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Notes On Some Equations of Prime Rings With Symmetric Bi-(α, α)-Derivations

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Abstract

Let R be a prime ring, I a nonzero ideal of R, α an automorphism and $D : R \times R \to R$ a symmetric bi-(α, α)-derivation. In the present paper, we shall prove that R satisfying some idendities involving symmetric bi-(α, α)-derivation on ideals must be commutative ring.

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1 Introduction

Let R be an associative ring with center Z. A ring R is said to be prime if $xRy = (0)$ implies that either $x = 0$ or $y = 0$, where $x, y \in R$. For any $x, y \in R$, the symbol $[x, y]$ stands for the commutator $xy - yx$ and the symbol $x \circ y$ stands for the commutator $xy + yx$. For any $x, y \in R$ and α a mapping from R into itself, we write $[x, y]_{\alpha,\alpha}$ and $(x \circ y)_{\alpha,\alpha}$, for $x\alpha(y) - \alpha(y)x$ and $x\alpha(y) + \alpha(y)x$ respectively. We set $C_{\alpha,\alpha}=\{c \in R \mid c\alpha(x) = \alpha(x)c$ for all $x \in R\}$ and call (α, α)-center of R. An additive mapping $d : R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$.

The study of centralizing and commuting mappings on prime rings was initiated by the result of Posner [11] which states that the existence of a nonzero centralizing derivation on a prime ring implies that the ring has to be commutative. Through the years, many authors have proved commutativity theorems for prime rings admitting various types of additive maps like automurphisms, derivations, bi-derivations. A mapping $D(.,.) : R \times R \to R$ is said to be symmetric if $D(x,y) = D(y,x)$ for all $x, y \in R$. A mapping $d : R \to R$ is called the trace of $D(.,.)$ if $d(x) = D(x,x)$ for all $x \in R$. It is obvious that if $D(.,.)$ is bi-additive (i.e., additive in both arguments), then the trace d of $D(.,.)$ satisfies the identity $d(x+y) = d(x) + d(y) + 2D(x,y)$, for all $x, y \in R$. If $D(.,.)$ is bi-additive and satisfies the identities

$$D(xy,z) = D(x,z)y + xD(y,z)$$

and

$$D(x,yz) = D(x,y)z + yD(x,z),$$

for all $x, y, z \in R$. Then $D(.,.)$ is called a symmetric bi-derivation.

The concept of bi-derivation was introduced by Maksa in [6]. It is shown in [7] that symmetric bi-derivations are related to general solution of some functional equations. Some results concerning symmetric bi-derivations in prime rings can found in [9], [1], [8], [2] and [10]. Typical examples are mappings of the form $(x,y) \longmapsto \lambda[x,y]$ where $\lambda \in C$, the extended centroid of R. We shall call such maps inner bi-derivations.

Inspired by the definition symmetric bi-derivation, we introduce the notion of symmetric bi-(α,α)-derivation as follow:

Let α be an any automorphism of R. A bi-additive mapping $D(.,.) : R \times R \to R$ is said to be bi-(α,α)-derivation if it satisfies the identities

$$D(xy,z) = D(x,z)\alpha(y) + \alpha(x)D(y,z)$$

and

$$D(x,yz) = D(x,y)\alpha(z) + \alpha(y)D(x,z),$$

for all $x, y, z \in R$. Of course a symmetric bi-$(1,1)$-derivation where 1 is the identity map on R is symmetric bi-derivation.

In this paper, we extend the results in [4] concerning of ideals in prime rings with bi-(α,α)-derivation. Also we gave the true proof in [4, Theorem 2.4]. We shall make use of the following basic identities without any specific mention:

i) $[x,yz] = y[x,z] + [x,y]z$
ii) $[xy,z] = [x,z]y + x[y,z]$
iii) $xyoz = (xoz)y + x[y,z] = x(yoz) - [x,z]y$
iv) $xoyz = y(xoz) + [x,y]z = (xoy)z - y[z,x]$
v) $[xy,z]_{\alpha,\alpha} = x\,[y,z]_{\alpha,\alpha} + [x,\alpha(z)]\,y = x[y,\alpha(z)] + [x,z]_{\alpha,\alpha}\,y$
vi) $[x,yz]_{\alpha,\alpha} = \alpha(y)\,[x,z]_{\alpha,\alpha} + [x,y]_{\alpha,\alpha}\,\alpha(z)$
vii) $(xz \circ y)_{\alpha,\alpha} = x(z \circ y)_{\alpha,\alpha} - [x,\alpha(y)]z.$

2 Results

Lemma 2.1 *[5, Theorem 4] Let* R *be a prime ring with* $charR \neq 2$, d_1 *and* d_2 *are nonzero derivations of* R. *If* $d_1d_2(R) \subseteq Z$, *then* R *is commutative ring.*

Lemma 2.2 *[3, Lemma 2.3] Let* R *be a prime ring and* I *be a nonzero left ideal of* R. *Then the following conditions are equiavalents:*
i) $[I, I] = (0)$
ii) $I \subseteq Z(R)$
iii) R *is commutative.*

Lemma 2.3 *Let* R *be a prime ring,* I *a nonzero ideal of* R *and* $a, b \in R$. *If* $aIb = (0)$, *then* $a = 0$ *or* $b = 0$.

Proof 2.4 *We get*

$$axb = 0, \text{ for all } x \in I.$$

Replacing x *by* xr, $r \in R$ *in this equation, we have*

$$axrb = 0, \text{ for all } x \in I, \; r \in R.$$

That is $axRb = (0)$. *Since* R *is prime ring, we have* $ax = 0$ *or* $b = 0$. *In the former case, we get* $ax = 0$, *for all* $x \in I$. *Replacing* x *by* $rx, r \in R$ *in last equation, we have* $aRx = (0)$. *Since* I *is nonzero ideal of a prime ring* R, *we have* $a = 0$. *Hence we conclude that* $a = 0$ *or* $b = 0$.

Lemma 2.5 *Let* R *be a prime ring,* I *a nonzero ideal of* R *and* D *a nonzero symmetric bi-*(α, α)*-derivation of* R. *If* $D(x, y) = 0$ *for all* $x, y \in I$, *then* $D = 0$.

Proof 2.6 *Replacing* x *by* $xr, r \in R$ *in the hypothesis, we see that*

$$\alpha(x)D(r, y) = 0, \text{ for all } x, y \in I, r \in R.$$

This implies that

$$VD(r, y) = (0), \text{ for all } y \in I, r \in R,$$

where $\alpha(I) = V$ *is ideal of* R. *By Lemma 2.3, we have*

$$D(r, y) = 0, \text{ for all } x \in I, r \in R.$$

Writing y *by* $ys, s \in R$ *in this equation and using this, we arrive at* $D = 0$.

