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The Discrete Poisson-Rani Distribution with Properties and Applications

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Abstract

To analyse count data, a new discrete Poisson-Rani distribution has been established in this study by combining two distributions, Poisson and Rani, using a compounding procedure. The developed distribution's structural characteristics have been derived and investigated. Different graphics have been used to demonstrate the behaviour of the probability mass function (pmf) and the cumulative distribution function (cdf). The well-known approach of maximum likelihood estimation is utilised to estimate the parameter of the formulated model. Eventually, two data sets were utilised to monitor the effectiveness of the constructed model to the Poisson and Poisson-Lindley distribution.

Keywords: Compound technique, Poisson distribution, Rani distribution, reliability analysis, order statistics, maximum likelihood estimator.

Mathematics subject classification: 60-XX, 62-XX, 11-KXX.

1 Introduction

In statistical distributions, discrete distributions are extremely important. Researchers have made a lot of effort in recent years to establish new discrete models for interpreting count data. There are several approaches in the statistical literature for developing new distributions and providing extensions to classical distributions by adding additional parameters to them. Transmutation, discretization of a continuous distribution, the Marshall-Olkin technique, compounding, and so on are examples. Classical distributions frequently fail to offer an appropriate fit to observable data. It has become urgent for scholars to investigate new probability models in order to overcome the constraints of traditional distributions. Over the last decade, researchers' attention has been drawn to the compounding of distributions. The compounding approach is most commonly used when the parameter of one distribution is a random variable and follows another distribution, as in the case of count data. The combining of two separate distributions yields the compounding of distributions. It makes no difference whether they are discrete or continuous. Depending on the parent distribution, the resultant distribution may be continuous or discrete. As in this study, we used the compounding approach to establish a new distribution by combining the Poisson and Rani distributions. The newly established distribution is "Discrete Poisson-Rani distribution". There are numerous applications of compounding distributions in various fields of bio-medicine, insurance, engineering and communications etc. Researchers in this field have put forth a lot of effort. Greenwood and Yule are credited with the invention of compounding models [1]. They used a Poisson distribution combined with a negative binomial distribution, with the Poisson model's parameter as the gamma variate. Sankaran [2] developed the Poisson-Lindley distribution by using compounding mechanism and studied its several properties. Mahmodi et al. [3] expanded the Poisson-Lindley distribution and shown that it produces superior results when analysing count data. Zamiani and Ismail [4] compounded negative binomial-Lindley distribution and studied its different properties. Gupta and Ong [5] developed new generalization of the negative binomial distribution by compounding the mean of the Poisson distribution with generalized gamma distribution. The resultant distribution has been applied to a variety of life time data and has proven to be a superior alternative to the negative binomial distribution. Shanker [6] has proposed discrete Poisson-Akash distribution. Wanbo lu ad Daiman. Shi [7] has proposed a new compounding life time distribution the Weibull-Poisson distribution. Subhradev sen [8] has formulated the Poisson quasi-xgamma distribution and discussed its several properties. Giovani carrara Rodrigues et al [9] used different methods of estimation for Poisson exponential distribution. Shanker et al [10] proposed a generalization of Poisson- Sujatha distribution. This study proposes a new discrete Poisson-Rani distribution probability model which is constructed by employing a compounding method, and its different mathematical aspects are addressed. Finally, the newly constructed model's efficacy was tested using two data sets.

2 Definition of Established Model Discrete Poisson-Rani Distribution

Consider a random variable Y follows Poisson distribution i:e $Y \sim P(\lambda)$ and assuming that the parameter of $P(\lambda)$ follows Rani distribution with parameter θ. The distribution formed by combining the Poisson and Rani distributions is a discrete distribution with a probability mass function abbreviated as PRD(Y,θ). The probability function of the obtained model PRD(θ) is given by the following theorem.

