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Practical output feedback stabilisation for non linear time varying delayed uncertain systems

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Abstract

***I*n this paper, we treat the problem of practical output feedback stabilization of nonlinear uncertain systems with time varying delay. Furthermore, based on Lyapunov-Krasovskii functionals, we propose an output feedback controller that guarantees global uniform practical stability of the closed loop system.**

Keywords: *Time varying delay, Nonlinear uncertain systems, Practical output feedback stabilization, Lyapunov-Krasovskii functional.*

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1 Introduction

The phenomena of time-delays are often encountered in physical systems like communication systems, aircraft stabilization, nuclear reactors, and process control systems and so on. The presence of a time delay component in the state vector has an adverse impact not only on the system performance, but also on its stability; and therefore, neglecting the effects of delay in the analysis may lead to instability and incorrect design calculations (see, e.g., [10] and [11]). Hence, various methods have been proposed in the recent past for the analysis and design of time delayed systems; and mostly, the stability criterion for the time delayed systems is formulated in the linear matrix inequality (LMI) framework, that can be readily solved using standard numerical packages. The problem of stabilization for uncertain systems has been widely investigated for many years [6],[3]. In these studies, the origin was not supposed to be an equilibrium point of the uncertain system. So we can no longer expect to design a controller that guarantee the stability of the origin as an equilibrium point.

Most of the recent nonlinear controllers are designed for an uncertain system that has a nominal linear part and the controller is designed based on the knowledge of the upper bound, possibly time varying and state dependent, of the uncertainties vector norm. Such a class of systems is important because it may represent many physical systems. In [6], [15] authors investigated the state feedback stabilization problem for these systems. In this paper, we will synthesis an output feedback controller for this class of systems. It should be noted that output feedback stabilization problem for uncertain system with linear nominal part has been discussed in [3], [7], [14], [8].Under the assumption that the uncertain part is bounded by a known function that depends only on the output, they construct an output feedback controller that guarantees global exponential stability of the closed loop system.

From practical point of view, systems with perturbations cause instability. Many researches have devoted themselves to design effective control approaches to guarantee the system stability [4], [8], [5].

In this work, we have treated the case where the system presents a perturbation term, which could result from errors in modeling the nonlinear system aging of parameters, or uncertainties and disturbances which exist in any realistic problems [12], [13]. We do not know the term perturbation, but we know some information about it, like knowing an upper bound on it. The main contribution of this paper lies in the following aspects. Firstly, compared with [2] an extended class of nonlinear time-varying dynamical system under time-varying delay and a new control law are presented. Here, we will suppose that the unknown part is bounded by a function that depends on the delayed output. We will design an output feedback controller that guarantees global uniform practical stability of the closed loop system.

The rest of this paper are organized as follows: In Section 2, the practical stability definition is presented. In Section 3, the system description is shown. According to the designed an output feedback controller law, some assumptions are provided in order to prove the global uniform exponential practical stability of the controlled system. Finally, an illustrative example is described and the simulation results are presented in order to show the performances of the suggested control strategy.

2 Preliminaries

We consider a differential delay equation

$$\dot{x}(t) = F(t, x(t), x(t-\tau)) \tag{1}$$

where $t \in \mathbb{R}_+$, $x \in \mathbb{R}^n$ where $\tau > 0$ is the delay time. The knowledge of x at time $t = 0$ does not allow to deduce x at time t. Thus, the initial condition is specified as a continuous function $\chi : [t_0 - \tau, t_0] \longrightarrow \mathbb{R}^n$. The state of equation (1) at time t can be described as a function segment x_t defined by

$$x_t(h) = x(t+h),\ \ h \in [t_0 - \tau, t_0].$$

Therefore, delay equations form a special class of functional differential equations:

$$\dot{x} = \tilde{F}(t, x_t) \tag{2}$$

where $\tilde{F} : [0, +\infty[\times \mathcal{C} \longrightarrow \mathbb{R}^n$; $\mathcal{C}$ denotes the Banach space of continuous functions mapping the interval $[t_0 - \tau, t_0]$ into $\mathbb{R}^n$ equipped with the supremum-norm:

$$\forall \chi \in \mathcal{C},\ \ |\chi| = \sup_{s \in [t_0 - \tau, t_0]} \|\chi(s)\|$$

where $\|.\|$ is the Euclidean norm. As system (1) is a special case of (2).