Lemma 2.7 *Let* R *be a prime ring and* I *a nonzero ideal of* R. *If* $[I, I] \subseteq Z$, *then* R *is a commutative ring.*

Proof 2.8 *By the hypothesis, we get*

$$[x,y] \in Z, \text{ for all } x,y \in I.$$

Replacing y *by* yx *in above expression, we have*

$$[x,y]x \in Z, \text{ for all } x,y \in I.$$

Commuting this term with r, $r \in R$, *we obtain that*

$$[[x,y]x,r] = 0, \text{ for all } x,y \in I, r \in R.$$

Using the hypothesis in the last expression, we get

$$[x,y][x,r] = 0, \text{ for all } x,y \in I, r \in R.$$

Replacing r *by* ry *in the above equation and using this expression, we see that*

$$[x,y]R[x,y] = (0), \text{ for all } x,y \in I.$$

Since R *is prime ring, we get*

$$[x,y] = 0, \text{ for all } x,y \in I.$$

That is, $[I,I] = (0)$. *By Lemma 2.2, we conclude that* R *is a commutative ring. This completes the proof.*

Lemma 2.9 *Let* R *be a prime ring and* I *a nonzero ideal of* R. *If* $I \circ I \subseteq Z$, *then* R *is a commutative ring.*

Proof 2.10 *We get*

$$x \circ y \in Z, \text{ for all } x,y \in I.$$

Replacing y *by* yx *in the last expression, we obtain that*

$$(x \circ y)\, x \in Z, \text{ for all } x,y \in I.$$

This implies that

$$[(x \circ y)\, x, r] = 0, \text{ for all } x,y \in I,$$

and so

$$(x \circ y)[x,r] = 0, \text{ for all } x,y \in I, r \in R.$$

Replacing y *by* $yt, t \in R$ *in the above expression and using this, we get*

$$[x,y]t[x,r] = 0, \text{ for all } x,y \in I, r,t \in R.$$

Replacing r *by* y *in this equation, we have*

$$[x,y]R[x,y] = (0), \text{ for all } x,y \in I.$$

Since R *is prime ring, we get*

$$[x,y] = 0, \text{ for all } x,y \in I.$$

By Lemma 2.2, we conclude that R *is a commutative ring. This completes the proof.*

Lemma 2.11 *Let R be a prime ring, I a nonzero ideal of R and α an automorphism of R. If $I \circ \alpha(I) \subseteq Z$, then R is a commutative ring.*

Proof 2.12 *We get*

$$x \circ \alpha(y) \in Z, \text{ for all } x, y \in I.$$

Replacing x by $x\alpha(y)$ in the last expression, we obtain that

$$(xo\alpha(y))\alpha(y) \in Z, \text{ for all } x, y \in I.$$

Since $x \circ \alpha(y) \in Z$ and R is prime ring, we have

$$x \circ \alpha(y) = 0 \text{ or } \alpha(y) \in Z \text{ for all } x, y \in I.$$

Let us $x \circ \alpha(y) = 0$, for all $x, y \in I$. Replacing y by $yr, r \in R$ in this equation and using this, we find that

$$\alpha(y)[x, \alpha(r)] = 0, \text{ for all } x, y \in I, r \in R.$$

Again taking y by $yt, t \in R$ in the above expression and using this, we get

$$\alpha(y)\alpha(t)[x, \alpha(r)] = 0.$$

Since α is onto, we have

$$\alpha(y)R[x, \alpha(r)] = (0).$$

Since R is prime ring, we get

$$\alpha(y) = 0 \text{ or } x \in Z.$$

Hence we conclude that $\alpha(y) \in Z$ or $x \in Z$, for all $x, y \in I$. By Lemma 2.2, we arrive at R is a commutative ring for any cases. This completes the proof.

Lemma 2.13 *Let R be a prime ring, I a nonzero ideal of R and D a nonzero symmetric bi-(α, α)-derivation of R. Then $D(x, t) \in C_{\alpha,\alpha}$ and $D(x, t) \in Z$ for all $x \in Z, t \in I$.*

Proof 2.14 *Let $x \in Z$. We know that*

$$xy = yx, \text{ for all } y \in R$$

and so

$$D(xy, t) = D(yx, t), \text{ for all } t \in I.$$

Expanding this equation and using $\alpha(x) \in Z$, we get

$$\begin{aligned} D(x,t)\alpha(y) + \alpha(x)D(y,t) &= D(y,t)\alpha(x) + \alpha(y)D(x,t) \\ D(x,t)\alpha(y) &= \alpha(y)D(x,t), \ \textit{for all } t \in I, y \in R. \end{aligned}$$

It means $D(x,t) \in C_{\alpha,\alpha}$ for all $x \in Z, t \in I$.

Since α is an automorphism of R, the last equation gives that

$$D(x,t)y = yD(x,t), \ \textit{for all } y,t \in R$$

and so $D(x,t) \in Z$ for all $x \in Z, t \in I$.

Lemma 2.15 *Let R be a prime ring, I a nonzero ideal of R and D a nonzero symmetric bi-(α,α)-derivation of R. If $D(I,I) \subseteq C_{\alpha,\alpha}$, then R is commutative ring.*

Proof 2.16 *By the hypothesis, we see that*

$$[D(x,t),r]_{\alpha,\alpha} = 0, \ \textit{for all } x,t \in I, r \in R.$$

Replacing x by xy in the last expression and using this, we have

$$D(x,t)\alpha([y,r]) + \alpha([x,r])D(y,t) = 0, \ \textit{for all } x,t \in I, r \in R.$$

Taking r by x in this equation, we see that

$$D(x,t)\alpha([y,x]) = 0, \ \textit{for all } x,y,t \in I. \tag{1}$$

Replacing y by yz in the last equation and using this, we obtain that

$$D(x,t)\alpha(y)\alpha([z,x]) = 0, \ \textit{for all } x,y,z,t \in I.$$

This implies that

$$D(x,t)V\alpha([z,x]) = (0), \ \textit{for all } x,z,t \in I.$$

where $\alpha(I) = V$ is a nonzero ideal of R. By Lemma 2.3, we have

$$D(x,t) = 0 \ \ \textit{or} \ \ [z,x] = 0, \ \textit{for all } x,t \in I. \tag{2}$$

Let $K = \{x \in I \mid D(x,t) = 0, \textit{for all } t \in I\}$ and $L = \{x \in I \mid [z,x] = 0\}$ of additive subgroups of I. Morever, I is the set-theoretic union of K and L. But a group can not be the set-theoretic union of two proper subgroups, hence $K = I$ or $L = I$. In the former case, we get $D = 0$ by Lemma 2.4, a contradiction. In the latter case, $[I,I] = (0)$. By Lemma 2.2, R is commutative ring. This completes the proof.