Theorem 2.1 The probability mass function of a discrete Poisson-Rani distribution PRD(Y,θ) is given as

$$P(Y=y)=\frac{\theta^5}{(\theta^5+24)}\left[\frac{y^4+10y^3+35y^2+50y+24+\theta(\theta+1)^4}{(\theta+1)^{y+5}}\right]\ ;\ y=0,1,2..,\theta>0.$$

Proof The probability mass function of discrete Poisson- Rani distribution (PRD) (Y,θ) may be obtained as follows

If $Y \sim P(\lambda)$, then the probability mass function (pmf) of Poisson distribution is given by

$$f(Y|\lambda)=\frac{e^{-\lambda}\lambda^y}{y!};y=0,1,2,...\ ;\lambda>0.$$

As the parameter λ follows Rani distribution with probability density function (pdf)

$$g(\lambda;\theta)=\frac{\theta^5}{\theta^5+24}(\theta+\lambda^4)e^{-\theta\lambda};\lambda>0,\theta>0.$$

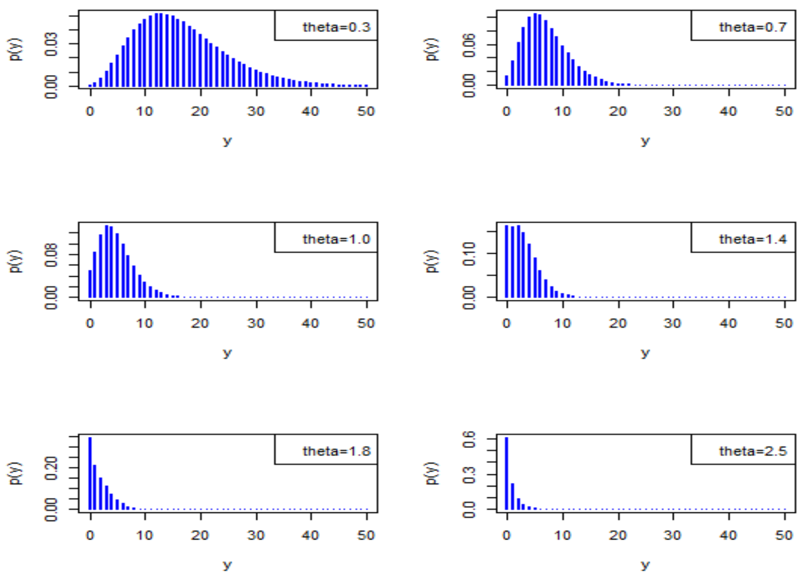
We have

$$P(Y=y)=\int_0^\infty f(Y|\lambda).g(\lambda;\theta)d\lambda=\int_0^\infty\frac{e^{-\lambda}\lambda^y}{y!}\frac{\theta^5}{(\theta^5+24)}(\theta+\lambda^4)e^{-\theta\lambda}d\lambda$$

$$=\frac{\theta^5}{y!(\theta^5+24)}\int_0^\infty(\theta\lambda^y+\lambda^{y+4})e^{-(1+\theta)\lambda}d\lambda$$

$$=\frac{\theta^5}{(\theta^5+24)}\left[\frac{y^4+10y^3+35y^2+50y+24+\theta(\theta+1)^4}{(\theta+1)^{y+5}}\right];\ y=0,1,2,...;\theta>0.\quad(2.1)$$

The following graphs depict the behaviour of pmf for different values of parameters



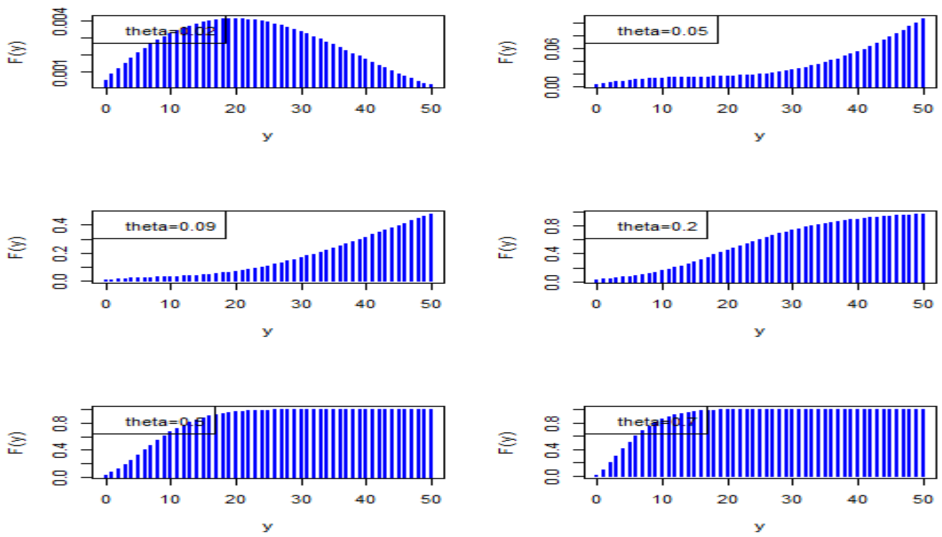
The corresponding cumulative distribution function (c.d.f) of discrete Poisson-Rani distribution is given as

