup \Sigma_f$ where Σ_f the part of the fluid border that is not in contact with the structure. Vectors n_f and n_s are respectively the unit normal vector pointing outwards of the fluid and of the structure.

2.1 Numerical modeling in the structure code

In the context of this study we consider the determination of the motion of an flexible structure in contact with a compressible fluid. We assume a linear response of the structure because the displacements are small in this case. We consider the linear elasticity theory, then the linearized strain tensor ε is

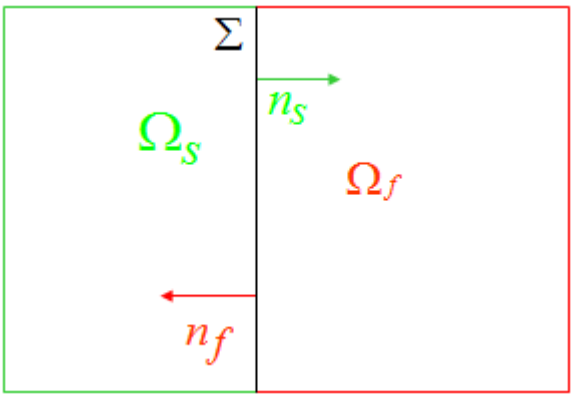

Figure 1: Geometrical notations.

related to the displacement u as follows

$$\varepsilon_{kh} = \frac{1}{2}(u_{kh} + u_{hk}) \tag{1}$$

The behavior of the structure material is considered to be elastic, linear, isotropic and homogeneous. The constitutive law, linking the Cauchy stress tensor σ to the tensor of the linearized strains ε, is reduces to the Hooke's law

$$\sigma_{ij} = a_{ijkh}\varepsilon_{kh} \tag{2}$$

where a_{ijkh} are the elasticity coefficients of the structure.
The stress tensor can be expressed as a displacement function, if one replaces ε_{kh} by its expression in (1) in the constitutive law (2) he obtains

$$\sigma_{ij}(u) = a_{ijkh}\varepsilon_{kh}(u) = \frac{a_{ijkh}}{2}(u_{k,h} + u_{h,k}) \tag{3}$$

Under theses conditions, the displacement u satisfies the elasto-dynamic equation

$$\sigma_{ij,i}(u) - \rho\frac{\partial^2 u_i}{\partial t^2} = 0 \qquad \text{on } \Omega_s \tag{4}$$

In this study, the structure is excited cyclically by a harmonic force that can be formulated at point M and at time t as follows : $F(M,t) = F(M)e^{i\omega t}$, with $\omega > 0$ the pulsation of the source.
The expected response to such excitation is of the harmonic type $u(M,t) = u(M)e^{i\omega t}$.
Thus, for $\omega > 0$ and F defined on $\partial\Omega_s \setminus \Sigma_s$, the problem is written in the following form

$$\begin{cases} \text{search } u \text{ so that} \\ \sigma_{ij,i}(u) - \rho\omega^2 u_i = 0 \qquad \text{in } \Omega_s \\ \sigma(u).n_s = F \qquad \text{on } \partial\Omega_s \setminus \Sigma_s \\ u_i = 0 \qquad \Sigma_s \end{cases} \tag{5}$$

For the sake of simplicity, Dirichlet homogeneous boundary conditions are considered on Σ_s

2.1.1 Numerical approximation

The numerical approximation of the problem (5) is done by the finite element method. We consider that $\omega > 0$ and $F \in (L^2(\Omega_s))^3$, the weak formulation of the problem (5) is

$$\left\{ \begin{array}{l} \text{search } u \in (H^1_{0,\Sigma_s}(\Omega_s))^3 \text{ so that} \\ \displaystyle\int_{\Omega_s} \sigma_{ij}(u)\varepsilon_{ij}(v) - \omega \int_{\Omega_s} \rho u.v\, dx \;=\; \int_{\partial\Omega_s \setminus \Sigma_s} F.v\, d\sigma \qquad \forall v \in (H^1_{0,\Sigma_s}(\Omega_s))^3 \end{array} \right. \tag{6}$$

where $H^1_{0,\Sigma_s}(\Omega_s)$ is the sobolev space defined by $H^1_{0,\Sigma_s}(\Omega_s) = \{v \in H^1(\Omega_s) : v|_{\Sigma_s} = 0\}$.
We approximate the weak formulation (6) by the finite element method. We denote respectively by U and V the vectors of $\mathbb{R}^{n_s}$ (n_s is the number of equations of the discretized system) of components u_i and v_i, by F the force vector. The discretized expressions of the bilinear and linear forms occurring in the variational equation (6) lead to the mass M_s and the stiffness K_s matrices, that are symmetric, regular $n_s \times n_s$, defined, thanks to linearity assumptions, by