We consider now system (2), we will recall the definition of exponential stability of the origin of system (2). Assume that $\tilde{F}$ is Lipschitz on bounded sets and satisfies $\tilde{F}(t, 0) = 0$. For $\chi \in \mathcal{C}$, we denote by $x(t, \chi)$ or shortly $x(t)$ the solution of (2) that satisfies $x_0 = \chi$.

Definition 2.1 *The system (2) is said to be exponentially practically stable, if there exist positive numbers $k \geq 0$, $r \geq 0$ and $\alpha > 0$, such that every solution $x(t, \chi)$ of the system (2) satisfies*

$$\|x(t, \chi)\| \leq k|\chi| e^{-\alpha(t - t_0)} + r, \quad \forall t \geq t_0. \tag{3}$$

Sufficient conditions for stability of time-delay systems are provided by the theory of Lyapunov-Krasovskii functionals [9], a generalization of the classical Lyapunov theory of ordinary differential equations [1]. The following theorem gives sufficient conditions to ensure that the origin of system (2) is globally practically exponentially stable.

3 Output Feedback Stabilisation

We consider the dynamical system

$$\left\{ \begin{array}{l} \dot{x} = F(t, x) + G(t, x)\left[u + \xi(t, x(t), x(t - \tau(t)), u)\right] \\ y = h(t, x) \end{array} \right. \tag{4}$$

where $t \in \mathbb{R}$, is the time $x \in \mathbb{R}^n$, is the state $u(.) \in \mathbb{R}^m$ is the control vector. $\tau(t)$ is the time varying delay in state which is differentiable function satisfying: $0 < \tau(t) < \bar{\tau}$, and there exist a parameter $\varepsilon > 0$, such that

$$\dot{\tau}(t) < 1 - \frac{1}{\varepsilon}, \quad \forall t \geq t_0. \tag{5}$$

The knowledge of x at time $t = 0$ does not allow to deduce x at time t. $F(.,.) : [0, +\infty[\times\mathbb{R}^n \longrightarrow \mathbb{R}^n$, $G(.,.) : [0, +\infty[\times\mathbb{R}^n \longrightarrow \mathbb{R}^{n\times m}$ and $h(.,.) : [0, +\infty[\times\mathbb{R}^n \longrightarrow \mathbb{R}^p$ are known nonlinear functions which are assumed, without loss of generality, to be continuous, uniformly bounded with respect to time t, locally uniformly bounded with respect to the state x satisfying $F(t, 0) = 0$ and $h(t, 0) = 0$ for all $t \in \mathbb{R}_+$.
$\xi(.,.,.) : [0, +\infty[\times\mathbb{R}^n \times \mathbb{R}^m \longrightarrow \mathbb{R}^m$ represents uncertainties in the plant. The nominal system corresponding to system (4) is given by

$$\dot{x} = F(t, x) + G(t, x)u \tag{6}$$

Our aim is to design an output delayed feedback controller such that system (4) to be globally exponentially practically stable. Before giving our synthesis approach, we introduce for system (4) the following assumption.