Lemma 2.17 *Let R be a prime ring, I a nonzero ideal of R and D a nonzero symmetric bi-(α, α)-derivation of R. If $D(I, I) \subseteq Z$, then R is commutative ring.*

Proof 2.18 *By the hypothesis, we have*

$$[D(x, y), r] = 0, \textit{for all } x, y \in I, r \in R.$$

Taking $xt, t \in I$ instead of x in this equation and using this, we find that

$$D(x, y)[\alpha(t), r] + [\alpha(x), r]D(t, y) = 0, \textit{for all } x, y, t \in I, r \in R.$$

Replacing r by $\alpha(x)$ in this equation, we get

$$D(x, y)[\alpha(t), \alpha(x)] = 0, \textit{for all } x, y, t \in I.$$

Taking $st, s \in I$ instead of t and using this, we have

$$D(x, y)\alpha(s)[\alpha(t), \alpha(x)] = 0, \textit{for all } x, y, t, s \in I.$$

We obtain that

$$D(x, y)V[\alpha(t), \alpha(x)] = (0), \textit{for all } x, y, t \in I,$$

where $\alpha(I) = V$ is ideal of R. By Lemma 2.3, we obtain that

$$D(x, y) = 0 \ \textit{or} \ [t, x] = 0, \textit{for all } x, y, t \in I.$$

Using the same arguments after (2) in the proof of Lemma 2.9, we get the required results.

Theorem 2.19 *Let R be a prime ring, I a nonzero ideal of R and D a nonzero symmetric bi-(α, α)-derivation of R. If $D([x, y], t) = 0$ for all $x, y, t \in I$, then R is commutative ring.*

Proof 2.20 *By the hypothesis, we get*

$$D([x, y], t) = 0, \ \textit{for all } x, y, t \in I.$$

Replacing x by xy in the last expression and using this, we obtain that

$$\alpha([x, y])D(y, t) = 0, \ \textit{for all } x, y, t \in I.$$

Taking xk, $k \in I$ instead of x in this equation and using this, we see that

$$\alpha([x, y])\alpha(k)D(y, t) = 0, \ \textit{for all } x, y, t, k \in I$$

and so

$$\alpha([x, y])VD(y, t) = (0), \ \textit{for all } x, y, t \in I$$

where $\alpha(I) = V$ is a nonzero ideal of R. By Lemma 2.3, we have

$$D(y, t) = 0 \quad \textit{or} \quad [x, y] = 0, \ \textit{for all } x, t \in I.$$

Using the same arguments after (2) in the proof of Lemma 2.9, we get the required results.

Theorem 2.21 *Let* R *be a prime ring,* I *a nonzero ideal of* R *and* D *a nonzero symmetric bi-*(α,α)*-derivation of* R*. If* $D([x,y],t)\in Z$ *for all* $x,y,t\in I$ *and* $D(Z,I)\neq 0$*, then* R *is commutative ring.*

Proof 2.22 *Since* $D(Z,I)\neq 0$ *by the hypothesis, there exists* $z\in Z$*,* $t\in I$ *such that* $D(z,t)\neq 0$*. By Lemma 2.8, we get* $D(z,t)\in Z$*. Replacing* x *by* zx *in the hypothesis, we have*

$$\begin{aligned}D([zx,y],t)&\in Z\\ D([zx,y],t)&=D([z,y]x+z[x,y],t)\\ &=D(z[x,y],t)\in Z\end{aligned}$$

and so

$$D(z,t)\alpha([x,y])+\alpha(z)D([x,y],t)\in Z,\ \textit{for all } x,y,t\in I.$$

Using the hypothesis and $\alpha(z)\in Z$*, we obtain that*

$$D(z,t)\alpha([x,y])\in Z,\ \textit{for all } x,y,t\in I. \tag{3}$$

Since $0\neq D(z,t)\in Z$ *and* R *is prime ring, we have*

$$\alpha([x,y])\in Z$$

and so

$$[I,I]\subseteq Z.$$

By Lemma 2.5, we obtain that R *is commutative ring. This completes the proof.*

Theorem 2.23 *Let* R *be a prime ring,* I *a nonzero ideal of* R *and* D *a nonzero symmetric bi-*(α,α)*-derivation of* R*. If* $D([x,y],t)\in C_{\alpha,\alpha}$ *for all* $x,y,t\in I$ *and* $D(Z,I)\neq 0$*, then* R *is commutative ring.*

Proof 2.24 *Since* $D(Z,I)\neq 0$ *by the hypothesis, there exists* $z\in Z$*,* $t\in I$ *such that* $D(z,t)\neq 0$*. By Lemma 2.8, we get* $D(z,t)\in Z$ *and* $D(z,t)\in C_{\alpha,\alpha}$*. Replacing* x *by* zx *in the hypothesis, we have*

$$\begin{aligned}D([zx,y],t)&\in C_{\alpha,\alpha}\\ D([zx,y],t)&=D([z,y]x+z[x,y],t)\\ &=D(z[x,y],t)\in C_{\alpha,\alpha}\end{aligned}$$

and so

$$D(z,t)\alpha([x,y])+\alpha(z)D([x,y],t)\in C_{\alpha,\alpha},\ \textit{for all } x,y,t\in I.$$

Commuting this term with $r \in R$, *we obtain that*

$$[D(z,t)\alpha([x,y]) + \alpha(z)D([x,y],t), r]_{\alpha,\alpha} = 0.$$

Expanding this equation and using the hypothesis, $\alpha(z) \in Z$, $D(z,t) \in C_{\alpha,\alpha}$, *we arrive at*

$$D(z,t)\alpha([[x,y],r]) = 0, \text{ for all } x,y,t \in I, r \in R. \tag{4}$$

Since $0 \neq D(z,t) \in Z$ *and* R *is prime ring, we have*

$$\alpha([x,y]) \in Z$$

and so

$$[I,I] \subseteq Z.$$

By Lemma 2.5, we find that R *is commutative ring. This completes the proof.*

Theorem 2.25 *Let* R *be a prime ring,* I *a nonzero ideal of* R *and* D *a nonzero symmetric bi-*(α,α)*-derivation of* R. *If* $D(xoy,t) = 0$ *for all* $x,y,t \in I$, *then* R *is commutative ring.*

