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Nearly Metacompact in Bitopological Space

JAMAL OUDETALLAH

Department of Mathematics, Irbid National University
P.O.Box 2600 Irbid, Jordan
e-mail:dr_jamal@inu.edu.jo
jamalayasrah12@gmail.com

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Abstract

This study describes nearly metacompact spaces, pairwise nearly metacompact spaces, and looks not only into their properties but also their associations with other bitopological spaces. A number of examples were considered and there was a generalization of several well-known theorems with reference to nearly metacompact spaces.

Keywords: *Nearly metacompact, pairwise nearly metacompact, pairwise compact space, pairwise metacompact, bitopological, radial space.*

2010 Mathematics Subject Classification: 54A05.

1 Introduction

A bitopological space (Y, y_1, y_2) is a non-empty set Y with equipped with two arbitrary topologies y_1, y_2. The concept of bitopological space was brought about by topologies generalized by the sets

$$B_{\rho_\epsilon} = \{b \in Y \mid g(a,b) \leq \epsilon\}, \tag{1}$$

and

$$B_{\delta_\epsilon} = \{b \in Y \mid z(a,b) \leq \epsilon\}, \tag{2}$$

On the introduction of the bitopological space model by Kellay (1963), a number of topological properties in single topology are generalized into bitopological spaces, such as compactness, paracompactness, separation axioms, connectedness, types of functions, and other topics. Bitopological spaces have gained considerable attention from numerous authors after the concept had been put forward by Kellay (1963), see [4,5,6]. The aim of this work presents some properties of pairwise perfect and pairwise countably perfect mappings and uses them to obtain finite product theorems concerning pairwise expandable and almost pairwise expandable space. If (Y, y) is a topological space and $B \subseteq Y$ with $CL(B)$ and $Int(B)$ depicting the closure and interior of B respectively. If B is a subset of a bitopological space $Y = (Y, y_1, y_2)$, then the relative topology (subspace topology) on the set B inherited by y will be denoted by y_B. The cardinality of a set Δ will be denoted by $|\Delta|, Z, R, N, Q$ represent the sets of integers, real numbers, natural numbers, and rational numbers respectively; ω_0, ω_1 denote the first uncountable ordinal respectively; and m denotes an infinite cardinal in general. And the usual, the cofinite, the discrete, and co-countable topology will be denoted by $y_u, y_{cof}, y_{dis}, y_{coc}$ respectively.

2 Problem Formulations

Several definitions and preliminaries associated with the topological and ebitopological space for completeness, and are stated as follows:

Definition 2.1 *[1] A topological space (Y, y) is described as meta-Lindelöf if each open cover $\tilde{R}$ of Y has a finite subcover of $\tilde{R}$ that covers Y. Given the above open cover $\tilde{R}$, another open cover $\tilde{S}$ (which is a refinement of $\tilde{R}$) such that $Sa = s \in S, a \in s$ is finite (countable) for all $a \in D$. The set $D \subseteq Y$ is said to be dense.*

Definition 2.2 *If $\tilde{R}$ is a pairwise open cover of Y then the bitopological space (Y, y_1, y_2) is called pairwise nearly meta-compact spaces and as such, there exist a_m^y-dense set $D \subseteq Y$ and an y_n-refinement S of $\tilde{R}$ such that $S_a = \{s \in S, a \in S\}$ is finite (countable) for all $a \in D$ and $m \neq n\,(m, n = 1, 2)$.*

Definition 2.3 *[5] The cover $\tilde{R}$ of a bitopological space (Y, y_1, y_2) is said to be open y_1y_2 if $\tilde{R} \subset y_1 \cup y_2$, and also if $\tilde{R}$ contains no less than one non-empty element of y_1 and no less than one non-empty element of y_2, it is referred to as P—open.*

Definition 2.4 *[7] A pairwise open cover is known as pairwise point finite in a bitopological space (Y, y_1, y_2) assuming for every $a \in Y$ there is a finite number of y_1-open members of $\tilde{R}$, or it contained in a finite number of y_2-open members of $\tilde{R}$.*

Definition 2.5 *[3] A bitopological space (Y, y_1, y_2) is said to be pairwise compact (P-compact) if all P open cover of the space (Y, y_1, y_2) has a finite subcover.*

Definition 2.6 *[2] A bitopological space (Y, y_1, y_2) is pairwise $T_2(P - T_2)$ space if for each pair of distinct points $a, b \in Y$, there is a y_1-open set R containing a but not b and a $y_2$open set S including b but not a.*

Definition 2.7 *[9] A bitopological space (Y, y_1, y_2) is pairwise $T_2(P - T_2)$ space on the condition that for every pair of distinct points $a, b \in Y$, there are two open-sets ($y_1$open set R and a $y_2$open set S disjoint from R), such that $a \in R$, $b \in S$.*