($\mathcal{H}_1$) $\dot{x} = F(t, x) + G(t, x)u$ is globally uniformly practically output exponentially stablilizable. There exist an output feedback $\alpha(t, y)$ such that: there exists a $\mathcal{C}^1$ function $V_1(.,.) : \mathbb{R}_+ \times \mathbb{R}^n \longrightarrow \mathbb{R}_+$ which satisfies

$$\lambda_1\|x\|^2 \leq V_1(t, x) \leq \lambda_2\|x\|^2,$$

$$\frac{\partial V_1(t, x)}{\partial t} + \nabla_x^T V_1(t, x)[F(t, x) + G(t, x)\alpha(t, y)] \leq -\lambda_3 V_1(t, x) + \mu$$

$$\|\nabla_x^T V_1(t, x)G(t, x)\| \leq \lambda_4\|x\|.$$

for all $(t, x) \in \mathbb{R}_+ \times \mathbb{R}^n$, where $\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4$ and μ are positive scalar constants and

$$\nabla_x^T V_1(t, x) = \frac{\partial V_1(t, x)}{\partial x}$$

($\mathcal{H}_2$) There exists a continuous function $\phi(.,.) : [0, +\infty[\times\mathbb{R}^p \longrightarrow \mathbb{R}^m$ such that

$$\nabla_x^T V_1(t, x)G(t, x) = \phi(t, y).$$

($\mathcal{H}_3$) There exists a positive real scalar functions $\varrho_1(.,.)$, $\varrho_2(.,.)$ and $\gamma(.)$ such that

$$\|\xi(t, x(t), x(t - \tau(t)), u)\| \leq \varrho_1(t, y) + \varrho_2(t - \tau(t), y(t - \tau(t))), \tag{7}$$

$\forall t \in \mathbb{R}_+,\ \forall u \in \mathbb{R}^m,\ \forall y \in \mathbb{R}^p$, where

$$\varrho_2(t,y) \leq \gamma(t)\|\phi(t,y)\|,$$

$$\int_0^{+\infty} \gamma^2(s)\,ds \leq M_\gamma < +\infty$$

and

$$\int_0^{+\infty} \gamma^4(s)\,ds \leq N_\gamma < +\infty$$

Theorem 3.1 *Under assumptions ($\mathcal{H}_1$), ($\mathcal{H}_2$), ($\mathcal{H}_3$) and the fact*

$$\gamma^2(t) < \frac{(\lambda_3\lambda_1)^2}{4\lambda_4^4\varepsilon e^{\lambda_3\bar{\tau}}}, \tag{8}$$

the uncertain system described by (4) *in closed-loop with the feedback*

$$u(t,y) = \alpha(t,y) - \frac{(\phi(t,y))^T(\varrho_1(t,y))^2}{\|\phi(t,y)\|\varrho_1(t,y) + r(t)} \tag{9}$$

is globally uniformly practically exponentially stable.
Where $r(t) > 0$ is a strictly positive continuous bonded function. Denote r the upper bound of $r(t)$.

proof

Let $\eta \in \mathbb{R}$, we consider the following Lyapunov-Krasovskii functional:

$$W(t,x_t) := V_1(t,x_t) + \eta^2 V_2(t,x_t),$$

where

$$V_2(t,x_t) := \int_{t-\tau(t)}^{t} e^{-\lambda_3(t-s)}(\varrho_2(s,y(s)))^2\,ds$$

The derivative of V_1 along the trajectories of system (4) is given by:

$$\dot{V}_1(t) \leq -\lambda_3 V_1(t) + \mu - \nabla_x^T V_1(t,x) G(t,x) \left[\frac{(\phi(t,y))^T (\varrho_1(t,y))^2}{\|\phi(t,y)\| \varrho_1(t,y) + r(t)} \right]$$
$$+ \|\nabla_x^T V_1(t,x) G(t,x)\| [\varrho_1(t,y) + \varrho_2(t-\tau(t), y(t-\tau(t))]$$

$$\leq -\lambda_3 V_1 + \mu - \phi(t,y) \left[\frac{(\phi(t,y))^T (\varrho_1(t,y))^2}{\|\phi(t,y)\| \varrho_1(t,y) + r(t)} \right] + \|\phi(t,y)\| \varrho_1(t,y)$$
$$+ \|\phi(t,y)\| \varrho_2(t-\tau(t), y(t-\tau(t))$$