$$\int_{\Omega_s} \sigma_{ij}(u)\varepsilon_{ij}(v)\, dx \quad \Rightarrow \quad V^T\, K_s\, U$$

$$\int_{\Omega_s} \rho u.v\, dx \qquad \Rightarrow \qquad V^T\, M_s\, U$$

$$\int_{\partial\Omega_s \setminus \Sigma_s} F.v\, d\sigma \qquad \Rightarrow \qquad V^T\, F$$

The discretized form of the system (5) is then written:

$$V^T\, K_s\, U - \omega^2 V^T\, M_s\, U \;=\; V^T\, F \qquad\qquad \forall\, V \in \mathbb{R}^{n_s}$$

hence, the resulted system is

$$K_s\, U - \omega^2 M_s\, U \;=\; F \tag{7}$$

2.2 Numerical modeling in the fluid code

Mechanical models derived from physics, which represent the phenomena of wave propagation in a fluid, are presented in the form of equations or systems of partial differential equations of hyperbolic type. One of these models, based

on the theory of linear acoustics, is the scalar equation of acoustic pressure waves, otherwise known as the Helmholtz pressure equation. This model has been used in the acoustic code.

We consider a small movements in perfect, compressible, homogeneous fluid. The perfect fluid condition implies that the description of the stress tensor is made by a pressure field within the fluid. Thus, under this condition and under the other hypotheses, linear mechanics tells us that the pressure field p is related to the displacement vector u as follows

$$p = \rho c^2 div\, u \tag{8}$$

where ρ is the mass density of the fluid at rest, and c is the speed of sound in the fluid.
The behavior of a perfect fluid is governed by the Euler equation

$$\nabla p = -\rho \frac{\partial^2 u}{\partial t^2} \qquad \text{in } \Omega_f \tag{9}$$

As we have mentioned above, it is not the problems in time or transient type that interest us, but problems in frequency. The time dependence is imposed a priori through a source for example. So we have

$$\begin{cases} p(M,t) = p(M)e^{i\omega t} & \omega > 0 \text{ the pulsation} \\ u(M,t) = u(M)e^{i\omega t} \end{cases}$$

The time t is no longer a parameter and the unknown sought is no more than a function of spatial variables. Thus, the linearized Euler equation becomes

$$\nabla p - \rho\omega^2 u = 0 \qquad \text{in } \Omega_f \tag{10}$$

It follows $u = \nabla(\frac{p}{\rho\omega^2})$. In other words $\varphi = \frac{p}{\rho\omega^2}$ represents a displacement potential such that $u = \nabla\varphi$.
Thanks to this potential, it is possible to re-express (8) and (10) in the form

$$\begin{cases} \nabla p - \rho\omega^2 \nabla\varphi = 0 & \text{in } \Omega_f \quad \text{(a)} \\ p + \rho c^2 \triangle\varphi = 0 & \text{in } \Omega_f \quad \text{(b)} \end{cases} \tag{11}$$

Taking the divergence of (11-a) and injecting into it (11-b), one obtains

$$\triangle p + \frac{\omega^2}{c^2} p = 0 \quad \text{in } \Omega_f \tag{12}$$

This partial differential equation in p gives us the Helmholtz scalar equation in linear acoustic pressure. For the sake of simplicity, Neumann boundary conditions are added.

$$\begin{cases} \Delta p + k^2 p = 0 & \text{in } \Omega_f \quad \text{where } k = \frac{\omega}{c} > 0 \\ \frac{\partial p}{\partial n_f} = 0 & \text{on } \Sigma_f \end{cases} \tag{13}$$

2.2.1 Numerical approximation

The numerical approximation of the problem (13) is done by the finite element method. We suppose that $k = \frac{\omega}{c} > 0$, the weak formulation of the problem (13) is written as

$$\left\{ \begin{array}{l} \text{search } p \in H^1(\Omega_f) \text{ so that} \\ \displaystyle\int_{\Omega_f} \nabla p.\nabla q \; dx - k^2 \int_{\Omega_f} p\, q \; dx \;=\; 0 \qquad \forall q \in H^1(\Omega_f) \end{array} \right. \tag{14}$$