$$F(Y=y)=p_r(Y\le y)=1-p_r(Y>y)=1-\sum_{w=y+1}^{\infty}P(w).$$

$$=1-\frac{\left\{\begin{array}{l} y^4\theta^4+5y^3\theta^3(\theta+1)+y^2\theta^2\left(41\theta^2+48\theta+12\right) \\ +y\theta\left(70\theta^4+94\theta^3+164\theta^2+84\theta+24\right) \\ +(\theta+1)\left(36\theta^4+117\theta^3+109\theta^2+95\theta+24\right)+24\theta^4+\theta(\theta+1)^4 \end{array}\right\}}{\left(\theta^5+24\right)(\theta+1)^{y+5}}$$

$$y=0,1,2,..;\theta>0. \qquad (2.2)$$

The following graphs depict the behaviour of pmf for different values of parameters



3 Statistical Measures of Discrete Poisson-Rani Distribution

In this section several statistical measures of the discrete Poisson-Rani distribution has been studied. Which are moments, moment generating function (Mgf) and probability generation function (Pgf).

3.1 Moments of Discrete Poisson-Rani Distribution

The r^{th} moment of the discrete Poisson-Rani distribution is denoted as μ_r' and can be obtained by

$$\mu_r' = E\left[E\left(Y^{(r)}|\lambda\right)\right], \text{ where } Y^{(r)} = Y(Y-1)(Y-2)...(Y-r+1)$$

$$= \frac{\theta^5}{\theta^5+24}\int_0^\infty \left[\sum_{y=0}^{\infty} y^{(r)} \frac{e^{-\lambda}\lambda^y}{y!}\right]\left(\theta+\lambda^4\right)e^{-\theta\lambda}\,d\lambda$$

$$= \frac{\theta^5}{\theta^5+24}\int_0^\infty \left[\lambda^r \sum_{y=r}^{\infty} \frac{e^{-\lambda}\lambda^{y-r}}{(y-r)!}\right]\left(\theta+\lambda^4\right)e^{-\theta\lambda}\,d\lambda.$$

Taking $y+r$ in place of y within the bracket, we get

$$\mu_r' = \frac{\theta^5}{\theta^5+24}\int_0^\infty \left[\lambda^r\left(\sum_{y=0}^{\infty}\frac{e^{-\lambda}\lambda^y}{y!}\right)\right]\left(\theta+\lambda^4\right)e^{-\theta\lambda}d\lambda. = \frac{\theta^5}{\theta^5+24}\int_0^\infty \lambda^r\left(\theta+\lambda^4\right)e^{-\theta\lambda}d\lambda$$

$$\mu_r' = \frac{\theta^5}{\theta^5+24}\int_0^\infty \left(\theta\lambda^r+\lambda^{r+4}\right)e^{-\theta\lambda}d\lambda = \frac{r!\left\{\theta^5+(r+4)(r+3)(r+2)(r+1)\right\}}{\theta^r\left(\theta^5+24\right)}. \qquad (3.1)$$

Substitute $r = 1,2,3,4$ in (3.1), the first four factorial moments can be obtained and using the relationship between factorial moments and moments about origin, the first four moments about origin of the PRD (2.1), are obtained as

$$\mu_1' = \frac{\theta^5+120}{\theta\left(\theta^5+24\right)}$$

$$\mu_2' = \frac{\theta^6+2\theta^5+120\theta+720}{\theta^2\left(\theta^5+24\right)}$$

$$\mu_3' = \frac{\theta^7+6\theta^6+6\theta^5+120\theta^2+2160\theta+5040}{\theta^3\left(\theta^5+24\right)}$$

$$\mu_4' = \frac{\theta^8+14\theta^7+36\theta^6+24\theta^5+120\theta^3+5040\theta^2+30240\theta+40320}{\theta^4\left(\theta^5+24\right)}.$$

