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Triple Sequence Space of Poisson Matrix

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Abstract

We define the concept of triple sequence space of poisson matrix and their analytical structures have been discussed and also devoted to the set of all non-zero multiplicative functions is a group.

***Keywords**: Arithmetic functions, completely multiplicative functions multiplicative functions, poisson matrix, triple sequences.*

1 Introduction

A triple sequence (real or complex) can be defined as a function $x: \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{R}(\mathbb{C})$, where $\mathbb{N}, \mathbb{R}$ and $\mathbb{C}$ denote the set of natural numbers, real numbers and complex numbers respectively. The different types of notions of triple sequence was introduced and investigated at the initial by Esi et al. [1-7], Dutta et al. [8], Debnath et al. [9], Sahiner et al. [10-11], Subramanian and Esi [12] and many others. Throughout w, χ and Λ denote the classes of all, gai and analytic scalar valued single sequences, respectively. We write w^3 for the set of all complex triple sequences (x_{mnk}), where $m, n, k \in \mathbb{N}$, the set of positive integers.Then, w^3 is a linear space under the coordinate wise addition and scalar multiplication.

Let (x_{mnk}) be a triple sequence of real or complex numbers. Then the series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ is called a triple series. The triple series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ is said to be convergent if and only if the triple sequence (S_{mnk}) is convergent, where

$$S_{mnk} = \sum_{i,j,q=1}^{m,n,k} x_{ijq}(m, n, k = 1,2,3, \ldots) .$$

A sequence $x = (x_{mnk})$is said to be triple analytic if

$$sup_{m,n,k}|x_{mnk}|^{\frac{1}{m+n+k}} < \infty.$$

The vector space of all triple analytic sequences are usually denoted by Λ^3. A sequence $x = (x_{mnk})$ is called triple entire sequence if

$$|x_{mnk}|^{\frac{1}{m+n+k}} \to 0 \text{ as } m,n,k \to \infty.$$

The vector space of all triple entire sequences are usually denoted by Γ^3. The space Λ^3 and Γ^3 is a metric space with the metric

$$d(x,y) = sup_{m,n,k}\left\{|x_{mnk} - y_{mnk}|^{\frac{1}{m+n+k}} : m,n,k : 1,2,3,\ldots\right\}, \tag{1.1}$$

for all $x = \{x_{mnk}\}$and$y = \{y_{mnk}\}\ in\ \ \Gamma^3$.

A sequence $x = (x_{mnk})$ is called triple gai sequence if $((m+n+k)!\,|x_{mnk}|)^{\frac{1}{m+n+k}} \to 0$ as $m,n,k \to \infty$. The triple gai sequences will be denoted by χ^3.

Consider a triple sequence $x = (x_{mnk})$. The $(m,n,k)^{th}$ section $x^{[m,n,k]}$ of the sequence is defined by $x^{[m,n,k]} = \sum_{i,j,q=0}^{m,n,k} x_{ijq}\delta_{ijq}$ for all $m,n,k \in \mathbb{N}$,

$$\delta_{mnk} = \begin{pmatrix} 0 & 0 & \ldots 0 & 0 & \ldots \\ 0 & 0 & \ldots 0 & 0 & \ldots \\ . & & & & \\ . & & & & \\ . & & & & \\ 0 & 0 & \ldots 1 & 0 & \ldots \\ 0 & 0 & \ldots 0 & 0 & \ldots \end{pmatrix}$$

δ_{mnk} has 1 in the $(m,n,k)^{th}$ position, and zero otherwise.
The Poisson matrix is defined by $A = T \otimes I + I \otimes T$.

Example: If $T = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then
$A = T \otimes I + I \otimes T = \begin{bmatrix} T+2I & -I & 0 \\ -I & T+2I & -I \\ 0 & -I & T+2I \end{bmatrix}$.

2 Properties of Poisson matrix of Eigen values and Eigen vectors

$\boldsymbol{A = T \otimes I + I \otimes T}$

(1) We have $A \quad x_{jk} = \lambda \quad x_{jk}$ for $j, k = 1,2,3,\cdots,m$
(2) The eigen vector is orthogonal;
(3) A is symmetric;
(4) A is positive definite.
Example: If $A = \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix}$.