This weak formulation (14) is approximated using the standard Lagrange finite elements. We denote respectively by P and Q the vectors of $\mathbb{R}^{n_f}$ (n_f is the number of equations of the fluid system) of components p_i and q_i. The discrete expressions of the bilinear and linear forms in (14) lead to the matrices M_f and K_f, that are symmetric, regular, with dimensions $n_f \times n_f$, defined by

$$\begin{array}{ll} \displaystyle\int_{\Omega_f} \nabla p{\cdot}\nabla q \; dx & \Longrightarrow \quad Q^T \, K_f \, P \\ \displaystyle\int_{\Omega_f} p\, q \; dx & \Longrightarrow \quad Q^T \, M_f \, P \end{array}$$

The discretized form of (14) is then written

$$Q^T \, K_f \, P \;-\; k^2 \, Q^T \, M_f \, P \;=\; 0 \qquad \forall Q \in \mathbb{R}^{n_f}$$

Then the resulting algebraic system is

$$K_F \, P \;-\; k^2 M_f \, P \;=\; 0 \tag{15}$$

3 Coupling conditions

Coupling conditions consist in taking into account the effect of the fluid on the structure and by imposing a slippery condition at the interface. The perfect fluid condition implies that the description of the stress tensor is made by a pressure field within the fluid, it is deduced that

$$\sigma{\cdot}n_s \;=\; -\; p n_s \quad \text{ on } \Sigma \tag{16}$$

This kinetic condition relates the structure stress σ on the interface to the pressure p exerted by the fluid (action-reaction principle). This Neumann condition links also the fluid pressure p to the displacement of the structure on the interface Σ, it reflects the effect of the acoustic pressure of the fluid on the structure.

The second one is a kinematic condition which states that fluid and structure are in contact at the interface without friction. It is expressed by

$$u.n_f \;=\; u_f.n_f \quad \text{ on } \Sigma \tag{17}$$

u and u_f denote respectively the displacement of the structure and of the fluid. This slippery condition, due to the equation (10), can be written as follows

$$\frac{\partial p}{\partial n_f} \;=\; w^2 \rho u.n_f \quad \text{ on } \Sigma \tag{18}$$

In order to independently take these conditions into account in each of the two codes, we introduce the operator C defined by

$$(C\,p, u) = \int_\Sigma u.n_f\, p\, d\sigma \tag{19}$$

The effect of the fluid on the structure is then translated by an operator C_{SF} defined by

$$(C_{SF}\,p, u) = \int_\Sigma p n_f.u\, d\sigma = -(C\,p, u) \tag{20}$$

The sliding of a perfect fluid against a moving wall is taken into account by the operator C_{FS} defined by

$$(C_{FS}\,u, p) = -w^2\rho \int_\Sigma u.n_f\, p\, d\sigma = -w^2\rho(p, C^T u) \tag{21}$$

It follows that the fluid-structure interactions is reduced to a one term, which depends on both of the pressure p of the fluid and the displacement u of the structure. The most important contribution of this paper is to provide a finite element approximation of this term, using computational method that is as accurate as possible and efficient in terms of data exchange and algorithmic and implementation complexity.

3.1 Coupling term approximation

The precision of the finite element approximation and the related computation cost of the coupling term have to be optimized so that this does not negatively affect the accuracy or/and the cost of the coupling approach. A particular attention has to be made in the treatment of the interaction when the discrete structure and fluid models do not necessarily match over the interface. Non-matching meshes can arise for several reasons, for example the fluid or the structure may require a finer mesh than another for accurate results. It is necessary, in this case, to establish the neighborhood relations between the

two meshes of the common interface in order to extrapolate fields and carry out data exchanges.

To ensure an accurate finite element approximation of the coupling term, we perform this approximation in a refined mesh of the interface; to obtain an exact numerical integration of the product of a polynomial function on Γ_1 and a polynomial function on Γ_2 we integrate this product on Γ_{12} (see figure 2). This method can be easily implemented and only requires knowing how to interpolate the values of the external fields at the Gauss points of the local refined surface mesh.