Definition 2.8 *[9] A space (Y, y_1, y_2) is described as $y_1$regular as regards τ_2 if for every point $a \in Y$ and each $y_1$closed set F such that $a \notin F$, there is $y_1$open set R and a $y_2$open set S such that $a \in R$, $F \subset S$ and $R \cap S$. A bitopological space (Y, y_1, y_2) is pairwise $T_3(P - T_3)$space, if it is P-T_1 and p-regular space.*

Definition 2.9 *[6] A bitopological space (Y, y_1, y_2) is called P-countably compact if every countable $P-$open cover of the space (Y, y_1, y_2) has a finite subcover.*

Definition 2.10 *[10] A topological space (Y, y_1, y_2) is said to be radial space pro- vided for every $a \in Y$ and $C \subseteq Y$ be y_m-subset of Y we have $a \in cl(C)$ if and only if there is a cardinal k and function $f : k \to c$ such that d converges it a $\forall_m = 1, 2$.*

Definition 2.11 *A bitopological space (Y, y_1, y_2) is said to be radial space provided for every $\{a \in Y\}$ and $C \subseteq Y$ be y_m- subset of Y we have $a \in cl(C)$ if and only if there is a cardinal k and function $f : k \to c$ such that d converges it a $\forall_m = 1, 2$.*

3 Pairwise nearly metacompac spaces

Theorem 3.1 *A space (Y, y_1, y_2) is pairwise metacompact Lindelöf, provided that each directed pairwise open cover $\tilde{R}$ has a pairwise open refinement, which is point finite countable on a number of dense subsets of Y.*

proof 1 *Assuming $\tilde{R}$ is a pairwise open covering of Y and $RF = \cup R'$ is a countable subset of $\tilde{R}$. Evidently, RF is directed pairwise open cover of Y; such that $\tilde{E}$ is a pairwise refinement of RF (that is to say another open cover of Y) which is point finite on the pairwise dense set C. For every $E \in \tilde{E}$, let $H(E)$ be a finite subset of R such that $E \subseteq \cup H(E)$ and $W(E) = \{E \cap R, R \in H(E)\}$, since $E = \cup W(E)$ for all $E \in \tilde{E}$, then the collection $W = \cup\{W(E), E \in \tilde{E}\}$ is a pairwise open refinement of R. suppose that $a \in C$ and $(\tilde{E})n = \{E1, \cdots, En\}$. Then $(W)_x \subseteq W(E_1) \cup \cdots W(E_n)$) and is hence finite. For this reason, W is a pairwise open refinement of R, a pairwise point finite one. This is the same as the meta-Lindelöf version.*

Definition 3.1 *A space (Y, y_1, y_2) can be described as pairwise absolutely countably compact, on the condition that for each pairwise open cover $\tilde{R}$ of Y and each dense subset $D \subset Y$, the cover $\{(a, \tilde{R}), a \in D\}$ has a finite pairwise subcover.*

Remark 3.2 *All pairwise compact (T_2) spaces are countably compact*

Theorem 3.3 *If (Y, y_1, y_2) is a pairwise nearly meta-Lindelöf, then pairwise absolutely countably compact (T_2)spaces is said to be compact.*

Theorem 3.4 *A theory to showcase that both nearly meta-Lindelöf T_3- Space and pairwise countably compact are pairwise compact.*

proof 2 *Open cover R is assumed is as pairwise nearly meta-Lindelöf T_3- space Y. Let $\tilde{S}$ be a pairwise open refinement of R, which is a pairwise point countable on some dense set D. Considering that Y is pairwise T_3, supposing for every $s \in \tilde{S}$ there is a $R(S) \in \tilde{S}$ given $\tilde{S} \subseteq \cup R(S)$. The result indicates that D has a sub-collection of a pairwise R over it. However, if D has no sub-collection of a pairwise R over it and as such by induction, for every m is greater than ω, the point as indicated can be picked $d_m \in D$ and list $Sd_m = \{S_{m,n} : n\, is\, less\, than\, \omega\}$, in that for every $x < \omega d_{x+1} \in D/\cup\{S_{m,n} : k, n \leq n\}$. Considering $d_x t$ are distinct and Y is pairwise countably compact, then the set $\{d_x : x < \omega\}$ has a complete accumulation point $x \in Y$. Let $s \in \tilde{S}$, such that $x \in S$ and note that $A = \{x < w : d_x \in S\}$ is infinite. Then, $x \in E$ and n is less than W in that $S = x, n$, thus, $k \in E$ in that k is greater than $max(x, nj)$ and considering that $dk \notin S_x n = S$ represent a contradiction. Let $\tilde{S}'$ be a finite pairwise sub-collection of $\tilde{S}$ that covers D. Considering that $Y = D$ is covered by $\{\tilde{S} : S \in \tilde{S}'\}$, $\{R(S) : S \in \tilde{S}'\}$ is a finite pairwise sub-cover of $\tilde{S}$.*