Hence

$$\begin{bmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix},$$

$$\begin{bmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \text{ and so on.}$$

3 Definitions and preliminaries

3.1 Definition

Any function $f: \mathbb{Z}^+ \longrightarrow \mathbb{C}$ is called a complex valued arithmetic function of poission matrix of A and the cardinality of poission matrix of $A = 2^{|\mathbb{Z}^+|}$.

3.2 Definition

Function g is called a paranorm on X linear space if
(i) $g: X \longrightarrow R$ with $g(x) \geq 0$,
(ii) $g(x) = 0 \Leftrightarrow x = 0$,
(iii) $g(x) = g(-x)$ for $\forall x \in X$,
(iv) $g(x+y) \leq g(x) + g(y)$ for $\forall x, y \in X$,
(v) If $\lambda_{rst}, \lambda_0 \in C$ with $\lambda_{rst} \longrightarrow \lambda_0 (r,s,t \longrightarrow \infty)$ and if $x_{rst}, a \in X$ with $x_{rst} \longrightarrow a (r,s,t \rightarrow \infty)$ in the sense that $g(x_{rst} - a) \longrightarrow 0 (r,s,t \longrightarrow \infty)$, then $\lambda_{rst} x_{rst} \longrightarrow \lambda_0 a (r,s,t \longrightarrow \infty)$ in the sense that $g(\lambda_{rst} x_{rst} - \lambda_0 a) \longrightarrow 0 (r,s,t \longrightarrow \infty)$.

3.3 Note

Define for each $f = (f_{mnk}) \in A$.

$$p_f = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{1}{2^{m+n+k}} \frac{|f_{mnk}|}{1+|f_{mnk}|} \leq \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{1}{2^{m+n+k}} < \infty,$$

where p is a paranorm of the poisson matrix of A.

3.4 Definition

Let $g(uvw, rst) = 1$ with $u, v, w \in \mathbb{Z}^+$. A triple sequence space of poisson matrix of the function $f \in A$ is called a multiplicative function if

$$\left(f_{mnk}^{(uvw,rst)}\right) = \left(f_{mnk}^{(uvw)}\right)\left(f_{mnk}^{(rst)}\right) \qquad (3.1)$$

Let M be a multiplicative function but M is not a linear sub-space of triple sequence space of poisson matrix of A.

3.5 Definition (Cauchy Product)

Consider

$$\left(f_{mnk}^{(rst)} \circ g_{mnk}^{(rst)}\right) = \sum_{m=0}^{r} \sum_{n=0}^{s} \sum_{k=0}^{t} (f_{mnk})(g_{rst-mnk}). \quad (3.2)$$

In equation (3.2), rst can take the value zero. We have

$$\left(f_{mnk}^{(rst)} \circ g_{mnk}^{(rst)}\right) = (f_{000})(g_{rst-000}) + (f_{111})(g_{rst-111}) + (f_{222})(g_{rst-222}) + \cdots + (f_{rst})(g_{000}).$$

3.6 Definition (Dirichlet Product)

If f and g are two triple sequences then

$$\left(f_{mnk}^{((rst))} - g_{mnk}^{(rst)}\right) = \sum_{\frac{d}{r}} \sum_{\frac{d}{s}} \sum_{\frac{d}{t}} f_{mnk,d}\, g\left(\frac{rst}{d}\right), \quad (3.3)$$

where the summation is over all divisors d of (rst).

3.7 Definition

If $\left(f_{mnk}^{(rst)}\right) \neq 0, \forall r, s, t,$ and f, g are two triple sequences then $f_{mnk}, g_{mnk} \in G$, $\left(f_{mnk}^{(rst)} * g_{mnk}^{(rst)}\right) = \left(f_{mnk}^{(rst)}\right) * \left(g_{mnk}^{(rst)}\right)$.

3.8 Definition

The triple sequence space of (f_{mnk}) is completely multiplicative if (i) $(f_{mnk,1}) = 1$, (ii) $\left(f_{mnk}^{(uvw,rst)}\right) = \left(f_{mnk}^{(uvw)}\right)\left(f_{mnk}^{(rst)}\right)$ if $g(uvw, rst) = 1$,

(iii) $f_{mnk}^{p} = (f_{mnk})^p$, where p is a prime and m, n, k are positive integers.
Let $\mathbb{C}$ be all completely multiplicative function and $\mathbb{C} \subset M$.