The moments about mean of the Poisson-Rani distribution are obtained by using relationship between moments about mean and moments about origin

$$\mu_2 = \frac{\theta^{11}+\theta^{10}+144\theta^6+528\theta^5+2880\theta+2880}{\theta^2\left(\theta^5+24\right)^2}$$

$$\mu_3 = \frac{\left\{\begin{array}{l}\theta^{17}+3\theta^{16}+2\theta^{15}+168\theta^{12}+1656\theta^{11}+3024\theta^{10}+6286\theta^7\\ -57324\theta^6+106836\theta^5+63120\theta^2+99360\theta-113760\end{array}\right\}}{\theta^3\left(\theta^5+24\right)^3}$$

$$\mu_4 = \frac{\left\{\begin{array}{l} \theta^{23} + 10\theta^{22} + 21\theta^{21} + 33\theta^{20} - 24\theta^{19} + 192\theta^{18} + 4896\theta^{17} - 13824\theta^{16} \\ + 42048\theta^{15} + 10368\theta^{13} + 281088\theta^{12} + 949644\theta^{11} + 528768\theta^{10} \\ + 13824\theta^{8} + 5577696\theta^{7} + 12939264\theta^{6} + 1114496\theta^{5} + 165880\theta^{3} \\ + 36495360\theta^{2} + 69672960\theta - 748984320 \end{array}\right\}}{\theta^4\left(\theta^5 + 24\right)^4}$$

The standard deviation (S.D), coefficient of variation (c.v), coefficient of skewness $\left(\sqrt{\beta_1}\right)$, coefficient of kurtosis $\left(\beta_2\right)$, index of dispersion $\left(\gamma\right)$ of Poisson-Rani distribution are determined as

$$\sigma = \frac{\sqrt{\theta^{11} + \theta^{10} + 144\theta^6 + 528\theta^5 + 2880\theta + 2880}}{\theta\left(\theta^5 + 24\right)}$$

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\theta^{11} + \theta^{10} + 144\theta^6 + 528\theta^5 + 2880\theta + 2880}}{\left(\theta^5 + 120\right)}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\left(\mu_2\right)^{\frac{3}{2}}} = \frac{\left\{\begin{array}{l} \theta^{17} + 3\theta^{16} + 2\theta^{15} + 168\theta^{12} + 1656\theta^{11} + 3024\theta^{10} + 6286\theta^{7} \\ - 57324\theta^{6} + 106836\theta^{5} + 63120\theta^{2} + 99360\theta - 113760 \end{array}\right\}}{\theta^{11} + \theta^{10} + 144\theta^6 + 528\theta^5 + 2880\theta + 2880}$$

$$\beta_2 = \frac{\mu_4}{\left(\mu_2\right)} = \frac{\left\{\begin{array}{l} \theta^{23} + 10\theta^{22} + 21\theta^{21} + 33\theta^{20} - 24\theta^{19} + 192\theta^{18} + 4896\theta^{17} - 13824\theta^{16} \\ + 42048\theta^{15} + 10368\theta^{13} + 281088\theta^{12} + 949644\theta^{11} + 528768\theta^{10} \\ + 13824\theta^{8} + 5577696\theta^{7} + 12939264\theta^{6} + 1114496\theta^{5} + 165880\theta^{3} \\ + 36495360\theta^{2} + 69672960\theta - 748984320 \end{array}\right\}}{\left(\theta^{11} + \theta^{10} + 144\theta^6 + 528\theta^5 + 2880\theta + 2880\right)^2}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{\left(\theta^{11} + \theta^{10} + 144\theta^6 + 528\theta^5 + 2880\theta + 2880\right)}{\theta\left(\theta^5 + 24\right)\left(\theta^5 + 120\right)}.$$

3.2 Generating Functions (Pgf, Mgf, Cgf) of Discrete Poisson-Rani Distribution

We obtain (Pgf,Mgf,Cgf) through following theorems.