$$\leq -\lambda_3 V_1 + \mu + \left[\frac{-(\|\phi(t,y)\| \varrho_1(t,y))^2 + (\|\phi(t,y)\| \varrho_1(t,y))^2}{\|\phi(t,y)\| \varrho_1(t,y) + r(t)} \right] +$$
$$+ \frac{r(t) \|\phi(t,y)\| \varrho_1(t,y)}{\|\phi(t,y)\| \varrho_1(t,y) + r(t)} + \phi(t,y) \varrho_2(t-\tau(t), y(t-\tau(t))$$

$$\leq -\lambda_3 V_1 + \mu + \frac{r(t) \|\phi(t,y)\| \varrho_1(t,y)}{\|\phi(t,y)\| \varrho_1(t,y) + r(t)} + \phi(t,y) \varrho_2(t-\tau(t), y(t-\tau(t))$$

$$\leq -\lambda_3 V_1 + \mu + r(t) + \|\phi(t,y)\| \varrho_2(t-\tau(t), y(t-\tau(t))$$

The derivative of V_2 along the trajectories of system (4) is given by:

$$\dot{V}_2(t) = -\lambda_3 V_2 + [\varrho_2(t, y(t))]^2 - (1 - \dot{\tau}(t)) e^{-\lambda_3 \tau(t)} [\varrho_2(t-\tau(t), y(t-\tau(t)))]^2$$

Then

$$\dot{W}(t) \leq -\lambda_3 [V_1 + \eta^2 V_2] + \|\phi(t,y)\| \varrho_2(t-\tau(t), y(t-\tau(t)) + \eta^2 [\varrho_2(t, y(t))]^2 -$$
$$\eta^2 (1 - \dot{\tau}(t)) e^{-\lambda_3 \tau(t)} [\varrho_2(t-\tau(t), y(t-\tau(t)))]^2 + \mu + r(t)$$

$$\leq -\lambda_3 [V_1 + \eta^2 V_2] - \left[\frac{e^{\frac{\lambda_3 \tau(t)}{2}}}{2\eta \sqrt{1 - \dot{\tau}(t)}} \|\phi(t,y)\| - \eta \sqrt{1 - \dot{\tau}(t)} e^{-\frac{\lambda_3}{2} \tau(t)} \varrho_2(t, y(t)) \right]^2 +$$
$$\frac{e^{\lambda_3 \tau(t)}}{4\eta^2 (1 - \dot{\tau}(t))} \|\phi(t,y)\|^2 + \eta^2 (\varrho_2(t, y(t)))^2 + \mu + r(t)$$

$$\leq -\lambda_3 W(t) + \frac{e^{\lambda_3 \tau(t)}}{4\eta^2 (1 - \dot{\tau}(t))} \|\phi(t,y)\|^2 + \eta^2 (\varrho_2(t, y(t)))^2 + \mu + r(t)$$

Which implies that

$$\dot{W}(t) \leq -\lambda_3 W(t) + \frac{e^{\lambda_3 \tau(t)}}{4\eta^2(1-\dot{\tau}(t))}\lambda_4^2\|x\|^2 + \eta^2(\gamma(t))^2(\|\phi(t,y)\|)^2 + \mu + r(t)$$

$$\leq -\lambda_3 W(t) + \frac{e^{\lambda_3 \tau(t)}}{4\eta^2(1-\dot{\tau}(t))}\lambda_4^2\|x\|^2 + \eta^2\lambda_4^2(\gamma(t))^2\|x\|^2 + \mu + r(t)$$

$$\leq -\lambda_3 W(t) + \frac{e^{\lambda_3 \tau(t)}}{4\eta^2(1-\dot{\tau}(t))}\frac{\lambda_4^2}{\lambda_1}V_1(t) + \frac{(\lambda_4\eta\gamma(t))^2}{\lambda_1}V_1(t) + \mu + r(t)$$