Theorem 3.5 *If Y is meta-Lindelöf then Y is a pairwise nearly metacompact radial space.*

proof 3 *Given $\tilde{S}$ is a pairwise open covering of Y and $\tilde{S}$ a pairwise open refinement point-finite on a number of the dense set (D). It should be noted that in a situation where q is in the closure of a finite $E \subseteq D$, in that case, $\tilde{S}q$ is pairwise finite given that $\tilde{S}q = SSe : e \in E$ and this is a countable set in view of the fact that $\tilde{S}$ is a point-finite on D. Let's assume that $q \in Y \setminus D$, so $\tilde{S}q$ is infinite. Given that Y is pairwise radial then a cardinal () and a function $(d : k \to D)$ is present, so that d converges to q. For every $m < \omega$, we make use of a distinct $S_i \in \tilde{S}_x$ and $y_m < k$, so that $\{d(\alpha) : y_m \leq \alpha < k\} \subseteq S_m$. If $\lambda = sup\{y_m : m < \omega\} < k$, subsequently $d(\lambda) \in \cap_{n<\omega} S_m$ is inconsistent with the hypothesis (on $\tilde{S}$). Therefore, $k = sup\{y_m : m < \omega\}$ and as a result x is in the closure of the countable set $\{d(y_m) : m < \omega\} \subseteq D$. Consequently, by the observation above, $\tilde{S}q$ is pairwise finite, and therefore $\tilde{S}$ is pairwise point-finite.*

Theorem 3.6 *A closed subspace is a root of each bitopological space* (Y, y_1, y_2) *as indicated in a nearly metacompact space* Y.

proof 4 *Given that all pairwise finite* T_1 *Space is pairwise metacompact, assuming* Y *as pairwise* T_1 *Space as well as* $|Y| = k > \omega_0$. *Order* Y. *Supposing* $Y = \{y\alpha :$*is greater than* $k\}$. *Therefore* $\tilde{E} = \{Ea : a < k\}$ *be a non-empty subset of* k *pairwise collection with the countable intersection characteristic, So that for every infinite* $I \subseteq k$, $T\{Ea : a\, is\, an\, element\, of\, I\} = \phi$. *For every* $a < k$ *let* $\tilde{F}a$ *be a local base at* ya *and let* $\tilde{E}(a = \{e \in \tilde{E} : E \subseteq Ea\})$. *Let* $Z = Y \cup (Y \times k)$ *and define* $ay_1 = y_2$ *on* Z *along these lines:*
(1) The points of $Y \times k$ *are isolated.*
(2) For each $\alpha < k$, *a local base is formed by the set* $F \cup (F \times E) : F \in \tilde{F}a$ *and* $E \in \tilde{E}\alpha$ *at the point* $q\alpha$ *in* Z.
Given that $\phi \notin \tilde{E}$, *the set* $Y \times k$ *is simply seen to be dense in* Z. *Assuming* $\tilde{R}$ *is a pairwise open covering of* Z. *When* α *is greater than then* $F(\alpha) \in \tilde{F}\alpha$. *Let* $E(\alpha) \in \tilde{E}\alpha$, *consequently* $S(\alpha) = F(\alpha) \cup (F(\alpha) \times E(\alpha)) \subseteq R$ *specific to* $R \in \tilde{R}$. $\tilde{S} = \{S)(\alpha) : \alpha < k\} \cup \{\{y\} : y \in Z \setminus \cup \{S(\alpha) : a < k\}\}$. *Obviously, is a pairwise open refinement of . Assume* $\alpha, \lambda < k$. *Therefore* $I = \mu < k : (x_\alpha, \lambda) \in S(\mu)$. *In the case where* $\lambda \in E(\mu) \subseteq E\mu$, *this would only have occurred if* $\mu \in I$, *consequently,* $\lambda \in \cap\{E\mu : \mu \in I\}$ *and for that reason,* I *without a doubt is countable/finite. Therefore, for everyone* $\alpha, \beta < k$, $(y\alpha, \beta)$ *is the only countably numerous elements of* $\tilde{S}$. *Thus,* $\tilde{S}$ *is pairwise point-finite on the set* Y *(which is dense).*

4 Open Problem

From the perspective of the presented theories, one might further investigate and explore certain conditions that can guarantee the pairwise nearly metacompact space to become pairwise metacompact space.

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