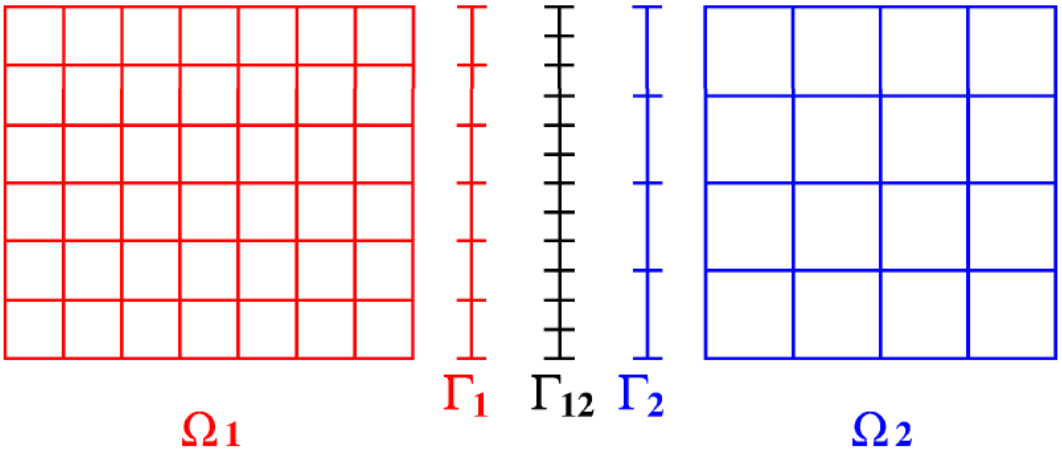

Figure 2: Non-matching meshes of the shared interface in 2D configuration.

3.1.1 Mesh refinement

Without loss of generality, we consider the 3D case where the mesh are made with hexahedra in each code. Therefore the two meshes of the common interface are made by quadrilateral elements. The coupling term is approximated in a refined mesh of the interface Γ. To obtain this refined mesh, an artificial triangulation of the mesh of the shared interface is carried out; each quadrangle element is cut into four triangles (figure 3). Then, if necessary, the triangular mesh is refined; each triangle is divided into four triangles (figure 3). Obviously, this last refinement can be repeated as many times as desired until a satisfactory mesh is obtained.

From an algorithmic point of view, a mesh is defined by a list of points (the nodes) and a list of elements (the geometry). The mesh of the interface, derived from each of the two codes, is formed by quadrilateral elements.

- To triangularize the mesh of the interface, for each quadrangle we introduce a fifth node, which is identic to the barycenter, and then we

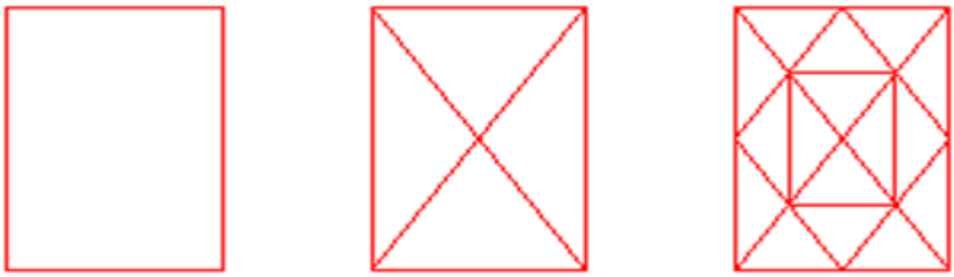

Figure 3: Mesh triangulation and refinement.

construct four triangular elements with this point and two successive points of this quadrangle.

- To refine the triangular mesh, we first construct the edges of the mesh. The difficulty in this construction is to avoid - or eliminate - duplicates (most often an edge belongs to two triangles). Next, for the three midpoints of the edges of a triangle, we associate three new nodes, then we cut the element into four triangles as shown in the figure 3.

In the case of the finite element polynomial approximation, the fields are defined on the nodes. It is therefore necessary to define these fields on the nodes of the refined mesh using their values in the initial mesh, to do that it is necessary to perform an interpolation of order 2. First at the nodes constructed from the barycenters, its value is the average of the values at the four nodes, and then at the nodes constructed from the edges its value is the average of values on the nodes of the edge.

The approximation of the coupling term is carried out in each one of the codes, we will illustrate this in the fluid code for example.