Using 3.1 we have

$$\frac{e^{\lambda_3 \tau(t)}}{4\eta^2(1-\dot{\tau}(t))} \leq \varepsilon e^{\lambda_3 \bar{\tau}},$$

then

$$\dot{W}(t) \leq -\left[\lambda_3 - \frac{\lambda_4^2}{\lambda_1}(\frac{\varepsilon e^{\lambda_3\bar{\tau}}}{4\eta^2} - \eta^2(\gamma(t))^2\right] W(t) + \mu + r(t).$$

Choose η such that

$$\lambda_3 - \frac{\lambda_4^2 \varepsilon e^{\lambda_3\bar{\tau}}}{4\eta^2\lambda_1} > 0,$$

take

$$\eta^2 := \frac{\lambda_4^2 \varepsilon e^{\lambda_3\bar{\tau}}}{2\lambda_1\lambda_3}.$$

Which imply that

$$\dot{W}(t) \leq -\left(\frac{\lambda_3}{2} - \frac{\left[\lambda_4\eta\gamma(t)\right]^2}{\lambda_1}\right) W(t) + \mu + r$$

Using 8 and let

$$\alpha(t) := \frac{\lambda_3}{2} - \frac{\left[\lambda_4\eta\right]^2}{\lambda_1}\gamma(t)^2 > 0$$

and $\delta := \mu + r > 0$, then

$$\dot{W}(t) \leq -\alpha(t)W(t) + \delta$$

Let $y(t) = W(t)e^{\int_{t_0}^{t}\alpha(s)ds}$, it follows that,

$$\begin{aligned} \dot{y}(t) &= (\dot{W}(t) + \alpha(t)W(t))e^{\int_{t_0}^{t}\alpha(s)\,ds} \\ &\leq \delta e^{\int_{t_0}^{t}\alpha(s)\,ds} \end{aligned}$$

Integrating between t_0 and t, one obtains $\forall t \geq t_0$,

$$y(t) \leq y(t_0) + \int_{t_0}^{t} \delta e^{\int_{t_0}^{s} \alpha(\tau)\, d\tau} ds$$

Then,

$$W(t) \;\leq\; W(t_0) e^{-\int_{t_0}^{t} \alpha(s)\, ds} + \left[\int_{t_0}^{t} \delta e^{\int_{t_0}^{s} \alpha(\tau)\, d\tau} ds \right] e^{-\int_{t_0}^{t} \alpha(s)\, ds}.$$

therefore,

$$\int_{t_0}^{t} \alpha(s)\, ds = \frac{\lambda_3}{2}(t - t_0) - e^{\frac{[\lambda_4 \eta]^2}{\lambda_1}} \int_{t_0}^{t} \gamma(s)^2\, ds$$

which implies that

$$e^{-\int_{t_0}^{t} \alpha(s)\, ds} \leq e^{\frac{[\lambda_4 \eta]^2 M_\gamma}{\lambda_1}} e^{-\frac{\lambda_3}{2}(t - t_0)}$$

and

$$\left[\int_{t_0}^{t} e^{\int_{t_0}^{s} \alpha(\tau)\, d\tau} ds \right] e^{-\int_{t_0}^{t} \alpha(s)\, ds} \leq \frac{2}{\lambda_3} e^{\frac{[\lambda_4 \eta]^2 M_\gamma}{\lambda_1}}$$

thus

$$W(t) \leq e^{\frac{[\lambda_4 \eta]^2 M_\gamma}{\lambda_1}} W(t_0)\, e^{-\frac{\lambda_3}{2}(t - t_0)} + \frac{2\delta}{\lambda_3} e^{\frac{[\lambda_4 \eta]^2 M_\gamma}{\lambda_1}}$$