Proof 2.26 *By the hypothesis, we get*

$$D(xoy,t) = 0, \text{ for all } x,y,t \in I.$$

Replacing x *by* xy *in this equation and using this, we obtain that*

$$\begin{aligned} 0 &= D(xyoy,t) = D((xoy)y,t) \\ &= D(xoy,t)\alpha(y) + \alpha(xoy)D(y,t) \end{aligned}$$

and so

$$\alpha(xoy)D(y,t) = 0, \text{ for all } x,y,t \in I.$$

Taking xz, $z \in I$ *instead of* x *in this equation and using this, we see that*

$$\alpha([x,y])\alpha(z)D(y,t) = 0, \text{ for all } x,y,t,z \in I$$

and so

$$\alpha([x,y])VD(y,t) = (0), \text{ for all } x,y,t,z \in I$$

where $\alpha(I) = V$ *is a nonzero ideal of* R. *By Lemma 2.3, we have*

$$D(y,t) = 0 \text{ or } [x,y] = 0, \text{ for all } x,t \in I.$$

Using the same arguments after (2) in the proof of Lemma 2.9, we get the required results.

Theorem 2.27 *Let R be a prime ring, I a nonzero ideal of R and D a nonzero symmetric bi-(α,α)-derivation of R. If $D(xoy,t)\in Z$ for all $x,y,t\in I$ and $D(Z,I)\neq 0$, then R is commutative ring.*

Proof 2.28 *Since $D(Z,I)\neq 0$ by the hypothesis, there exists $z\in Z, t\in I$ such that $D(z,t)\neq 0$. By Lemma 2.8, we get $D(z,t)\in Z$. Replacing x by zx in the hypothesis, we have*

$$\begin{aligned} D(zxoy,t)&\in Z\\ D(zxoy),t)&=D(z(xoy)-[z,y]x,t)\\ &=D(z(xoy),t)\in Z \end{aligned}$$

and so

$$D(z,t)\alpha(xoy)+\alpha(z)D(xoy,t)\in Z,\ \textit{for all } x,y,t\in I.$$

Using the hypothesis and $\alpha(z)\in Z$, we obtain that

$$D(z,t)\alpha(xoy)\in Z,\ \textit{for all } x,y,t\in I. \tag{5}$$

Since $0\neq D(z,t)\in Z$ and R is prime ring, we have

$$\alpha(xoy)\in Z$$

and so,

$$IoI\subseteq Z.$$

By Lemma 2.6, we obtain that R is commutative ring. This completes the proof.

Theorem 2.29 *Let R be a prime ring, I a nonzero ideal of R and D a nonzero symmetric bi-(α,α)-derivation of R. If $D(xoy,t)\in C_{\alpha,\alpha}$ for all $x,y,t\in I$ and $D(Z,I)\neq 0$, then R is commutative ring.*

Proof 2.30 *Using the same methods in the beginning of Theorem 2.5, we have*

$$D(z,t)\alpha(xoy)+\alpha(z)D(xoy,t)\in C_{\alpha,\alpha},\ \textit{for all } x,y,t\in I.$$

Commuting this term with $r\in R$, we obtain that

$$[D(z,t)\alpha(xoy)+\alpha(z)D(xoy,t),r]_{\alpha,\alpha}=0,\ \textit{for all } x,y,t\in I.$$

Expanding this equation and using the hypothesis, $\alpha(z)\in Z$, $D(z,t)\in C_{\alpha,\alpha}$, we arrive at

$$D(z,t)\alpha([xoy,r])=0,\ \textit{for all } x,y,t\in I, r\in R.$$

Since $0 \neq D(z,t) \in Z$ *and* R *is prime ring, we have*

$$\alpha([xoy,r]) = 0, \text{ for all } x,y \in I, r \in R$$

and so,

$$IoI \subseteq Z.$$

By Lemma 2.6, we obtain that R *is commutative ring. This completes the proof.*

Theorem 2.31 *Let* R *be a prime ring,* I *a nonzero ideal of* R *and* D *a nonzero symmetric bi-*(α,α)*-derivation of* R*. If* $D([x,y],t) - \alpha([x,y]) \in Z$ *for all* $x,y,t \in I$ *and* $D(Z,I) \neq 0$*, then* R *is commutative ring.*

Proof 2.32 *Since* $D(Z,I) \neq 0$ *by the hypothesis, there exists* $z \in Z, t \in I$ *such that* $D(z,t) \neq 0$*. By Lemma 2.8, we get* $D(z,t) \in Z$*. Replacing* x *by* zx *in the hypothesis, we have*

$$D([zx,y],t) - \alpha([zx,y]) \in Z$$

and so

$$D(z[x,y],t) - \alpha(z[x,y]) \in Z.$$

That is

$$D(z,t)\alpha([x,y]) + \alpha(z)(D([x,y],t) - \alpha([x,y])) \in Z.$$

Commuting this term with $r \in R$*, we obtain that*

$$[D(z,t)\alpha([x,y]) + \alpha(z)(D([x,y],t) - \alpha([x,y])), r] = 0, \text{ for all } x,y,t \in I.$$

Expanding this equation and using the hypothesis, $\alpha(z) \in Z$*,* $D(z,t) \in C_{\alpha,\alpha}$*, we find that*

$$D(z,t)\alpha([[x,y],r]) = 0, \text{ for all } x,y,t \in I, r \in R. \tag{6}$$

Applying the same methods after the equation (4) in the proof of Theorem 2.3, we get the required results.

Theorem 2.33 *Let* R *be a prime ring,* I *a nonzero ideal of* R *and* D *a nonzero symmetric bi-*(α,α)*-derivation of* R *If* $D([x,y],t) - \alpha([x,y]) \in C_{\alpha,\alpha}$ *for all* $x,y,t \in I$ *and* $D(Z,I) \neq 0$*, then* R *is commutative ring.*

Proof 2.34 *Applying the same arguments in the beginning of Theorem 2.7, we get*

$$D(z,t)\alpha([x,y]) + \alpha(z)(D([x,y],t) - \alpha([x,y])) \in C_{\alpha,\alpha}.$$

Commuting this term with $r \in R$ *and expanding this equation with the hypothesis,* $\alpha(z) \in Z$*,* $D(z,t) \in C_{\alpha,\alpha}$*, we find that*

$$D(z,t)\alpha([[x,y],r]) = 0, \text{ for all } x,y,t \in I.$$

Arguing the same methods after (4) in the proof of Theorem 2.3, we get the required results.