Theorem 3.2.1 If $Y \sim PRD(\theta)$ then the probability generating function $P_Y(t)$ is

$$p_Y(t)=\frac{\theta^5}{\left(\theta^5+24\right)\left(\theta+1\right)^4}\left[\begin{array}{l}\frac{\left\{\theta(\theta+1)^4+24(\theta+1)\right\}t}{(\theta+1)(\theta+1-t)}+\frac{24t^4}{(\theta+1-t)^5}+\frac{73t^3}{(\theta+1-t)^4}\\+\frac{144t^2}{(\theta+1-t)^3}+\frac{96t}{(\theta+1-t)^2}\end{array}\right]$$

Proof The probability generating function (P.g.f) of discrete Poisson-Rani distribution is defined as

$$P_Y(t)=E(t)=\sum_{y=0}^{\infty}t^yP(y)$$

$$P_Y(t)=\frac{\theta^5}{\left(\theta^5+24\right)\left(\theta+1\right)^5}\left[\sum_{y=0}^{\infty}t^y\frac{y^4+10y^3+35y^2+50y+24+\theta(\theta+1)^4}{(\theta+1)^y}\right]$$

$$P_Y(t)=\frac{\theta^5}{\left(\theta^5+24\right)\left(\theta+1\right)^5}\left[\begin{array}{l}\sum_{y=0}^{\infty}y^4\left(\frac{t}{\theta+1}\right)^y+10\sum_{y=0}^{\infty}y^3\left(\frac{t}{\theta+1}\right)^y+35\sum_{y=0}^{\infty}y^2\left(\frac{t}{\theta+1}\right)^y\\+50\sum_{y=0}^{\infty}y\left(\frac{t}{\theta+1}\right)^y+\left\{24+\theta(\theta+1)^4\right\}\sum_{y=0}^{\infty}\left(\frac{t}{\theta+1}\right)^y\end{array}\right]$$

$$=\frac{\theta^5}{\left(\theta^5+24\right)\left(\theta+1\right)^5}\left[\begin{array}{l}\frac{\left\{\theta(\theta+1)^4+24(\theta+1)\right\}t}{(\theta+1)(\theta+1-t)}+\frac{t^4+11t^3(\theta+1)+11t^2(\theta+1)^2+t(\theta+1)}{(\theta+1-t)^5}\\10\frac{t^3+4t^2(\theta+1)+t(\theta+1)^2}{(\theta+1-t)^4}+35\frac{t^2+t(\theta+1)}{(\theta+1-t)^3}+50\frac{t}{(\theta+1-t)^2}\end{array}\right]$$

$$p_Y(t)=\frac{\theta^5}{\left(\theta^5+24\right)\left(\theta+1\right)^4}\left[\begin{array}{l}\frac{\left\{\theta(\theta+1)^4+24(\theta+1)\right\}t}{(\theta+1)(\theta+1-t)}+\frac{24t^4}{(\theta+1-t)^5}+\frac{73t^3}{(\theta+1-t)^4}\\+\frac{144t^2}{(\theta+1-t)^3}+\frac{96t}{(\theta+1-t)^2}\end{array}\right]$$

Theorem 3.2.2 If $Y\sim PRD(\theta)$, then the moment generating function $M_Y(t)$ is

$$M_Y(t)=\frac{\theta^5}{\left(\theta^5+24\right)\left(\theta+1\right)^4}\left[\begin{array}{l}\frac{\left\{\theta\left(\theta+1\right)^4+24\left(\theta+1\right)\right\}e^t}{\left(\theta+1\right)\left(\theta+1-e^t\right)}+\frac{24e^{4t}}{\left(\theta+1-e^t\right)^5}+\frac{73e^{3t}}{\left(\theta+1-e^t\right)^4}\\+\frac{144e^{2t}}{\left(\theta+1-e^t\right)^3}+\frac{96e^t}{\left(\theta+1-e^t\right)^2}\end{array}\right]$$

Proof Since the moment generating function is a generalization of probability

generating function with relationship given by

$$M_Y(t)=P_Y\left(e^t\right)$$

So that

$$M_Y(t)=\frac{\theta^5}{\left(\theta^5+24\right)\left(\theta+1\right)^4}\left[\begin{array}{l}\frac{\left\{\theta\left(\theta+1\right)^4+24\left(\theta+1\right)\right\}e^t}{\left(\theta+1\right)\left(\theta+1-e^t\right)}+\frac{24e^{4t}}{\left(\theta+1-e^t\right)^5}+\frac{73e^{3t}}{\left(\theta+1-e^t\right)^4}\\+\frac{144e^{2t}}{\left(\theta+1-e^t\right)^3}+\frac{96e^t}{\left(\theta+1-e^t\right)^2}\end{array}\right]$$