Let's note, p_i, the basis functions related to the finite element approximation of the pressure on the common interface, we have

$$\begin{aligned}(C\,p_i, u) &= \int_{\sum} u.n_f\, p_i\, d\sigma \\ &= \sum_{j=1}^{J} \int_{\Delta_j} p_i\,(u.n_f) d\sigma\end{aligned}$$

summation on the elements Δ_j included in the support of the basis functions p_i.

$$(C\,p_i, u) = \sum_{j=1}^{J} \frac{Aire(\Delta_j)}{3}[p_i(u.n_f)(\Delta_{j1}) + p_i(u.n_f)(\Delta_{j2}) + p_i(u.n_f)(\Delta_{j3})]$$

where Δ_{j1}, Δ_{j2} and Δ_{j3} are the nodes of the triangle Δ_j.

One can also write

$$(C\, p_i, u) = \sum_{j=1}^{N} coef(i,j)\, (u.n_f)(s_j)$$

summation on the nodes includes in the support of the basis function p_i (see figure 4), where

$$coef(i,j) = p_i(s_j) \left(\sum_{l=1}^{J} \frac{Aire(A_l)}{3} \right)$$

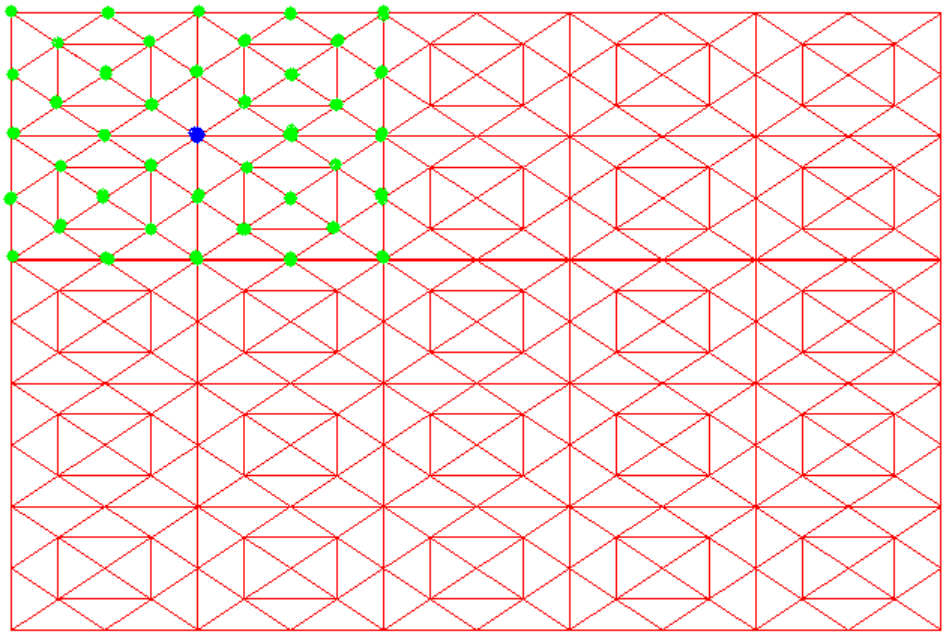

Figure 4: The nodes, of the refined mesh, contained in the support of a basis function associated to the initial mesh.

In order to optimize the later calculations and consequently of the computation of coupling term, it is therefore necessary, for each node s_i, to have the nodes that are in the support of the basis function p_i as well as the values of $coef(i,j)$ and $n_f(s_j)$. In the following section, we propose a method for this optimization using a better adapted data structure.

3.1.2 A suitable mesh structure

The conventional data structure of a mesh allows to know directly all the vertices of an element of the mesh, for example. Otherwise, we do not have the

elements containing a given vertex and a fortiori we do not have the integration points contained in the support of a basis function, which is a necessary information in the previous calculation. It is possible to develop algorithms to search for this information, but their algorithmic complexities are generally high. The idea we propose here is to create a data structure for the mesh that contains all the necessary information on the new meshes, which are obtained by the operations of triangulation and refinement of the mesh of the shared interface, in particular information used in the evaluation of the coupling term. This structure contains especially the unit outer normal defined on the nodes and on the elements, the area of each element, the elements of the refined mesh contained in the support of each basis function associated with the initial mesh, and the elements containing each node.

Having a such rich mesh structure, that contains enough information and which does not incur any additional cost to build, allows to avoid the use of search algorithms on the mesh. These algorithms usually require loops on all vertices or all elements of the mesh, which may impact negatively the complexity of this approach. Therefore, this computational optimization of the coupling term is crucial so that coupling term computation does not penalize the total computation cost.