Example: Define $\left(f_{mnk}^{(rst)}\right) = rst$.
$\Rightarrow \left(f_{mnk,p}\right) = p^{mnk} = (f_{mnk})^p$.
$\Rightarrow f$ is completely multiplicative.

4 Main Results

4.1 Proposition

$p(f_{rst}) \longrightarrow 0 \Leftrightarrow \left|f_{rst,m,n,k}\right| \longrightarrow 0 \quad \forall m,n,k$ as $r,s,t \longrightarrow \infty$.
Proof: $\frac{1}{2^{m+n+k}} \frac{|f_{mnk}|}{1+|f_{mnk}|} \leq p_f \Rightarrow \frac{|f_{mnk}|}{1+|f_{mnk}|} \leq 2^{m+n+k} p_f$.
Suppose that $f_{rst} \longrightarrow 0 \forall m, n, k$ as $r, s, t \longrightarrow \infty$
$\Rightarrow 2^{m+n+k} p_{f_{rst}} \longrightarrow 0$.
$\Rightarrow \left|f_{rst,m,n,k}\right| \longrightarrow 0 \quad \forall m,n,k$ as $r,s,t \longrightarrow \infty$.
Similarly the converse holds.

4.2 Theorem

The triple sequence space of poisson matrix of A is a linear metric space.
Proof: **Step 1:** Let $f, g \in A$. Define $d(f,g) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{f_{mnk} - g_{mnk}}{2^{m+n+k}(1+|f_{mnk} - g_{mnk}|)}$. Then d is a metric of triple sequence space of poisson matrix for A.
$\Rightarrow |f_{mnk} - h_{mnk} - g_{mnk} + h_{mnk}| = |f_{mnk} - g_{mnk}| \quad \forall m,n,k \in \mathbb{Z}^+$.
$\Rightarrow d$ is translation invariant.
Step 2: Also, $d(x+y, a+b) \leq d(x,a) + d(y,b)$
$\Rightarrow$ Addition is continuous in (A, d).
Step 3: Let $\lambda_0 \in \mathbb{C}$ and $f \in A$. We shall prove that $d\left(f^{(rst)}, f\right) \longrightarrow 0 (r,s,t \longrightarrow \infty)$.
$\Rightarrow f_{mnk}^{(rst)} \longrightarrow f_{mnk} (r,s,t \rightarrow \infty, \forall m,n,k \in \mathbb{Z}^+)$.
Then $\lambda_{rst} \longrightarrow \lambda_0, f^{(rst)} \longrightarrow f (r,s,t \longrightarrow \infty)$.
$\Rightarrow \lambda_{rst} f_{mnk}^{(rst)} \longrightarrow \lambda_0 f_{mnk} (r,s,t \longrightarrow \infty, \quad \forall m,n,k \in \mathbb{Z}^+)$.
$\Rightarrow \lambda_{rst} f^{(rst)} \longrightarrow \lambda_0 f (r,s,t \longrightarrow \infty)$.
$Therefore$ scalar multiplication is continuous.
Step 4: suppose that $d\left(f^{(rst)}, f\right) \longrightarrow 0 (r,s,t \longrightarrow \infty)$.
Let $b_{mnk}^{(rst)} = \left|f_{mnk}^{(rst)} - f_{mnk}\right|$, for any $m,n,k \in \mathbb{Z}^+$.
Then $\epsilon_{mnk}^{(rst)} = \frac{b_{mnk}^{(rst)}}{1+b_{mnk}^{(rst)}} \leq 2^{m+n+k} \cdot d(f^{rst}, f) \longrightarrow 0$ as $r,s,t \longrightarrow \infty$.

But $0 \le \epsilon_{mnk}^{(rst)} < 1$, and hence $b_{mnk}^{(rst)} = \frac{\epsilon_{mnk}^{(rst)}}{1+\epsilon_{mnk}^{(rst)}}$.

$\Rightarrow b_{mnk}^{(rst)} \longrightarrow 0$ as $(r, s, t \longrightarrow \infty)$.

$\Rightarrow f_{mnk}^{(rst)} \longrightarrow f_{mnk}(r, s, t \longrightarrow \infty, \forall m, n, k \in \mathbb{Z}^{+})$.