Similarily the relationship between M.g.f and C.g.f is given by

$$M_Y(t)=\phi_Y(it)$$

$$M_Y(t)=\frac{\theta^5}{\left(\theta^5+24\right)\left(\theta+1\right)^4}\left[\begin{array}{l}\frac{\left\{\theta\left(\theta+1\right)^4+24\left(\theta+1\right)\right\}e^{it}}{\left(\theta+1\right)\left(\theta+1-e^{it}\right)}+\frac{24e^{4it}}{\left(\theta+1-e^{it}\right)^5}+\frac{73e^{3it}}{\left(\theta+1-e^{it}\right)^4}\\+\frac{144e^{2it}}{\left(\theta+1-e^{it}\right)^3}+\frac{96e^{it}}{\left(\theta+1-e^{it}\right)^2}\end{array}\right]$$

4 Order Statistics of Discrete Poisson-Rani Distribution

Consider $Y_1, Y_2, \ldots Y_n$ be random samples from discrete Poisson-Rani distribution. Let $Y_{(1)} < Y_{(2)} < \ldots < Y_{(n)}$ be the corresponding order statistics. Then the p.d.f of the r^{th} order statistics of the Poisson-Rani distribution, say $X = Y_{(r)}$

$$f_X(x)=\frac{n!}{(r-1)!(n-r)!}F^{r-1}(x)\left[1-F(x)\right]^{n-r}f(x).$$

$$= \frac{n!}{(r-1)!(n-r)!}\sum_{k=0}^{\infty}\binom{n-r}{k}(-1)^k F^{r+k-1}(x) f(x)$$

$$= \frac{n!\left\{\theta^5\left(y^4+10y^3+35y^2+50y+24+\theta(\theta+1)^4\right)\right\}}{(r-1)!(n-r)!\left(\theta^5+24\right)(\theta+1)^{y+5}}$$

$$\sum_{k=0}^{\infty}\binom{n-r}{k}(-1)^k\left[1-\frac{\left\{\begin{array}{l} y^4\theta^4+5y^3\theta^3(\theta+1)+y^2\theta^2\left(41\theta^2+48\theta+12\right) \\ +y\theta\left(70\theta^4+94\theta^3+164\theta^2+84\theta+24\right) \\ +(\theta+1)\left(36\theta^4+117\theta^3+109\theta^2+95\theta+24\right)+24\theta^4+\theta(\theta+1)^4 \end{array}\right\}}{\left(\theta^5+24\right)(\theta+1)^{y+5}}\right]^{r+k-1}$$

The corresponding c.d.f of X is given by

$$F_X(x) = \sum_{j=r}^{n} F^j(x)\left[1-F(x)\right]^{n-j}$$

$$F_X(x) = \sum_{j=r}^{n}\sum_{k=0}^{n-j}\binom{n}{j}\binom{n-j}{k}(-1)^k F^{j+k}(x)$$

$$= \sum_{j=r}^{n}\sum_{k=0}^{n-j}\binom{n}{j}\binom{n-j}{k}(-1)^k\left[1-\frac{\left\{\begin{array}{l} y^4\theta^4+5y^3\theta^3(\theta+1)+y^2\theta^2\left(41\theta^2+48\theta+12\right) \\ +y\theta\left(70\theta^4+94\theta^3+164\theta^2+84\theta+24\right) \\ +(\theta+1)\left(36\theta^4+117\theta^3+109\theta^2+95\theta+24\right)+24\theta^4+\theta(\theta+1)^4 \end{array}\right\}}{\left(\theta^5+24\right)(\theta+1)^{y+5}}\right]^{j+k}$$

5 Recurrence Relation of Discrete Poisson-Rani Distribution

If $Y \sim PRD(\theta)$ then probability mass function of Y is

$$P(Y=y) = \frac{\theta^5}{\left(\theta^5+24\right)}\left[\frac{y^4+10y^3+35y^2+50y+24+\theta(\theta+1)^4}{(\theta+1)^{y+5}}\right] \; ; y = 0,1,2,\ldots;\theta > 0.$$

The recurrence relation of discrete Poisson-Rani distribution is given by

$$\frac{P(y+1)}{P(y)}=\frac{\left\{(y+1)^4+10(y+1)^3+35(y+1)^2+50(y+1)+24+\theta(\theta+1)^4\right\}}{\left\{y^4+10y^3+35y^2+50y+24+\theta(\theta+1)^4\right\}}\left\{\frac{(\theta+1)^{y+5}}{(\theta+1)^{y+6}}\right\}$$

$$P(y+1)=\frac{\left\{(y+1)^4+10(y+1)^3+35(y+1)^2+50(y+1)+24+\theta(\theta+1)^4\right\}}{\left\{y^4+10y^3+35y^2+50y+24+\theta(\theta+1)^4\right\}}\frac{1}{(\theta+1)}P(y)$$