In addition, since

$$V_1(t_0, x_{t_0}) \leq \lambda_2 |\chi|^2$$

and

$$\begin{aligned}
V_2(t_0, x_{t_0}) &= \int_{t_0 - \tau(t_0)}^{t_0} e^{-\lambda_3 (t_0 - s)} \varrho_2(s, y(s))^2\, ds \\
&\leq \int_{t_0 - \bar{\tau}}^{t_0} e^{-\lambda_3 (t_0 - s)} \varrho_2(s, y(s))^2\, ds \\
&\leq \lambda_4^2 \int_{t_0 - \bar{\tau}}^{t_0} e^{-\lambda_3 (t_0 - s)} \gamma(s)^2 |\chi|^2\, ds \\
&\leq \lambda_4^2 \sqrt{N_\gamma} \frac{1}{\sqrt{2\lambda_3}} \left(1 - e^{-2\lambda_3 \bar{\tau}}\right) |\chi|^2 \\
&\leq \frac{\lambda_4^2 \sqrt{N_\gamma}}{\sqrt{2\lambda_3}} |\chi|^2,
\end{aligned}$$

We get

$$W(t_0) \leq \left(\lambda_2 + \eta^2 \frac{\lambda_4^2 \sqrt{N_\gamma}}{\sqrt{2\lambda_3}} \right) |\chi|^2. \tag{10}$$

Deduce that,

$$W(t) \leq e^{\dfrac{[\lambda_4 \eta]^2 M_\gamma}{\lambda_1}} \left(\lambda_2 + \eta^2 \frac{\lambda_4^2 \sqrt{N_\gamma}}{\sqrt{2\lambda_3}} \right) |\chi|^2 \, e^{-\dfrac{\lambda_3}{2}(t-t_0)} + \frac{2\delta}{\lambda_3} e^{\dfrac{[\lambda_4 \eta]^2 M_\gamma}{\lambda_1}}$$

Then

$$\|x(t,\chi)\|^2 \leq \frac{e^{\dfrac{[\lambda_4 \eta]^2 M_\gamma}{\lambda_1}}}{\lambda_1} \left(\lambda_2 + \eta^2 \frac{\lambda_4^2 \sqrt{N_\gamma}}{\sqrt{2\lambda_3}} \right) |\chi|^2 \, e^{-\dfrac{\lambda_3}{2}(t-t_0)} + \frac{2\delta}{\lambda_1 \lambda_3} e^{\dfrac{[\lambda_4 \eta]^2 M_\gamma}{\lambda_1}}$$

This yields, the global uniform exponential stability of B_κ, with

$$\kappa = \sqrt{\frac{2\delta}{\lambda_1 \lambda_3} e^{\dfrac{[\lambda_4 \eta]^2 M_\gamma}{\lambda_1}}}$$

Hence, the system (4) in closed-loop with the output feedback described by (9) is globally practically exponentially stable. □

4 Illustrative example

To illustrate the utilization of our approach, in this section, we consider the uncertain nonlinear system under time-variable delay:

$$\begin{cases} \dot{x}_1 = -x_1 - x_2 \; e^{-2t}, \\ \dot{x}_2 = x_1 + u + \cos(x_1(t)) + \; e^{-3(t-\tau)} \sin(x_2(t-\tau(t))), \\ y = x_2. \end{cases} \tag{11}$$

This system is of the form (4) with

$$F(t,x) = \begin{bmatrix} -x_1 - x_2 \; e^{-t} \\ x_1 \end{bmatrix}, \quad G(t,x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

and

$$\xi(t, x(t), x(t-\tau(t)), u) = \frac{\cos(x_1(t))}{1+t^2} + \; e^{-3(t-\tau)} \sin(x_2(t-\tau(t)))$$