3.2 Geometric processing on the shared interface

The management of the common interface is a tedious task because the meshes of this interface are in the majority of cases different. One important point of the coupling approach is the geometric processing and the establishment the neighborhood relations between the meshes i.e. the association of the nodes and the elements of one of the two meshes with nodes and elements of the other mesh, which is needed to extrapolate fields and carry out data exchanges. For this geometric processing we distinguish the cases of matching and non-matching meshes.

The two meshes are matching on the common interface if their nodes are themselves matching i.e. the two meshes of the interface, resulting from the two codes, are identical. In this case, pairs (node, node) are created. Each node of a given mesh is associated with the node of the other mesh that has the same coordinates.

As previously stated, the discrete structure and fluid models do not necessarily match over the common interface. Therefore, in this case pairs of (node,element) are created instead of (node,node) pairs used in the matching meshes case. We associate a node of one of the two meshes of the interface with the nearest element in the other mesh of this interface. This element is obtained by projection of the node on the other mesh, it is the one that minimizes the sum of the normal distance $d_1 = \|p - p_2\|$ and the tangential

distance $d_2 = \|p_1 - p_2\|$, as shown in figure 5.

After exchanging the meshes of the interface. The pairs (node-element) are formed i.e. for each node, the interpolation element is determined, which is the one that minimizes the distance $d = d_1 + d_2$ (see figure 5).

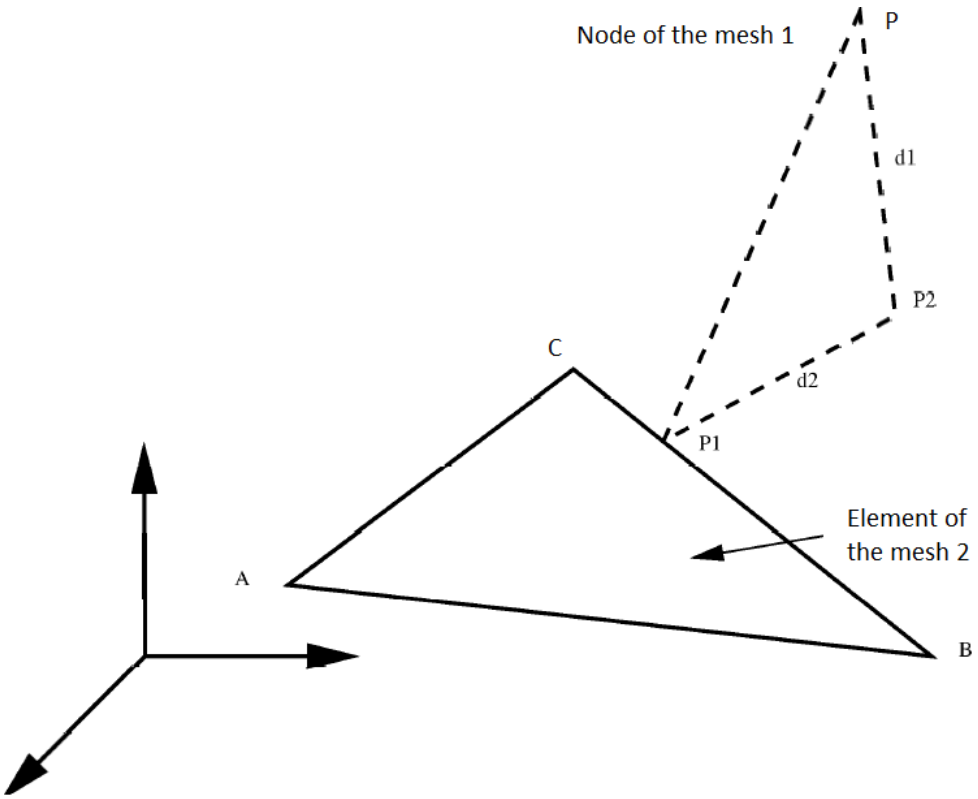

Figure 5: Interpolation element criterion.

These pairs allow to interpolate the data exchanged between the two solvers, if the node belongs to the plane of the element, interpolation coefficients are the barycentric coordinates u, v and $(1 - u - v)$. Otherwise the barycentric coordinates of its projection p_2 are used instead.

3.3 Data exchanges

We consider at first two sequential codes and each one of them is executed in a separate processor. The processors have to be connected and the MPI library is used for message transfer and reception. The meshes of the interface of the two codes are exchanged. Then, the geometric processing, which is detailed above, is performed in each processor. During resolution iterations, the interpolation of the exchanged data is carried out thanks to this geometric processing.