Theorem 2.35 *Let R be a prime ring, I a nonzero ideal of R and D a nonzero symmetric bi-(α,α)-derivation of R If $D(x,t)o\alpha(y)-\alpha(xoy)\in Z$ for all $x,y,t\in I$ and $D(Z,I)\neq 0$, then R is commutative ring.*

Proof 2.36 *Since $D(Z,I)\neq 0$ by the hypothesis, there exists $z\in Z,t\in I$ such that $D(z,t)\neq 0$. By Lemma 2.8, we get $D(z,t)\in Z$. Replacing x by zx in the hypothesis, we have*

$$D(zx,t)o\alpha(y)-\alpha(zxoy)\in Z$$
$$(D(z,t)\alpha(x)+\alpha(z)D(x,t))o\alpha(y)-\alpha(z(xoy))\in Z.$$

That is

$$D(z,t)(\alpha(x)o\alpha(y))-[D(z,t),\alpha(y)]\alpha(x)+\alpha(z)(D(x,t)o\alpha(y))-[\alpha(z),\alpha(y)]D(x,t)-\alpha(z(xoy))\in Z.$$

By the hypothesis and $\alpha(z),D(z,t)\in Z$, we get

$$D(z,t)\alpha(xoy)\in Z.$$

Using the same arguments after (5) in the proof of Theorem 2.5, we get the required results.

Theorem 2.37 *Let R be a prime ring, I a nonzero ideal of R and D a nonzero symmetric bi-(α,α)-derivation of R If $D(x,t)o\alpha(y)-\alpha(xoy)\in C_{\alpha,\alpha}$ for all $x,y,t\in I$ and $D(Z,I)\neq 0$, then R is commutative ring.*

Proof 2.38 *Using the same methods in the begining of Theorem 2.9, we have*

$$D(z,t)(\alpha(x)o\alpha(y)-[D(z,t),\alpha(y)]\alpha(x)+\alpha(z)(D(x,t)o\alpha(y))-[\alpha(z),\alpha(y)]D(x,t)-\alpha(z(xoy))\in C_{\alpha,\alpha}$$

That is

$$D(z,t)\alpha(xoy)+\alpha(z)(D(x,t)o\alpha(y))-\alpha(xoy)\in C_{\alpha,\alpha}.$$

Commuting this term with $r\in R$, we obtain that

$$[D(z,t)\alpha(xoy)+\alpha(z)(D(x,t)o\alpha(y))-\alpha(xoy)),r]_{\alpha,\alpha}=0,\ \textit{for all } x,y,t\in I.$$

Expanding this equation and using the hypothesis, $\alpha(z)\in Z$, $D(z,t)\in C_{\alpha,\alpha}$, we arrive at

$$D(z,t)\alpha([xoy,r])=0,\ \textit{for all } x,y,t\in I,r\in R.$$

Since $0\neq D(z,t)\in Z$ and R is prime ring, we have

$$\alpha([xoy,r])=0,\ \textit{for all } x,y\in I,r\in R$$

and so

$$IoI\subseteq Z.$$

By Lemma 2.6, we obtain that R is commutative ring. This completes the proof.

Theorem 2.39 *Let R be a prime ring, I a nonzero ideal of R and D a nonzero symmetric bi-(α,α)-derivation of R If $[x, D(y,t)] \in Z$ for all $x, y, t \in I$ and $D(Z, I) \neq 0$, then R is commutative ring.*

Proof 2.40 *Since $D(Z, I) \neq 0$ by the hypothesis, there exists $z \in Z, t \in I$ such that $D(z,t) \neq 0$. By Lemma 2.8, we get $D(z,t) \in Z$. By the hypothesis, we get*

$$[x, D(y,t)] \in Z, \text{ for all } x, y, t \in I.$$

Replacing y by zy in the hypothesis, we have

$$[x, D(zy,t)] \in Z$$

and so

$$[x, D(z,t)]\alpha(y) + D(z,t)[x,\alpha(y)] + [x,\alpha(z)]D(y,t) + \alpha(z)[x, D(y,t)] \in Z.$$

By the hypothesis and $\alpha(z), D(z,t) \in Z$, we get

$$D(z,t)[x,\alpha(y)] \in Z.$$

Since $0 \neq D(z,t) \in Z$ and R is prime ring, we have

$$[x,\alpha(y)] \in Z, \text{ for all } x, y \in I.$$

Now, writing x by $xt, t \in I$ in this equation, we see that

$$[x,\alpha(y)]t + x[t,\alpha(y)] \in Z, \text{ for all } x, y, t \in I.$$

Commuting this term with $t \in I$, we obtain that

$$[x,t][t,\alpha(y)] = 0, \text{ for all } x, y, t \in I.$$

Replacing x by $xs, s \in I$ in this equation, we get

$$[x,t]s[t,\alpha(y)] = 0$$

and so

$$[x,t]I[t,\alpha(y)] = 0, \text{ for all } x, y, t \in I.$$

By the primeness of R gives that

$$[x,t] = 0 \text{ or } [t,\alpha(y)] = 0, \text{ for all } x, y, t \in I.$$

Let $K = \{t \in I \mid [x,t] = 0, \text{ for all } x \in I\}$ and $L = \{t \in I \mid [t,\alpha(y)] = 0, \text{ for all } y \in I\}$ of additive subgroups of I. Morever, I is the set-theoretic union of K and L. But a group can not be the set-theoretic union of two proper subgroups, hence $K = I$ or $L = I$. In the former case, we get R is commutative ring by

Lemma 2.2. In the latter case, we have $[t,\alpha(y)]=0$, *for all* $y\in I$. *Writing* t *by* $tr, r\in R$, *we get*

$$t[r,\alpha(y)]=0,\ \textit{for all}\ t,y\in I, r\in R$$

and so, $[r,\alpha(y)]=0$. *Hence we find that* $\alpha(I)\subseteq Z$, *and so* R *is commutative ring by Lemma 2.2.*

Theorem 2.41 *Let* R *be a* $2-$*torsion free prime ring,* I *a nonzero ideal of* R *and* D *a nonzero symmetric bi-*(α,α)*-derivation of* R. *If* $[x,D(y,t)]\in C_{\alpha,\alpha}$ *for all* $x,y,t\in I$ *and* $D(Z,I)\neq 0$, *then* R *is commutative ring.*