$\Rightarrow b_{mnk}^{(rst)} \longrightarrow 0(r, s, t \longrightarrow \infty, \forall m, n, k \in \mathbb{Z}^{+})$.

$\Rightarrow d\left(f^{(rst)}, f\right) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{b_{mnk}^{(rst)}}{2^{m+n+k}\left(1+b_{mnk}^{(rst)}\right)} +$

$\sum_{m=r+1}^{\infty} \sum_{n=s+1}^{\infty} \sum_{k=t+1}^{\infty} \frac{b_{mnk}^{(rst)}}{2^{m+n+k}\left(1+b_{mnk}^{(rst)}\right)}$

$d\left(f^{(rst)}, f\right) \le \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{b_{mnk}^{(rst)}}{2^{m+n+k}\left(1+b_{mnk}^{(rst)}\right)} +$

$\sum_{m=r+1}^{\infty} \sum_{n=s+1}^{\infty} \sum_{k=t+1}^{\infty} \frac{1}{2^{m+n+k}}$

(4.1)

Let $\epsilon > 0$. Choose $m = m(\epsilon) \in \mathbb{Z}^{+}$ such that the sum of the second series in equation (4.1) is less that $\frac{\epsilon}{2}$, with this choice of m, the first sum in equation (4.1)$\longrightarrow 0$ as $r, s, t \longrightarrow \infty$. Since $b_{mnk}^{(rst)} \longrightarrow 0(r, s, t \longrightarrow \infty, \forall m, n, k \in \mathbb{Z}^{+})$. by our hypothesis, we have $d\left(f^{(rst)}, f\right) < \epsilon$ for sufficiently large r, s, t. Hence the triple sequence of Poisson matrix of A is a linear metric space.

4.3 Theorem

M is a closed subset of triple sequence space of poisson matrix of A.

Proof: Let $D(M)$ be the derived set of M and $(f_{mnk}) \in D(M)$ then there exists

$\Rightarrow \left(f_{mnk}^{(rst)}\right) \in M \longrightarrow f_{mnk}$.

$\Rightarrow p\left(f_{mnk}^{(rst)} - f_{mnk}\right) \longrightarrow 0$ as $r, s, t \longrightarrow \infty$.

$\Rightarrow \left|\left(f_{mnk}^{(rst)} - f_{mnk}\right)\right| \longrightarrow 0, \forall \quad m, n, k$.

$\Rightarrow \left(f_{mnk}^{(rst)}\right) \longrightarrow (f_{mnk}), \forall \quad m, n, k$

$\Rightarrow lim_{rst\to\infty}\left(f_{mnk}^{(rst)}\right) = (f_{mnk})$.

Let $\left((m, n, k), (uvw)\right) = 1$. Then

$\left(f_{mnk,uvw}\right) = lim_{rst\to\infty}\left(f_{mnk,uvw}^{(rst)}\right) = lim_{rst\to\infty}\left\{\left(f_{mnk}^{(rst)}\right)\left(f_{mnk}^{(uvw)}\right)\right\} =$

$lim_{rst\to\infty}\left(f_{mnk}^{(rst)}\right) \cdot lim_{rst\to\infty}\left(f_{mnk}^{(rst)}\right) = (f_{mnk}) \cdot (f_{uvw})$.

Therefore the triple sequence space of (f_{mnk}) is multiplicative. We have the triple sequence space of $f \in M$ and $D(M) \subset M$. Hence M is a closed subset of the triple sequence space of poisson matrix A.

4.4 Theorem

The triple sequence space of $(M,\circ)$ is an abelian group with the identity

$$(e_{rst-mnk}) = \begin{cases} 1 & \text{if} r,s,t = 0 \\ 0 & \text{if} r,s,t > 0 \end{cases}.$$

Proof: We shall verify

$\left(f_{mnk}^{(rst)} \circ e_{mnk}^{(rst)}\right) = (f_{000})(e_{rst-000}) + (f_{111})(e_{rst-111}) + (f_{222})(e_{rst-222}) + \cdots + (f_{rst})(e_{000})$

$= 0 + 0 + 0 + \cdots + (f_{rst}) \cdot 1 = (f_{rst}).$

Similarly $\left(e_{mnk}^{(rst)} \circ f_{mnk}^{(rst)}\right) = \left(e_{mnk}^{(rst)}\right)$. Hence e is the identity. Other axioms are verified.