Let consider now two parallel codes. To distinguish between the processors of the two codes in the global communicator, the two codes assign a label to their processors (it must not be the same in the two codes). Thus, to exchange the meshes of the interface, two processors having two different labels must exchange their meshes. Then, geometric processing is carried out to determine both the interpolation element and the interpolation coefficients from the received meshes.

In a first step, the same strategy can be followed for the communications between the processors during the resolution phase i.e. exchange the data between each pair of processors that having two different labels. However, since these communications occur at each coupling iteration, This may require a relatively high cost of communication. In the following we propose an optimization method to reduce these communications between processors.

Communication between processors is necessary to exchange their meshes at the interface, but this is only done at the beginning of the execution. On the other hand, data exchanges must be carried out at each iteration of the coupling, to this end it is necessary to reduce these communications as much as possible. For this purpose the provenance of a mesh received is also preserved during the exchanges of meshes. Then, instead of building pairs of (node,element), we create pairs of (node,(element+provenance)) i.e. we connect a node with the interpolation element together with the provenance of this element. The use of this additional information makes it possible, for each processor of each code, to construct a list of processors of the other code with which data exchange is necessary. By doing so, many unnecessary data exchanges are avoided, because data exchange will be done only between processors that really have a common part of the interface.

4 Conclusion

Codes coupling is an interesting approach for numerical modeling of FSI problems, this study describes a coupling method of two parallel finite element codes to model a vibro-acoustic problem. This method ensures the independence of the two softwares thanks to a coupling module developed symmetrically in each one of them.

Coupling conditions have been formulated as a coupling term, that depends on the pressure in the fluid and the vibrations of the structure.

The approximation of this term requires a geometric processing on the interface that enables handling spatially non-matching meshes. A projection method was used to determine the neighborhood relations in order to extrapolate fields and carry out data exchanges between the two solvers.

A refined mesh of the shared interface was employed to have an accurate finite element approximation of the coupling term. Furthermore, some imple-

mentation techniques have been detailed to optimize both the computing cost of this term and the data exchanges necessary for this computing particularly in parallel environment.

5 Open Problem

The open problem here is to assess the efficiency of the proposed coupling method in industrial environment. Moreover, some codes have several levels of parallelism that are not addressed in the current contribution.

References

[1] G. Hou, J. Wang and A. Layton, *Numerical Methods for Fluid-Structure Interaction - A Review*, Commun. Comput. Phys., 12 (2012), 337–377.

[2] H.-J. Bungartz, M. Mehl and M. Schfer, *Fluid Structure Interaction II: Modelling, Simulation, Optimization*, Springer Berlin Heidelberg, 53 (2010), 1439–7358. http://dx.doi.org/10.1007/978-3-642-14206-2

[3] H.J.-P. Morand and R. Ohayon, *Fluid Structure Interaction: Applied Numerical Methods*, Wiley, (1995).

[4] Y. Bazilevs, K. Takizawa and T. E. Tezduyar, *Computational Fluid-Structure Interaction: Methods and Applications*, Wiley, (2013).

[5] B. Hubner, E. Walhorn and D. Dinkler, *A monolithic approach to fluid-structure interaction using space-time finite elements*, Comput. Methods Appl. Mech. Engrg., 193(2004),2087–2104.

[6] C. Michler, S. J. Hulshoff, E. H. van Brummelen and R. de Borst, *A monolithic approach to fluid-structure interaction*, Comput. Fluids, 33(2004), 839–848.

[7] P. B. Ryzhakov, R. Rossi, S. R. Idelsohn and E. Oate, *A monolithic Lagrangian approach for fluid-structure interaction problems*, Comput. Mech., 46(2010), 883–899.

[8] J. Degroote, K.J. Bathe and J. Vierendeels, *Performance of a New Partitioned Procedure versus a Monolithic Procedure in Fluid-Structure Interaction*, Comput. Struct., 87 (2009), 793–801.

[9] J. Heinz, *Partitioned Fluid-Structure Interaction for Full Rotor Computations Using CFD*, Ph.D. Thesis, Technical University of Denmark, Denmark, (2013).

[10] R. Glowinski, T.-W. Pan, T. I. Hesla, D. D. Joseph and J. Periaux, *A distributed lagrange multiplier/fictitious domain method for the simulation of flows around moving rigid bodies: application to particulate flow*, Comput. Methods Appl. Mech. Engrg., 184(2000), 241–268.