Proof 2.42 *Using the same arguments in the begining of Theorem 2.11, we get*

$$D(z,t)[x,\alpha(y)]+\alpha(z)[x,D(y,t)]\in C_{\alpha,\alpha},\ \textit{for all}\ x,y,t\in I.$$

Commuting this term with $r\in R$, *we obtain that*

$$[D(z,t)[x,\alpha(y)]+\alpha(z)[x,D(y,t)],r]_{\alpha,\alpha}=0,\ \textit{for all}\ x,y,t\in I.$$

Expanding this equation and using the hypothesis, $\alpha(z)\in Z$, $D(z,t)\in C_{\alpha,\alpha}$, *we find that*

$$D(z,t)[[x,\alpha(y)],\alpha(r)]=0,\ \textit{for all}\ x,y,t\in I, r\in R.$$

Since $0\neq D(z,t)\in Z, \alpha$ *is an automorphism and* R *is prime ring, we have*

$$[[x,\alpha(y)],r]=0,\ \textit{for all}\ x,y,t\in I, r\in R.$$

Hence we get $\delta_x\delta_{\alpha(y)}(R)=(0)$ *where* $\delta_x(r)=[x,r], \delta_{\alpha(y)}(r)=[\alpha(y),r]$ *are inner derivations. It reduces to* $\delta_x=0$ *or* $\delta_{\alpha(y)}=0$ *by Lemma 2.1. If* $\delta_x=0$, *then* $[x,r]=0$, *for all* $x,\in I, r\in R$. *That is,* $I\subset Z$. *We have* R *is commutative ring by Lemma 2.2. If* $\delta_{\alpha(y)}=0$, *then* R *is commutative ring by Lemma 2.2. The proof is completed.*

Theorem 2.43 *Let* R *be a prime ring,* I *a nonzero ideal of* R *and* D *a nonzero symmetric bi-*(α,α)*-derivation of* R. *If* $xoD(y,t)\in Z$ *for all* $x,y,t\in I$ *and* $D(Z,I)\neq 0$, *then* R *is commutative ring.*

Proof 2.44 *Since* $D(Z,I)\neq 0$ *by the hypothesis, there exists* $z\in Z, t\in I$ *such that* $D(z,t)\neq 0$. *By Lemma 2.8, we get* $D(z,t)\in Z$. *By the hypothesis, we get*

$$xoD(y,t)\in Z,\ \textit{for all}\ x,y,t\in I.$$

Replacing y *by* zy *in the hypothesis, we have*

$$xoD(zy,t)\in Z$$

and so

$$D(z,t)(xo\alpha(y))+[x,D(z,t)]\alpha(y)+[x,\alpha(z)]D(y,t)+\alpha(z)(xoD(y,t))\in Z.$$

By the hypothesis and $\alpha(z), D(z,t)\in Z$, *we get*

$$D(z,t)(xo\alpha(y))\in Z,\ \textit{for all } x,y,t\in I.$$

Since $0\neq D(z,t)\in Z$ *and* R *is prime ring, we have*

$$xo\alpha(y)\in Z,\ \textit{for all } x,y,t\in I.$$

Hence we get R *is commutative ring by Lemma 2.7.*

Theorem 2.45 *Let* R *be a prime ring,* I *a nonzero ideal of* R *and* D *a nonzero symmetric bi-*(α,α)*-derivation of* R*. If* $xoD(y,t)\in C_{\alpha,\alpha}$ *for all* $x,y,t\in I$ *and* $D(Z,I)\neq 0$, *then* R *is commutative ring.*

Proof 2.46 *Applying the same arguments in the beginning of Theorem 2.13, we have*

$$D(z,t)(xo\alpha(y))+\alpha(z)(xoD(y,t)\in C_{\alpha,\alpha}.$$

Commuting this term with $r\in R$ *and using the hypothesis,* $\alpha(z)\in Z$, $D(z,t)\in C_{\alpha,\alpha}$, *we arrive at*

$$D(z,t)[xo\alpha(y),\alpha(r)]=0,\ \textit{for all } x,y,t\in I,r\in R.$$

Since $0\neq D(z,t)\in Z,\alpha$ *is an automorphism and* R *is prime ring, we have*

$$[xo\alpha(y),r]=0,\ \textit{for all } x,y\in I,r\in R.$$

That is $xo\alpha(y)\in Z$, *and so* R *is commutative ring by Lemma 2.7.*

Theorem 2.47 *Let* R *be a prime ring,* I *a nonzero ideal of* R *and* D *a nonzero symmetric bi-*(α,α)*-derivation of* R*. If* $[D(x,t),D(y,t)]-\alpha([x,y])\in Z$ *for all* $x,y,t\in I$ *and* $D(Z,I)\neq 0$, *then* R *is commutative ring.*

Proof 2.48 *Since* $D(Z,I)\neq 0$ *by the hypothesis, there exists* $z\in Z,t\in I$ *such that* $D(z,t)\neq 0$. *By Lemma 2.8, we get* $D(z,t)\in Z$. *Replacing* x *by* zx *in the hypothesis, we have*

$$[D(zx,t),D(y,t)]-\alpha([zx,y]))\in Z$$

and so

$$D(z,t)[\alpha(x),D(y,t)]+\alpha(z)[D(x,t),D(y,t)]-\alpha(z)\alpha([x,y])\in Z.$$

By the hypothesis and $\alpha(z) \in Z$*, we get*

$$D(z,t)[\alpha(x), D(y,t)] \in Z, \text{ for all } x, y, t \in I.$$

Since $0 \neq D(z,t) \in Z$ *and* R *is prime ring, we have*

$$[\alpha(x), D(y,t)] \in Z, \text{ for all } x, y, t \in I. \tag{7}$$

That is

$$[V, D(y,t)] \subseteq Z$$

where $\alpha(I) = V$ *is a nonzero ideal of* R*. Hence we get*

$$[xD(y,t), D(y,t)] \in Z, \text{ for all } x \in V, y, t \in I$$

and so

$$[x, D(y,t)]D(y,t) \in Z.$$

Since $[x, D(y,t)] \in Z$ *and* R *is prime ring, we have*

$$[x, D(y,t)] = 0 \text{ or } D(y,t) \in Z$$

and so,

$$[x, D(y,t)] = 0, \text{ for all } x, y, t \in I$$

and so

$$D(y,t) \in Z, \text{ for all } y, t \in I.$$

Hence we arrive at R *is commutative by Lemma 2.10.*

Theorem 2.49 *Let* R *be a prime ring,* I *a nonzero ideal of* R *and* D *a nonzero symmetric bi-*(α,α)*-derivation of* R*. If* $[D(x,t), D(y,t)] - \alpha([x,y]) \in C_{\alpha,\alpha}$ *for all* $x, y, t \in I$ *and* $D(Z, I) \neq 0$*, then* R *is commutative ring.*