Let $t_0 = 0$, the function $\tau(t)$ is defined as follows:

$$\tau(t) := \frac{\sin^2(t)}{2}, \quad 0 < \tau(t) < \frac{1}{2}, \quad \text{and} \quad \dot{\tau}(t) = \frac{\sin(2t)}{2} < 1, \forall t \geq t_0,$$

the initial conditions for the system are $x(t) = [3cos(t) \ \ cos(t)]^T$, $\forall t \in [-0.5, 0]$. The control law that stabilizes exponentially the nominal system

$$\begin{cases} \dot{x}_1 = -x_1 - x_2 \; e^{-2t}, \\ \dot{x}_2 = x_1 + u, \end{cases}$$

is defined as follows $\alpha(t, y) = -x_2 = -y$. It is readily seen that assumption $(\mathcal{H}_1)$ is satisfied with: $V_1(t, x) = \frac{1}{2}(x_1^2 + e^{-2t} \; x_2^2)$

$$\lambda_1 = \frac{1}{2}, \ \lambda_2 = 1, \ \lambda_3 = 2, \ \mu = 0 \ \text{ and } \ \lambda_4 = 1.$$

Furthermore,

$$\|\phi(t, y)\| = \left\| \frac{\partial V_1}{\partial x} G(t, x) \right\| = e^{-2t} \|y\|.$$

We note that assumption $(\mathcal{H}_3)$ is fulfilled with

$$\rho_1(t, y) = \frac{1}{1 + t^2}$$

and

$$\rho_2(t, y) = e^{-3t} \|y\| \leq e^{-t} \|\phi(t, y)\|,$$

and

$$\gamma(t) = e^{-t}, \ M_\gamma = \frac{1}{2}, \ N_\gamma = \frac{1}{4}$$

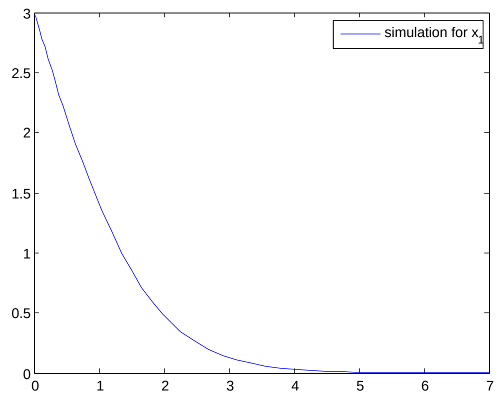
The Lyapunov-Krasovskii functional associated with this system is:

$$V_2(t, x_t) := \int_{t-\tau(t)}^{t} e^{-2(t-s)} e^{-6s} \; \|y(s)\|^2 \, ds = e^{-2t} \int_{t-\tau(t)}^{t} e^{-4s} \; \|y(s)\|^2 \, ds$$

Consequently, the assumptions of the Theorem3.1 are satisfied. Therefore, the output feedback that stabilizes exponentially practically the system (11) is defined as follows:

$$u(t, y) = \alpha(t, y) - \frac{e^{-2t} y}{e^{-2t} \|y\| + r e^{-t}(1 + t^2)},$$

where $r > 0$ and $r(t) = re^{-t}$,


simulation for x_1

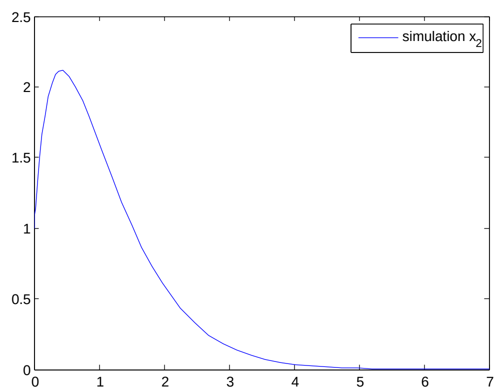

simulation x_2

Fig.1. Evolution of states x_1 and x_2 of the systems (11)

5 Open Problem

In this paper, a new control design for a class of nonlinear uncertain systems under time-varying delay is proposed. Based on Lyapunov's and Lyapunov-Krasovskii's stability theory, sufficient assumptions are given to ensure the practical and exponential stability of the suggested approach. Simulation results are shown in order to illustrate the good performances of the suggested stabilization methodology.

Currently, based on Lyapunov's and Lyapunov-Krasovskii's stability theory and sufficient assumptions for delays of any size, i am demonstrating the existence of an pratical observer which allows to obtain an practical exponential stability in the error between the state and its estimate.

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