4.5 Theorem

If f, g and h are triple sequences, then $(G,*)$ is an abelian group

Proof: Step 1: Let $(uvw, rst) = 1$ and $u, v, w, r, s, t \in \mathbb{Z}^+$.

$(f_{mnk} * g_{mnk})^{(uvw,rst)} = (f_{mnk})^{(uvw,rst)} \cdot (g_{mnk})^{(uvw,rst)}$

$= (f_{mnk})^{(uvw)} \cdot (f_{mnk})^{(rst)} \cdot (g_{mnk})^{(uvw)} \cdot (g_{mnk})^{(rst)}$

$= \left[(f_{mnk})^{(uvw)} \cdot (g_{mnk})^{(uvw)} \cdot\right]\left[(f_{mnk})^{(rst)} \cdot (g_{mnk})^{(rst)}\right]$

$= \left[(f_{mnk} * g_{mnk})^{(uvw)}\right]\left[(f_{mnk} * g_{mnk})^{(rst)}\right]$

$\Rightarrow (f_{mnk} * g_{mnk}) \in G$. Hence $(G,*)$ is closed.

Step 2: Let $f, g, h \in G$

$[f_{mnk} * (g_{mnk} * h_{mnk})]^{(rst)} = (f_{mnk})^{(rst)} \cdot (g_{mnk} * h_{mnk})^{(rst)}$

$= (f_{mnk})^{(rst)} \cdot \left[(g_{mnk})^{(rst)} \cdot (h_{mnk})^{(rst)} \cdot\right] \cdot (f_{mnk})^{(rst)}$

$= \left[(f_{mnk})^{(rst)} \cdot (g_{mnk})^{(rst)}\right] \cdot (h_{mnk})^{(rst)}$

$= \left[(f_{mnk} * g_{mnk})^{(rst)}\right] \cdot (h_{mnk})^{(rst)}$

$= [(f_{mnk} * g_{mnk}) * h_{mnk}]^{(rst)}$

$\Rightarrow [f_{mnk} * (g_{mnk} * h_{mnk})] = [(f_{mnk} * g_{mnk}) * h_{mnk}]$. Hence associativity holds.

Step 3: Define $e: \mathbb{Z}^+ \longrightarrow \mathbb{C}$ by $\left(e^{(rst)}\right) = 1 \forall r, s, t.$

$\Rightarrow \left(e^{(uvw,rst)}\right) = 1$

$= \left(e^{(uvw)}\right)\left(e^{(rst)}\right)$ with $\left((uvw), (rst)\right) = 1$.

$\Rightarrow e \in G.$

Also $(f_{mnk} * e)^{(rst)} = \left(f_{mnk}^{(rst)}\right) \cdot \left(e^{(rst)}\right) = \left(f_{mnk}^{(rst)}\right) \cdot 1 = \left(f_{mnk}^{(rst)}\right).$

$\Rightarrow f_{mnk} * e = f_{mnk}$. Similarly $e * f_{mnk} = f_{mnk}$, hence identity holds.

Step 4: Given $(f_{mnk}) \in G$, take g as $(g_{mnk})^{(rst)} = \frac{1}{(f_{mnk})^{(rst)}}$

$(g_{mnk})^{(uvw,rst)} = \frac{1}{(f_{mnk})^{(uvw,rst)}} = \frac{1}{(f_{mnk})^{(uvw)}(f_{mnk})^{(rst)}} = \frac{1}{(f_{mnk})^{(uvw)}} \cdot \frac{1}{(f_{mnk})^{(rst)}} = (g_{mnk})^{(uvw)} \cdot (g_{mnk})^{(rst)}.$

$\Rightarrow g \in G.$

Also $(f_{mnk} * g_{mnk})^{(rst)} = (f_{mnk})^{(rst)} \cdot (g_{mnk})^{(rst)} = \frac{(f_{mnk})^{(rst)}}{(f_{mnk})^{(rst)}} = e \cdot (r, s, t).$

$\Rightarrow (f_{mnk} * g_{mnk}) = e = (g_{mnk} * f_{mnk})$. Hence g is the inverse of triple sequence of f.

5 Open Problem

Whichever matrix or matrices are chosen intead of Poisson matrix, the conditions given in the article are satisfied?

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