Proof 2.50 *Applying the same arguments in the beginning of Theorem 2.15, we get*

$$D(z,t)[\alpha(x), D(y,t)] + \alpha(z)([D(x,t), D(y,t)] - \alpha([x,y])) \in C_{\alpha,\alpha}, \text{ for all } x, y, t \in I.$$

Commuting this term with $r \in R$*, we obtain that*

$$[D(z,t)[\alpha(x), D(y,t)] + \alpha(z)([D(x,t), D(y,t)] - \alpha([x,y])), r]_{\alpha,\alpha} = 0, \text{ for all } x, y, t \in I.$$

Expanding this equation and using the hypothesis, $\alpha(z) \in Z$*,* $D(z,t) \in C_{\alpha,\alpha}$*, we have*

$$D(z,t)[[\alpha(x), D(y,t)], \alpha(r)] = 0, \text{ for all } x, y, t \in I, r \in R.$$

Since $0 \neq D(z,t) \in Z, \alpha$ *is an automorphism and* R *is prime ring, we have*

$$[[\alpha(x), D(y,t)], r] = 0$$

and so

$$[\alpha(x), D(y,t)] \in Z, \text{ for all } x, y, t \in I.$$

The same arguing after the equation (7) of Theorem 2.15, we get the required result.

Theorem 2.51 *Let* R *be a prime ring,* I *a nonzero ideal of* R *and* D *a nonzero symmetric bi-*(α,α)*-derivation of* R. *If* $D(x,t)oD(y,t) - \alpha([x,y]) \in Z$ *for all* $x, y, t \in I$ *and* $D(Z, I) \neq 0$, *then* R *is commutative ring.*

Proof 2.52 *Since* $D(Z, I) \neq 0$ *by the hypothesis, there exists* $z \in Z, t \in I$ *such that* $D(z,t) \neq 0$. *By Lemma 2.8, we get* $D(z,t) \in Z$. *Replacing* x *by* zx *in the hypothesis, we have*

$$(D(zx,t)oD(y,t)) - \alpha([zx,y]) \in Z) \in Z$$

and so

$$D(z,t)(\alpha(x)oD(y,t)) - [D(z,t), D(y,t)]\alpha(x)$$
$$+\alpha(z)(D(x,t)oD(y,t)) - [\alpha(z), D(y,t)]D(x,t) - \alpha(z)\alpha([x,y]) \in Z.$$

By the hypothesis and $\alpha(z), D(z,t) \in Z$, *we get*

$$D(z,t)(\alpha(x)oD(y,t)) \in Z, \text{ for all } x, y, t \in I.$$

Since $0 \neq D(z,t) \in Z$ *and* R *is prime ring, we have*

$$\alpha(x)oD(y,t) \in Z, \text{ for all } x, y, t \in I. \tag{8}$$

That is

$$VoD(y,t) \subseteq Z$$

where $\alpha(I) = V$ *is a nonzero ideal of* R. *Hence we obtain that* R *is commutative by Theorem 2.13.*

Theorem 2.53 *Let* R *be a prime ring,* I *a nonzero ideal of* R *and* D *a nonzero symmetric bi-*(α,α)*-derivation of* R. *If* $D(x,t)oD(y,t) - \alpha([x,y]) \in C_{\alpha,\alpha}$ *for all* $x, y, t \in I$ *and* $D(Z, I) \neq 0$, *then* R *is commutative ring.*

Proof 2.54 *Using the same arguments in the beginning of Theorem 2.17, we get*

$$D(z,t)(\alpha(x)oD(y,t)) + \alpha(z)((D(x,t)oD(y,t) - \alpha([x,y])) \in C_{\alpha,\alpha}, \text{ for all } x, y, t \in I.$$

Commuting this term with $r \in R$*, we obtain that*

$$[D(z,t)(\alpha(x)oD(y,t)) + \alpha(z)((D(x,t)oD(y,t) - \alpha([x,y])), r]_{\alpha,\alpha} = 0, \text{ for all } x, y, t \in I.$$

Expanding this equation and using the hypothesis, $\alpha(z) \in Z$*,* $D(z,t) \in C_{\alpha,\alpha}$*, we arrive at*

$$D(z,t)[\alpha(x)oD(y,t), \alpha(r)] = 0, \text{ for all } x, y, t \in I, r \in R.$$

Since $0 \neq D(z,t) \in Z, \alpha$ *is an automorphism and* R *is prime ring, we have*

$$[\alpha(x)oD(y,t), r] = 0$$

and so

$$\alpha(x)oD(y,t) \in Z, \text{ for all } x, y, t \in I, r \in R$$

The same arguing after the equation (8) in the proof of Theorem 2.17, we get the required result.

Theorem 2.55 *Let* R *be a prime ring,* I *a nonzero ideal of* R *and* D *a nonzero symmetric bi-*(α,α)*-derivation of* R*. If* $D(x,t)oD(y,t) - \alpha(xoy) \in Z$ *for all* $x, y, t \in I$ *and* $D(Z,I) \neq 0$*, then* R *is commutative ring.*

Proof 2.56 *Since* $D(Z,I) \neq 0$ *by the hypothesis, there exists* $z \in Z, t \in I$ *such that* $D(z,t) \neq 0$*. By Lemma 2.8, we get* $D(z,t) \in Z$*. Replacing* x *by* zx *in the hypothesis, we have*

$$D(zx,t)oD(y,t) - \alpha(zxoy) \in Z$$

and so

$$D(z,t)(\alpha(x)oD(y,t)) - [D(z,t), D(y,t)]\alpha(x)$$
$$+\alpha(z)(D(x,t)oD(y,t)) - [\alpha(z), D(y,t)]D(x,t) - \alpha(z)\alpha(xoy) \in Z.$$

By the hypothesis and $\alpha(z), D(z,t) \in Z$*, we get*

$$D(z,t)(\alpha(x)oD(y,t)) \in Z, \text{ for all } x, y, t \in I.$$

Again using $0 \neq D(z,t) \in Z$ *and* R *is prime ring, we have*

$$\alpha(x)oD(y,t) \in Z, \text{ for all } x, y, t \in I.$$

Applying the same arguments after (8) in the proof of Theorem 2.17, we find that the required result.