This represents the recurrence relation

6 Reliability Measures of Discrete Poisson-Rani Distribution

Suppose Y be a continuous random variable with c.d.f $F(y); y>0$. Then its reliability function which is also called survival function is defined as

$$S(y)=p_r(Y>y)=\int_y^{\infty} f(y)dy=1-F(y).$$

The survival function of discrete Poisson-Rani distribution is given as

$$R(y,\theta)=S(y,\theta)=\frac{\left\{\begin{array}{l}y^4\theta^4+5y^3\theta^3(\theta+1)+y^2\theta^2\left(41\theta^2+48\theta+12\right)\\+y\theta\left(70\theta^4+94\theta^3+164\theta^2+84\theta+24\right)\\+(\theta+1)\left(36\theta^4+117\theta^3+109\theta^2+95\theta+24\right)+24\theta^4+\theta(\theta+1)^4\end{array}\right\}}{\left(\theta^5+24\right)(\theta+1)^{y+5}} \quad (6.1)$$

The hazard rate function of the random variable y is given as

$$H(y,\theta)=\frac{f(y,\theta)}{S(y,\theta)}. \quad (6.2)$$

Substituting (2.2) and (6.1), in to (6.2), we get

$$H(y,\theta)=\frac{\theta^5\left\{y^4+10y^3+35y^2+50y+24+\theta(\theta+1)^4\right\}}{\left\{\begin{array}{l}y^4\theta^4+5y^3\theta^3(\theta+1)+y^2\theta^2\left(41\theta^2+48\theta+12\right)\\+y\theta\left(70\theta^4+94\theta^3+164\theta^2+84\theta+24\right)\\+(\theta+1)\left(36\theta^4+117\theta^3+109\theta^2+95\theta+24\right)+24\theta^4+\theta(\theta+1)^4\end{array}\right\}}$$

The reverse hazard rate function denoted as h_r is given by

$$h_r(y,\theta)=\frac{f(y,\theta)}{F(y,\theta)}. \qquad (6.3)$$

Substituting equations(2.1),(2.2) into equation(6.3),we get

$$h_r(y,\theta)=\frac{\theta^5\left\{y^4+10y^3+35y^2+50y+24+\theta(\theta+1)^4\right\}}{\left(\theta^5+24\right)(\theta+1)^{y+5}-\left\{\begin{array}{l}y^4\theta^4+5y^3\theta^3(\theta+1)+y^2\theta^2\left(41\theta^2+48\theta+12\right)\\+y\theta\left(70\theta^4+94\theta^3+164\theta^2+84\theta+24\right)\\+(\theta+1)\left(36\theta^4+117\theta^3+109\theta^2+95\theta+24\right)+24\theta^4+\theta(\theta+1)^4\end{array}\right\}}$$

7 Maximum Likelihood Estimation of Discrete Poisson-Rani Distribution

Let $Y_1,Y_2,..,Y_n$ be a random sample of size n from discrete Poisson-Rani distribution then its likelihood function is given by

$$l=\left(\frac{\theta^5}{\theta^5+24}\right)^n\prod_{i=1}^{n}\frac{\left\{y_i^4+10y_i^3+35y_i^2+50y_i+24+\theta(\theta+1)^4\right\}}{(\theta+1)^{y_i+5n}}.$$

The log likelihood function is obtained as

$$\log l=5n\log\theta-n\log\left(\theta^5+24\right)+\sum_{i=1}^{n}\log\left(y_i^4+10y_i^3+35y_i^2+50y_i+24+\theta(\theta+1)^4\right)$$

$$-\sum_{i=1}^{n}\log(\theta+1)^{y_i+5n}.$$

Differentiate w.r.t to θ, we get

$$\frac{\partial \log l}{\partial \theta}=\frac{5n}{\theta}-\frac{5n\theta^4}{\left(\theta^5+24\right)}+\sum_{i=1}^{n}\frac{(5\theta+1)(\theta+1)^3}{y_i^4+10y_i^3+35y_i^2+50y_i+24+\theta(\theta+1)^4}-\sum_{i=1}^{n}\frac{(y_i+5n)}{(\theta+1)} \tag{7.1}$$

Substituting $\frac{\partial \log l}{\partial \theta}=0$ and solving the above non-linear equation, we get the required $\hat{\theta}_{mle}$.