Theorem 2.57 *Let* R *be a prime ring,* I *a nonzero ideal of* R *and* D *a nonzero symmetric bi-*(α,α)*-derivation of* R*. If* $D(x,t)oD(y,t) - \alpha(xoy) \in C_{\alpha,\alpha}$ *for all* $x, y, t \in I$ *and* $D(Z,I) \neq 0$*, then* R *is commutative ring.*

Proof 2.58 *Using the same arguments in the beginning of Theorem 2.19, we get*

$$D(z,t)(\alpha(x)oD(y,t))+\alpha(z)(D(x,t)oD(y,t)-\alpha(xoy))\in C_{\alpha,\alpha}.$$

Commuting this term with $r\in R$ and applying the same lines, we obtain that

$$D(z,t)[\alpha(x)oD(y,t),r]=0, \text{ for all } x,y,t\in I.$$

Since $0\neq D(z,t)\in Z$ and R is prime ring, we have

$$\alpha(x)oD(y,t)\in Z, \text{ for all } x,y,t\in I.$$

By the same arguing after (8) in the proof of Theorem 2.17 gives the result.

Theorem 2.59 *Let R be a prime ring, I a nonzero ideal of R and D a nonzero symmetric bi-(α,α)-derivation of R. If $[D(x,t),D(y,t)]-\alpha(xoy)\in Z$ for all $x,y,t\in I$ and $D(Z,I)\neq 0$, then R is commutative ring.*

Proof 2.60 *Since $D(Z,I)\neq 0$ by the hypothesis, there exists $z\in Z, t\in I$ such that $D(z,t)\neq 0$. By Lemma 2.8, we get $D(z,t)\in Z$. Replacing x by zx in the hypothesis, we have*

$$[D(zx,t),D(y,t)]-\alpha(zxoy)\in Z$$

and so

$$D(z,t)[\alpha(x),D(y,t)]+\alpha(z)[D(x,t),D(y,t)]-\alpha(z)\alpha(xoy)\in Z.$$

By the hypothesis and $\alpha(z)\in Z$, we get

$$D(z,t)[\alpha(x),D(y,t)]\in Z, \text{ for all } x,y,t\in I.$$

Since $0\neq D(z,t)\in Z$ and R is prime ring, we have

$$[\alpha(x),D(y,t)]=0, \text{ for all } x,y,t\in I.$$

By the same methods after (7) in the proof of Theorem 2.15, we get the required result.

Theorem 2.61 *Let R be a prime ring, I a nonzero ideal of R and D a nonzero symmetric bi-(α,α)-derivation of R. If $[D(x,t),D(y,t)]-\alpha(xoy)\in C_{\alpha,\alpha}$ for all $x,y,t\in I$ and $D(Z,I)\neq 0$, then R is commutative ring.*

Proof 2.62 *Applying the same arguments in the Theorem 2.21, we get*

$$D(z,t)[\alpha(x),D(y,t)]+\alpha(z)[D(x,t),D(y,t)]-\alpha(z)\alpha(xoy)\in C_{\alpha,\alpha} \text{ for all } x,y,t\in I.$$

Commuting this term with $r\in R$ *and appliying the same lines of Theorem 2.15 we obtain that*

$$D(z,t)[[\alpha(x),D(y,t)],\alpha(r)]=0, \text{ for all } x,y,t\in I, r\in R.$$

Since $0\neq D(z,t)\in Z$ *and* R *is prime ring, we have*

$$[[\alpha(x),D(y,t)],\alpha(r)]=0, \text{ for all } x,y,t\in I, r\in R.$$

and so

$$[\alpha(x),D(y,t)]\in Z, \text{ for all } x,y,t\in I.$$

The same arguing after the equation (7) of Theorem 2.15, we get the required result.

3 Open Problem

Symmetric bi-derivation has been introduced by Maksa in [6] and Vukman [9] . Several authors have proved commutativity theorems for prime rings admitting symmetric bi-derivations (see in [1], [8], [2], [4] and [10]). In this paper, we introduced the notion of symmetric bi-(α,α)-derivation and proved the results in [4] concerning of ideals in prime rings with bi-(α,α)-derivation. Also, we gave the true proof in [4, Theorem 2.4]. In future research, some well-known results in symmetric bi-(α,α)-derivation can be applied to and generalized symmetric bi-(α,β)-derivation. Besides, the findings herein could help to uncover properties of symmetric bi-(α,β)-derivation in Lie ideals or square-closed Lie ideals.

References

[1] A. Z. Ansari, F. Shujat, G. Alhendi: Ideals, centralizers and symmetric bi-derivations on prime rings. International Journal of Math. and Computer Science, Vol 15, (2019), 549-557.

[2] B. Dhara, F. Shujat: Symmetric Skew n-derivations in prime and semiprime rings. Southeast Asian Bull. of Math., Vol 42, (2018), 845-852.

[3] B. Fahid, D. Bennis, A. Mamouni: Dhara-Rehman-Raza's identities on left ideals od prime rings. Rend. Circ. Math. Palermo (2), 70, no:2, (2021), 875-882.

[4] K. H. Kim: On symmetric bi-derivations and commutativity of prime rings. Gulf Journal of Math., Vol 7, Issue 2, (2019), 22-23.

[5] P. H. Lee, T. K. Lee: On derivations of prime rings, Chinese J. Math. Vol.9(2), (1981), 107-110.

[6] Gy. Maksa: A remark on symmetric biadditive functions having non-negative diagonalization. Glasnik. Mat., 15 (35), (1980), 279–282.

[7] Gy. Maksa: On the trace of symmetric biderivations. C. R. Math. Rep. Acad. Sci. Canada, 9, (1987), 303–307.

[8] F. Shujat: Symmetric generazlized biderivations on prime rings. Bol. Soc. Paran. Math., Vol 39, Issue 4, (2021), 65-72.

[9] J. Vukman: Symmetric bi-derivations on prime and semiprime rings. Aequationes Math., 38, (1989), 245–254.

[10] J. Vukman: Two results concerning symmetric biderivations on prime rings. Aequationes Math., 40, (1990), 181–189.

[11] E.C. Posner: Derivations in prime rings. Proc. Amer. Soc., 8, (1957), 1093-1100.