8 Applications

In this section the goodness of fit of Poisson-Rani distribution (PRD) has been proposed for two real count data sets. And we show that established distribution perform better than Poisson and Poisson Lindley distribution. The first data set is regarding the distribution of Pyrausta nublilalis in 1937 taken from Das et al. [11]. Data set second taken from Shanker and Shukla [12] is about the number of European red mites on apple leaves.

In order to compare the above distribution models, we consider the criteria like AIC (Akaike information criterion), AICC (corrected Akaike information criterion), BIC (Bayesian information criterion). Among the above distributions, the better distribution is considered having lesser values of AIC, AICC, and BIC

Table 1: Distribution of Pyrausta nublilalis in 1937

No. of insects	observed frequency	Expected frequency		
		PRD	PD	PLD
0	33	33.3648741	26.45252695	31.5174669
1	12	12.2033741	19.83939522	14.1511265
2	6	5.3702605	7.43977321	6.0793090
3	3	2.6199305	1.85994330	2.5343370
4	1	1.2993961	0.34873937	1.0337411
5	1	0.6274065	0.05231091	0.4147467
Total	56	55.48524	55.99269	55.73073

ML estimates	2.445769	0.7500	1.811538
(Standard error)	(0.1883897)	(0.1157273)	(0.3067991)
$-2\log L$	133.7622	143.1647	133.9641
AIC	135.7622	145.1647	135.9641
AICC	135.8363	145.2388	136.0382
BIC	137.7876	147.1901	137.9895
χ^2	0.426615	24.08072	1.310285
d.f	2	2	2
p-value	0.807907489	5.90117e-06	0.519367947

Table 2: Distribution of European red mites on apple leaves.

	observed frequency	Expected frequency		
		PRD	PD	PLD
0	70	72.7976261	47.654082064	67.261105
1	38	33.4358912	54.643363318	38.887660
2	17	18.8123855	31.328870743	21.244459
3	10	11.2267490	11.974594076	11.186467
4	9	6.5188205	3.432717966	5.7400350
5	3	3.5900868	0.787236882	2.8894662
6	2	1.8736771	0.150449759	1.4332055
7	1	0.9322884	0.024645111	0.7025948
8	0	0.4453279	0.003532467	0.3411585
Total	150	149.6329	149.9995	149.6862
ML estimates		2.114577	1.146667	1.26016
(Standard error)		(0.08124686)	(0.08743248)	(0.1139965)
$-2\log L$		444.2586	485.6199	445.0218
AIC		446.2586	487.6199	447.0218

AICC	446.2856	487.6469	447.0488
BIC	449.2692	490.6305	450.0324
χ^2	2.539315	99.01727	3.652513
d.f	5	3	5
p-value	0.770562857	2.52807E-21	0.600446312

9 Discussions and Conclusion

In the present paper a new life time distribution has been established "The Discrete Poisson-Rani distribution" by compounding technique. Its several properties including moments, coefficient of variation, skewness, kurtosis, index of dispersion, moment generating function, survival function, hazard rate function, reverse hazard function and order statistics have been discussed. The parameters of the distribution have been estimated by known method of maximum likelihood estimator. Finally, the performance of the model has been examined through two data sets and compared with Poisson distribution and Poisson-Lindley distribution. As it is evident from table 1 and 2, that the discrete Poisson-Rani distribution have the lesser AIC, AICC, $-2\log L$ and BIC values as compared to Poisson and Poisson- Lindley distributions. Hence, we can conclude that Poisson-Rani distribution provides better fit than compared ones. The formulated distribution can be utilized for different discipline of science to analyse count data.

10 Open Problem

The following probability mass function (pmf) represents Poisson Size-biased Rani distribution.

$$P(Y=y)=\frac{\theta^6(y+1)}{(\theta^5+120)}\left[\frac{y^5+20y^4+113y^3+328y^2+1044y+720+\theta(\theta+1)^5}{(\theta+1)^{y+7}}\right];$$
$$y=0,1,2..\theta>0$$

Show that Poisson Size-Biased Rani distribution performs better than Poisson-Rani distribution while analysing count data.

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